Is the ρ Meson a Dynamical Gauge Boson of Hidden Local Symmetry?

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We suggest that the ρ meson is a dynamical gauge boson of a hidden local symmetry in the nonlinear chiral Lagrangian. The origin of the ρ-meson mass is understood as the Higgs mechanism of the hidden local symmetry. The low-energy dynamics of ρ, π, and matter fields, including the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation and ρ-coupling universality, is consistently described in this new framework. The electromagnetic interaction can be introduced in a unique manner, which gives a successful explanation of ρ dominance of the photon coupling.

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Since the discovery of hidden local symmetries in the supergravity theories,[1] it has become a popular understanding that any nonlinear sigma model based on the manifold G/H is gauge equivalent to another model with $G_{\text{global}} \otimes H_{\text{local}}$ symmetry and the gauge bosons corresponding to the hidden local symmetry, H_{local}, are composite fields.[1-3] Moreover, in some nonlinear sigma models in two-dimensional space-time, kinetic terms of the gauge fields of the hidden local symmetry are generated by quantum effects and poles of the gauge fields are developed accordingly.[2] Although the dynamical situation is not clear in four-dimensional models, it is certainly an intriguing possibility that four-dimensional nonlinear sigma models may also produce the kinetic terms of the hidden gauge fields.

As is well known, the $SU(2)_L \otimes SU(2)_R/SU(2)_V$ nonlinear sigma model is a low-energy effective Lagrangian of the massless two-flavored QCD whose global symmetry $G = SU(2)_L \otimes SU(2)_R$ is expected to be spontaneously broken to the diagonal subgroup $H = SU(2)_V$. If the above scenario of the dynamically generated hidden gauge fields is true, we can expect the phenomenological consequences of it in the low-energy hadron phenomena. In the case at hand, the immediate candidate for the hidden gauge field is the ρ meson.

In this Letter we suggest that the ρ meson is a dynamical gauge boson of a hidden local symmetry in the nonlinear chiral Lagrangian.

In the old days of the current algebra, the ρ meson was in fact treated as the "massive Yang-Mills" field[4-6], coupled to the nonlinear chiral Lagrangian, which achieved a partial success in explaining various couplings and masses among low-lying hadrons, π, ρ mesons and nucleons. Typical examples of such relations are the universality of ρ-meson couplings,[4] ρ dominance of the electromagnetic form factor,[4] and the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[7]

Of course the notion of "massive Yang-Mills" in the original form does not make sense, and nowadays the ρ meson is regarded simply as a bound state of quark and antiquark, so that one might think that the old idea of the ρ meson being a gauge field is at most a useful mnemonic of hadron phenomenology. Having a new concept of the hidden local symmetry, however, we can reformulate on a sound basis the old wisdom[4-6] to explain the low-energy dynamics associated with the ρ and π mesons. Once we assume that the gauge field of the hidden local symmetry develops its kinetic term, the low-energy Lagrangian is determined up to one parameter. The mass of the ρ meson is generated via the Higgs mechanism of the hidden local symmetry and the celebrated KSRF relation follows when the parameter is chosen in such a way that the ρ couples to pions with the same strength as to nucleons. A novel feature of our scheme is that the electromagnetic interaction can be uniquely introduced and ρ-γ mixing occurs in exactly the same manner as in the Glashow-Salam-Weinberg model. Most remarkably this yields precisely the ρ dominance of the photon coupling with the same parameter choice as above, providing us with an evidence of the dynamically generated gauge bosons.

Let us start with a nonlinear sigma model based on the manifold $G/H=\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R/\mathrm{SU}(2)_V$, the extension of which to $\mathrm{SU}(N)_L\otimes\mathrm{SU}(N)_R/\mathrm{SU}(N)_V$ is straightforward. The Lagrangian is given by

$$L=(f_\pi^2/4)\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger), \tag{1}$$

where U is written in terms of the Nambu-Goldstone (NG) pion fields as

$$U(x)\equiv\exp[2i\pi(x)/f_\pi],\quad \pi\equiv\pi^aT^a, \tag{2}$$

T^a being the SU(2) generators normalized as $\mathrm{Tr}(T^aT^b)=\frac{1}{2}\delta^{ab}$ and f_π ($\simeq 93$ MeV) the pion decay constant. $U(x)$ transforms under chiral $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$ as $U(x)\to g_LU(x)g_R^\dagger$, where g_L and g_R are the elements of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, respectively. The Lagrangian (1) may be cast into a form which possesses, besides global $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$, a local $\mathrm{SU}(2)_V$ symmetry—the hidden local symmetry. We introduce new [SU(2)-matrix valued] variables $\xi_L(x)$ and $\xi_R(x)$ and the SU(2) gauge field $V_\mu(x)$ such that

$$U(x)\equiv\xi_L^\dagger(x)\xi_R(x),\quad V_\mu(x)\equiv V_\mu^a(x)T^a. \tag{3}$$

[Here the non-Abelian gauge field $V_\mu(x)$ is introduced as an independent variable, but later will be given by

$$V_\mu=(1/2i)(\partial_\mu\xi_L\cdot\xi_L^\dagger+\partial_\mu\xi_R\cdot\xi_R^\dagger)$$

as a result of the equation of motion through the Lagrangian in (7).] The transformation properties of these variables under $[\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R]_{\mathrm{global}}\otimes[\mathrm{SU}(2)_V]_{\mathrm{local}}$ are

$$\xi_L(x)\to h(x)\xi_L(x)g_L^\dagger,\quad \xi_R(x)\to h(x)\xi_R(x)g_R^\dagger,$$
$$V_\mu(x)\to ih(x)\partial_\mu h(x)^\dagger+h(x)V_\mu(x)h(x)^\dagger, \tag{4}$$

where $h(x)$ is a parameter of the hidden local SU(2) transformation. We have two types of $[\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R]_{\mathrm{global}}\otimes[\mathrm{SU}(2)_V]_{\mathrm{local}}\otimes$ (parity) invariants made out of $\xi_{L,R}(x)$ and $D_\mu\xi_{L,R}(x)\equiv[\partial_\mu-iV_\mu(x)]\xi_{L,R}(x)$;

$$L_V=-\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\cdot\xi_L^\dagger+D_\mu\xi_R\cdot\xi_R^\dagger)^2, \tag{5}$$

$$L_A=-\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu\xi_L\cdot\xi_L^\dagger-D_\mu\xi_R\cdot\xi_R^\dagger)^2. \tag{6}$$

Any linear combination of (5) and (6),

$$L=L_A+aL_V, \tag{7}$$

is equivalent to the original Lagrangian (1). In fact, fixing the $[\mathrm{SU}(2)_V]_{\mathrm{local}}$ gauge by $\xi_L^\dagger=\xi_R\equiv\xi=\exp(-i\pi/f_\pi)$, we have

$$L_A=f_\pi^2\mathrm{Tr}\left\{\frac{1}{2i}(\partial_\mu\xi\cdot\xi^\dagger-\partial_\mu\xi^\dagger\cdot\xi)\right\}^2=\frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger),$$

$$L_V=f_\pi^2\mathrm{Tr}\left\{V_\mu-\frac{1}{2i}(\partial_\mu\xi\cdot\xi^\dagger+\partial_\mu\xi^\dagger\cdot\xi)\right\}^2.$$

L_A is identical to the Lagrangian (1), while L_V vanishes when we substitute the solution of the V_μ equation of motion:

$$V_\mu^a(x)=-i\,\mathrm{Tr}\{T^a(\partial_\mu\xi\cdot\xi^\dagger+\partial_\mu\xi^\dagger\cdot\xi)\}\simeq-(1/2f_\pi^2)(\boldsymbol{\pi}\times\partial\boldsymbol{\pi})^a+\dots. \tag{8}$$

So far we have had no kinetic terms of $V_\mu(x)$. Here we assume that the kinetic terms of $V_\mu(x)$ are generated by the underlying dynamics (QCD) or quantum effects at the composite level. This is our main assumption. Note that addition of the kinetic term does not affect the low-energy limit of the dynamics. Adding the kinetic term $-(1/4g^2)(\mathbf{F}_{\mu\nu})^2$, we have

$$L=L_A+aL_V-(4g^2)^{-1}(\mathbf{F}_{\mu\nu})^2\simeq\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2+\tfrac{1}{2}a\mathbf{V}^\mu\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}+\tfrac{1}{2}(af_\pi^2)\mathbf{V}_\mu^2-(4g^2)^{-1}(\mathbf{F}_{\mu\nu})^2+\dots, \tag{9}$$

where $\mathbf{F}_{\mu\nu}=\partial_\mu\mathbf{V}_\nu-\partial_\nu\mathbf{V}_\mu+\mathbf{V}_\mu\times\mathbf{V}_\nu$. After rescaling $g^{-1}\mathbf{V}_\mu\to\mathbf{V}_\mu$, we obtain

$$g_{\rho\pi\pi}=\tfrac{1}{2}ag, \tag{10}$$

$$m_\rho^2=ag^2f_\pi^2. \tag{11}$$

Equation (11) has the typical form of the Higgs mechanism and indeed the ρ-meson mass is acquired via spontaneous breakdown of the hidden local $\mathrm{SU}(2)_V$. Of course, here the unphysical NG modes absorbed into the ρ mesons are not pions π but scalars ϕ which were introduced in the new variables ξ_L and ξ_R as

$$\xi_{L,R}=\exp(i\phi/f_\pi)\exp(\pm i\pi/f_\pi).$$

The parameter a in Eq. (7) is left completely arbitrary within the symmetry considerations alone. However $a=2$ is a special value, since we then have $g_{\rho\pi\pi}=g$ and $m_\rho^2=2g_{\rho\pi\pi}^2f_\pi^2$, which are nothing but the universality[4,8] and the famous KSRF relation,[7,9] respectively.

It is to be noted that, after gauge fixing, our Lagrangian becomes identical to that of Weinberg's[6] which was proposed on a phenomenological basis without usage of the hidden local symmetry.

Let us now turn to the introduction of the electromagnetic interactions. The electromagnetic field couples to the charge $Q=I_3^{(L)}+I_3^{(R)}+Y/2$, with

$I_3^{(L,R)}$ and Y being the $[SU(2)_L \otimes SU(2)_R]_{\rm global}$ isospins and the hypercharge, respectively. Note that these charges are completely independent of the generators of $[SU(2)_V]_{\rm local}$ to which the ρ mesons couple. Thanks to this clear separation of ρ and photon source charges there are no complications in our framework to introduce electromagnetic interactions, in sharp contrast to the previous attempts.[5,6]

By denoting the electromagnetic $U(1)_Q$ gauge field by B_μ, the covariant derivatives for the fields $\xi_{L,R}$ (having $Y=0$) become

$$D_\mu \xi_{L(R)} = (\partial_\mu - igV_\mu)\xi_{L(R)} + ie_0\xi_{L(R)}B_\mu T^3, \quad (12)$$

with e_0 being a coupling constant of the $U(1)_Q$ gauge interaction. Then the previous Lagrangian (9) with this gauge interaction switched on now reads [with use of Eq. (11)]

$$L \sim \frac{f_\pi^2}{4}{\rm Tr}(\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{4}(\mathbf{F}_{\mu\nu})^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + m_\rho^2 2(\mathbf{V}_\mu)^2 - \frac{e_0 m_\rho^2}{g} V_\mu^3 B^\mu + \frac{1}{2}\left(\frac{e_0}{g}\right)^2 m_\rho^2 (B_\mu)^2 + \frac{a}{2} g\mathbf{V}_\mu \cdot (\boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi}) - \frac{a-2}{2} e_0 B_\mu \epsilon_{3ab}\pi^a \partial^\mu \pi^b + \ldots . \quad (13)$$

When $a=2$, the Lagrangian (13), describing ρ-γ mixing, just coincides with the one discussed in Sakurai's book,[4] and hence leads to the ρ dominance of the electromagnetic form factor of pions. We should emphasize here the following two points: (i) First, the particular form of the Lagrangian (13) which implies the ρ dominance is uniquely derived in our framework with no additional assumptions. (ii) Second, the special value $a=2$ explains simultaneously the three phenomenological facts, (a) the KSRF relation, (b) the universality of ρ coupling, and (c) ρ dominance of the photon coupling to pions.

Diagonalizing the mass matrix of V_μ^3 and B_μ, we obtain

$$m_\gamma^2 = 0, \quad m_{\rho^0}^2 = a(g^2 + e_0^2) f_\pi^2, \quad m_{\rho^\pm}^2 = a g^2 f_\pi^2, \quad (14)$$

where the mass eigenstates are

$$A_\mu = (g^2 + e_0^2)^{-1/2}(gB_\mu + e_0 V_\mu^3), \quad V_\mu^0 = (g^2 + e_0^2)^{-1/2}(gV_\mu^3 - e_0 B_\mu) \quad (15)$$

and the electromagnetic charge e is given by $e = ge_0/(g^2 + e_0^2)^{1/2} \simeq e_0$. We thus have encountered a quite analogous situation to the Glashow-Salam-Weinberg model. ρ-γ mixing in our model is the same as the Higgs mechanism of the $SU(2)_L \otimes U(1)_Y$ gauge symmetry; $[SU(2)_V]_{\rm hidden} \otimes U(1)_Q$ is spontaneously broken to the $U(1)_{\rm em}$, where the resultant conserving charges $Q_{\rm em}$ is given by $Q_{\rm em} = Q + I_3^{(\rm hidden)}$.

In conclusion, we have argued that the ρ meson is a dynamical gauge boson of the hidden $[SU(2)_V]_{\rm local}$ symmetry in the $SU(2)_L \otimes SU(2)_R/SU(2)_V$ nonlinear sigma model. The totality of the ρ-meson phenomenology seems to support our basic idea that the ρ meson is a dynamical gauge boson: (a) KSRF relation; (b) the universality of the ρ coupling to the pion as well as to matter fields, $g_{\rho\pi\pi} = g_{\rho AA} = g$, (c) ρ dominance of the photon coupling to pions; and (d) ρ-exchange dominance in pion-matter scattering. So we believe that the ρ meson is the first example of a dynamically generated gauge boson of hidden local symmetry. We further expect that this type of phenomena may occur rather generally in a wide class of strongly interacting systems not restricted to QCD. In this respect the weak bosons may also be such dynamical gauge bosons. In fact, such a possibility has been studied by three of the present authors within a supersymmetric $U(4n_f+2)/U(4n_f) \otimes SU(2)$ nonlinear sigma model.[10] A more exciting possibility would be a dynamical realization of hidden $SU(8)_{\rm local}$ symmetry in $N=8$ supergravity.

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[8]Precisely speaking, $g_{\rho\pi\pi} = g$ is not sufficient to conclude the ρ-coupling universality. If the ρ coupling to matter ψ_A is saturated only by the minimal term, $\bar\psi_A \gamma^\mu [\delta_{AB}\partial_\mu - igV_\mu^a (T^a)_{AB}]\psi_B$, then the universality $g_{\rho AA} = g$ really holds. But other possible terms like

$$\bar\psi_A \gamma^\mu (T^a)_{AB}\psi_B\, {\rm Tr}T^a (D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger),$$

which might be present with arbitrary strength, would violate this property. Such terms, however, cannot be appreciably large if π-ψ_A scattering is dominated by ρ exchange.

[9]It should be noted that, for the value $a=2$ leading to the KSRF relation, the π-π scattering amplitude is *not* saturated by the ρ-exchange contribution; that is, there remain direct 4π interaction terms, in sharp contrast to the pion-matter scatterings. Incidentally, performing a suitable point transformation of pion field $\boldsymbol{\pi} \rightarrow \boldsymbol{\pi}(1+c\boldsymbol{\pi}^2+\ldots)$, one can always eliminate either one of the two types of direct 4π terms $(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^2$ or $(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})^2$, but not both.

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