Dispersion Relation and the Low-Energy Behavior of the Heavy-Ion Optical Potential

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A dispersion relation is used to show that the "anomalous" behavior of the real part of the optical potential for $^{16}O + ^{208}Pb$ scattering at low energies is an example of a general property of heavy-ion optical potentials at energies approaching the top of the Coulomb barrier, where the flux into nonelastic channels is drastically reduced.

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It is shown in a recent paper[1] that the optical potential that describes $^{16}O + ^{208}Pb$ elastic scattering behaves in an apparently anomalous way (see Fig. 1) as the bombarding energy, E, approaches the Coulomb barrier ($E \approx 80$ MeV for this system). The imaginary potential decreases sharply in magnitude for $E \lesssim 90$ MeV, but this can be understood because the nonelastic channels are being effectively closed by the Coulomb barrier. However, at the same time, the real potential sharply increases in magnitude; by $E = 78$ MeV it has reached approximately 1.7 times the roughly constant value found for $E \gtrsim 130$ MeV. A similar "anomaly" has been ascribed[2] recently to the optical potential for the $^{32}S + ^{40}Ca$ system. We intend to show that these results are to be expected on very general grounds.

It is natural to ascribe such a behavior to the effects of couplings to the nonelastic channels, which can produce changes in the real potential even below the threshold where they are energetically closed. Such couplings are included in a very general way in the dispersion relation (DR) which connects the real and imaginary parts of the generalized optical potential $U(\mathbf{r}, \mathbf{r}';E)$. If we write $U = V + iW$, this DR has the form

$$V(\mathbf{r}, \mathbf{r}';E) = V_0(\mathbf{r}, \mathbf{r}') + \frac{P}{\pi}\int_0^\infty \frac{W(\mathbf{r}, \mathbf{r}';E')}{E' - E} dE' = V_0 + \Delta V(E), \quad (1)$$

say, where P denotes principal value, and V_0 is independent of E (though it may be nonlocal and hence momentum dependent). Such a relation is a consequence of causality and can be derived within the framework of general reaction theory.[3] It is the analog of the Kramers-Kronig relation in optics. It can be argued[4] plausibly that the DR is also obeyed by an equivalent, local, *model* potential $U_M(r,E)$, although in this case, the equivalent local $V_{M,0}$ and W_M terms will contain some "spurious" energy dependence due to nonlocality. The latter is expected to be rather small for heavy ions[5] and will be neglected here. Henceforth, we drop the subscript M, and U, V, and W will refer to the local model potential.

The effects on V of couplings to nonelastic channels are contained implicitly in the induced interaction ΔV. Further, any large, localized variation of $W(E)$ with E will be reflected in a similarly localized variation in $\Delta V(E)$. In particular, when we remember that W is negative, a moment's thought will show that a rapid decrease in $|W(E)|$ as E approaches the Coulomb barrier (or as E approaches zero for neutral particles) will

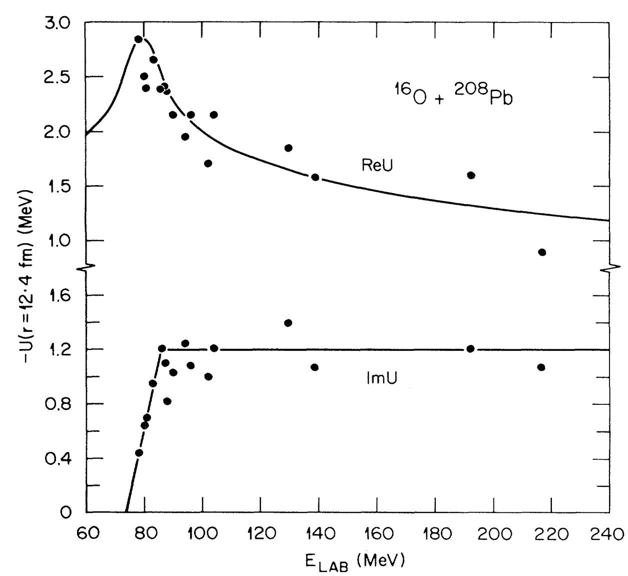

FIG. 1. Variation with bombarding energy of the real and imaginary parts of the optical potential U for $^{16}O + ^{208}Pb$ evaluated at the radius $r = 12.4$ fm. The dots represent empirical values (Ref. 1). The curve for ReU was obtained from the dispersion relation, using the fit to ImU that is shown.

result in a rapidly varying attractive contribution to ΔV in the same E region. Such an effect in nucleon-nucleus scattering has been invoked[6] in studies of the variation of the effective mass at energies near the Fermi surface.

Except at the lowest energies, the scattering of heavy ions is usually distinguished by strong absorption, and analyses of such scattering tend to determine the optical potential in a very limited radial region near some "strong absorption radius." This sensitive radius varies slowly with the energy E, whereas the DR relates values of the potential at a fixed radius. In the case of $^{16}O + ^{208}Pb$, one measure[7] of this strong absorption radius decreases from $r = 12.8$ fm at $E = 129.5$ MeV to $r = 12.1$ fm at $E = 312.6$ MeV, whereas the scattering at the lowest energy ($E = 78$ MeV) is dominated by the position of the Coulomb barrier at $r \approx 12.1$ fm. However, we believe that the slope of the potential in this region is known well enough that the extrapolation of the empirical values to a fixed radius is not attended by large errors. In accord with Ref. 1, we chose $r = 12.4$ fm for this radius.

The effect can be demonstrated by use of a very simple model[8] for $W(r,E)$ in which its energy dependence is represented by a series of linear segments. This has the advantage that the resulting $\Delta V(r,E)$ can be expressed in a simple analytic form. The behavior of $W(E)$ at high energies is not known. Since this has little effect on the *shape* of $\Delta V(E)$ at the low energies, one may use the DR in a subtracted form by normalizing V at some convenient energy E_s and absorbing the unknown contributions into the empirical value at that energy. Formally, this can be written as

$$V(r,E) = V(r,E_s) + \frac{P}{\pi}(E - E_s)\int_0^\infty \frac{W(r,E')\,dE'}{(E' - E_s)(E' - E)}. \quad (2)$$

For the $^{16}O + ^{208}Pb$, system, we chose two segments, as shown in Fig. 1, with $W(r = 12.4) = 0$ at $E = 74$ MeV and constant at -1.2 MeV for $E > 86$ MeV. There is some evidence[7,9] that $W(r = 12.4) \approx -1.0$ MeV at $E = 312.6$ MeV, and -0.8 MeV at $E = 400$ MeV, so that W may decrease slowly with increasing energy at the higher energies. However, even choosing the second segment so that W goes to zero at $E = 400$ MeV, instead of being constant, has almost no effect on the shape of the calculated $\Delta V(E)$ for the low energies. The calculated curve for V shown in the upper part of Fig. 1 was normalized to $V = -1.3$ MeV at $E = 200$ MeV. It reproduces very well both the shape and the magnitude of the increase in depth of ReU at the lower energies. At higher energies $-V(E)$ continues to decrease slowly. It is smaller than the empirical values at 312.6 and 400 MeV, but this may be due to an inappropriate choice for $W(E)$ in the high-energy region.

When the upper segment of $W(E)$ is chosen to be constant, the resulting $\Delta V(E)$ is symmetrical about the midpoint of the lower segment ($E = 80$ MeV in this case). The steepness of the lower segment of $W(E)$ determines the width of the peak in $\Delta V(E)$; the empirical values of V seem to demand that the cutoff on $W(E)$ near the Coulomb barrier be as sharp as that shown. The effects of a more realistic, rounded, shape for $W(E)$ are being investigated[10]; preliminary results indicate no qualitative changes in $\Delta V(E)$.

It is inevitable that $W(E)$ will decrease in magnitude as E approaches the Coulomb barrier. [In some cases such as $\alpha + ^{40}Ca$ and $^{16}O + ^{28}Si$, nuclear structural features[11,12] may result in reaction channels being effectively closed, and hence in a strong decrease of $|W(E)|$, at energies appreciably above the Coulomb barrier.] As we have indicated, this leads with equal inevitability to a localized energy dependence of $V(E)$ in the same energy region. It follows that "global" optical-model analyses (that is, simultaneous fits to data for a number of different energies) that keep $V =$ const, or at most allow a linear dependence on E, even at low energies, necessarily violate the dispersion relation. One example of such a violation occurs in a recent analysis[13] of $^{16}O + ^{28}Si$ scattering.

The same criticism may be made of attempts to understand fusion cross sections in terms of a simple barrier penetration model using a potential that is independent of the bombarding energy (for example, see Vaz, Alexander, and Satchler[14] and other references therein). For example, the $V(E)$ shown in Fig. 1 yields a fusion cross section at $E = 80$ MeV that is about 50 times larger than that which would be obtained if one had assumed that V remained fixed at the value shown for $E = 160$ MeV. Indeed, it has already been pointed out[15] that, within the barrier penetration model, the measured fusion cross sections for $^{16}O + ^{208}Pb$ imply that the real potential must be more attractive for energies close to the top of the Coulomb barrier than at higher energies. [However, we note that a fully consistent treatment of the effects on fusion of the channel couplings which give rise to $\Delta V(E)$ would also include the additional flux lost to fusion from the nonelastic channels themselves: see Jacobs and Smilansky, Udagawa and Tamura, and Rhoades-Brown and Prakash,[16] for example.]

In conclusion, we have pointed out that at energies close to the top of the Coulomb barrier a complementary behavior of the real and imaginary parts of the optical potential can be expected on very general grounds. This behavior is expressed within a dispersion relation. A simple model for the energy dependence of the imaginary potential allows one to reproduce the corresponding variation of the real potential in good agreement with empirical results for $^{16}O + ^{208}Pb$ scattering. A more detailed discussion of the relation, together with applications to other data, will be given elsewhere.[10]

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