# Chiral Fermions from Compactification of O(32) and E(8) ⊗ E(8) String Theories

Paul H. Frampton, Hendrik van Dam, and Katsuji Yamamoto
*Institute of Field Physics, Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514*
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Compactification to four dimensions of a low-energy approximation to ten-dimensional O(32) string theory leads only to grand unified theories with trivial replication of families. The E(8) ⊗ E(8) string is more promising and leads in four dimensions to SU(8) with six families. If compactification breaks the symmetry to lower-rank groups, e.g., SU(7) or E(6), there may be any number of families. In the SU(8) and SU(7) models there is nontrivial family symmetry.

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Recently there has been considerable interest in higher-dimensional gauge theory and its spontaneous compactification to four dimensions.[1] Solution of the Einstein-Yang-Mills field equations requires a nontrivial vacuum value for the gauge fields in the extra compact dimensions; this, in turn, has the advantage of allowing massless chiral fermions to survive in four dimensions.

This old approach had at least two shortcomings. First, the quantum corrections are uncontrolled since the theory is nonrenormalizable. Second, the gauge theory is not unified with gravitation, contrary to the original goal of Kaluza-Klein theory.[2]

Both of these objections may be overcome in a ten-dimensional string theory[3] with open and closed strings and based on a gauge group O(32) or E(8) ⊗ E(8). Such a theory is possibly finite. Also, the gauge coupling is unified with gravitation since the theory contains only one parameter, the string tension.

The string theory with open strings leads to a chiral anomaly in the hexagon loop diagram for any choice of gauge group[4]; recently,[5] in an interesting development by Green and Schwarz, it has been pointed out that for the two special choices of gauge group already mentioned the string incorporates a novel mechanism to cancel the loop anomaly by a pole term due to exchange of a massless tensor particle. This pole term induces a contact interaction which renders the relevant Green's functions gauge invariant. In the language of path integrals, the anomalous phase arising from a chiral transformation of the fermion measure is canceled by a phase arising from the gauge-dependent action. The classical action is, as usual, gauge invariant. The new term is proportional to Planck's constant.

This type of anomaly cancellation in higher-dimensional gauge theories means that the no-anomaly conditions widely used are unnecessarily restrictive; this more general aspect will be discussed elsewhere.[6] Here we shall focus on the two specific string theories. (Some remarks on this topic have been made already by Witten.[7])

Consider first the case O(32). Recall that the Majorana-Weyl fermions are in the adjoint of O(32). Here the embedding of a subgroup is specified by stating the transformation of the defining 32-dimensional representation. We cannot obtain E(6) since it is not a subgroup. To obtain O(10) with its sixteen-dimensional complex spinor involved would necessitate $\mathbf{32}=\mathbf{16}+\mathbf{16}^*$ since **32** is real. But then the adjoint **496** of O(32) contains no spinors since it is the antisymmetric part of $\mathbf{32}\times\mathbf{32}$. Thus, O(32) is too small to allow embedding of an O(10) grand unified theory (GUT) because the fermions are in the adjoint **496**. Of familiar GUT groups this leaves SU($N$) which we may embed as

$$\mathbf{32}=\mathbf{N}+\mathbf{N}^*+(32-2N)\mathbf{1}. \quad (1)$$

For $N\geqslant 7$ as is necessary to obtain nontrivial family unification, Eq. (1) is the only possible embedding of SU($N$). This means that the adjoint of O(32) contains only the $\mathbf{N}+\mathbf{N}^*$, the second-rank $[\mathbf{2}]+[\mathbf{2}]^*$, and the adjoint of SU($N$) as nontrivial representations. This means that the most general complex chiral representation on $M_4$ will be of the form

$$f\{[\mathbf{2}]+(N-4)[\mathbf{1}]^*\} \quad (2)$$

where $f$ is an integer. Here $[\mathbf{k}]$ is the $k$-rank antisymmetric representation. Equation (2) contains $f$ families but always trivially replicated. In this sense, SU($N$) family unification is impossible for the O(32) string. This result does not depend on the compact manifold $N_6$ involved in the reduction to $M_4\times N_6$. If we do take an SU($N$) subgroup of O(32) as the GUT group, there are still further potential problems beyond the lack of family unification. For example, if we were to choose SU(5) and the embedding O(32) → [SU(5) ⊗ U(1)] ⊗ O(22) the U(1) charges are $\mathbf{32}=\mathbf{5}_{+1}+\mathbf{5}^*_{-1}+22(\mathbf{1})_0$. Hence,

$$\mathbf{496}=(\mathbf{10}_{+2}+\mathbf{10}^*_{-2})+22(\mathbf{5}_{+1}+\mathbf{5}^*_{-1})+\mathbf{24}_0+132(\mathbf{1})_0$$

for both the fermions and the scalars. Thus although a coupling $\mathbf{10}_f\mathbf{5}^*_f\mathbf{5}^*_s$ occurs, $\mathbf{10}_f\mathbf{10}_f\mathbf{5}_s$ is excluded by the

O(32) symmetry and the up quarks would remain massless. Solution of this involves a more complete treatment of the spectrum of scalars on $M_4$ and is analogous to the familiar gauge hierarchy problem.

For the string with gauge group E(8) ⊗ E(8)′ the situation with respect to allowed GUT's is quite different and really more promising. Here, at least, there is a chance to say something about families.

Several maximal subalgebras of E(8) are potentially relevant. Here we describe only the more interesting cases which are based on E(8) → SU(3) ⊗ E(6), $\mathbf{248}=(\mathbf{8},1)+(1,\mathbf{78})+(\mathbf{3},\mathbf{27})+(\mathbf{3}^*,\mathbf{27}^*)$; on E(8) → O(16), $\mathbf{248}=\mathbf{120}+\mathbf{128}$; and on E(8) → SU(9), $\mathbf{248}=\mathbf{80}+\mathbf{84}+\mathbf{84}^*$.

Let us consider the sequence E(6) → O(10) → SU(5) of GUT groups first. For E(6) we consider the subalgebras E(8) → E(6) ⊗ SU(2) ⊗ $U(1)_1$ and E(8) → E(6) ⊗ $U(1)_2$ ⊗ $U(1)_1$ under which the complex **27**'s transform as

$$(\mathbf{27},2;1)+(\mathbf{27},1;-2) \tag{3a}$$

and

$$(\mathbf{27};1,1)+(\mathbf{27};1,-1)+(\mathbf{27};-2,0), \tag{3b}$$

respectively. Corresponding to (2a) we consider the compact manifold $N_6=S_4\times S_2$ with an SU(2) instanton (charge $I_1$) on $S_4$ and a U(1) monopole (charge $M_1$, an integer multiple of the minimum allowed Dirac charge) on $S_2$. The chiral Weyl spinors on $M_4$ are then simply $(I_1M_1)$**27**'s. For the simplest vacuum configuration $I_1=1$, $M_1=1$ this gives just one family. Corresponding to (2b) we consider $N_6=S_2\times S_2\times S_2$ with the three two-spheres labeled $i=1$, 2, 3, respectively. The gauge field is in a monopole configuration (charge $M_i$) on $(S_2)_i$ for the gauge generator

$$U(1)^i_{\text{mono}}=a_iU(1)_1+b_iU(1)_2. \tag{4}$$

From (3b) we find that the number of chiral **27**'s on $M_4$ is given by

$$\prod_{i-1}^{3}(a_i+b_i)+\prod_{i=1}^{3}(a_i-b_i)-8a_1a_2a_3=-6a_1a_2a_3+2(a_1b_2b_3+a_2b_3b_1+a_3b_1b_2). \tag{5}$$

The coefficients $a_i$ and $b_i$ are subject to the requirement that $2a_i$ and $a_i\pm b_i$ are integers. With any product of monopole charges $M=M_1M_2M_3$ including $M=1$, Eq. (4) can give any integer number of families.

With reduction of the GUT group still further to O(10) or SU(5) the number of families obviously remains unrestricted. Furthermore, in these rank-four and -five groups any family unification is known to be impossible.[8]

Another maximal subalgegra is E(8) → O(16) which might suggest spinors of larger orthogonal groups. Obviously O(18) is impossible. For O(14) we may take the subalgebra O(14) ⊗ U(1), and a manifold $N_6=(S_2)_1\times(S_2)_2\times(S_2)_3$ with monopole charges $M_1$, $M_2$, $M_3$. Since now the O(14) complex part [**120** of O(16) remains real] gives

$$\mathbf{128}=(\mathbf{64};1)+(\mathbf{64}^*;-1) \tag{6}$$

we obtain $M$ **64**'s and hence $2M$ families but here we will obtain also $2M$ mirror families.

For nontrivial incorporation of chiral families, without mirrors, the rank must be at least 6 and hence SU(7) and SU(8) are the only such subgroups; unfortunately we may not use an unbroken SU(9) since there is no room for the topological mechanism which yields chiral fermions on $M_4$.[1] Using SU(8) ⊗ U(1) and the usual monopoles on $N_6=(S_2)^3$ we have

$$\mathbf{248}=(\mathbf{63};0)+(\mathbf{8};+3)+(\mathbf{8}^*;-3)+(\mathbf{1};0)+(\mathbf{56};+1)+(\mathbf{56}^*;-1)+(\mathbf{28};-2)+(\mathbf{28}^*;+2). \tag{7}$$

This gives on $M_4$ the chiral fermions

$$M[\mathbf{56}+8(\mathbf{28}^*)+27(\mathbf{8})] \tag{8}$$

with $M=M_1M_2M_3=$ integer. This contains a multiple of *six* families; the only simple compactification we found which gives such a restriction is this one with SU(8).

If spontaneous compactification breaks the symmetry further to SU(7) ⊗ SU(2) ⊗ $U(1)_1$ or SU(7) ⊗ $U(1)_2$ ⊗ $U(1)_1$, we have

$$\mathbf{248}=(\mathbf{48};1;0)+(\mathbf{7},2;3)+(\mathbf{7}^*,2;-3)+(\mathbf{1},1;0)+(\mathbf{1},3;0)+(\mathbf{35},1;2)+(\mathbf{35}^*,1;-2)$$
$$+(\mathbf{21},2;-1)+(\mathbf{21}^*,2;+1)+(\mathbf{7},1;-4)+(\mathbf{7}^*,1;+4), \tag{9a}$$

and

$$\mathbf{248} = (48;0,0) + (7;3,\pm 1) + (7^*;-3,\mp 1) + 2(1;0,0) + (1;0,\pm 2) + (35;2,0) + (35^*;-2,0) + (21;-1,\pm 1) + (21^*;+1,\mp 1) + (7;-4,0) + (7^*;+4,0), \quad (9b)$$

respectively. Under compactification on $N_6 = S_4 \times S_2$, (9a) gives some multiple of $[21 + 3(7^*)]$ and hence trivial family replication [because the **35** is singlet under SU(2)]. Compactification of (9b) on $N_6 = (S_2)_1 \times (S_2)_2 \times (S_2)_3$ with $U(1)^i_{\text{mono}} = a_i U(1)_1 + b_i U(1)_2$ gives $M_4$ the chiral fermions

$$m[21 + 3(7^*)] + n[35 + 2(7^*)] \quad (10)$$

with $m + n$ families[8] where

$$m = -2(a_1 a_2 a_3 + a_1 b_2 b_3 + a_2 b_3 b_1 + a_3 b_1 b_2), \quad (11a)$$

$$n = 8 a_1 a_2 a_3. \quad (11b)$$

The number of families $(m+n)$ corresponds (within an unimportant sign) to that in the E(6) model of Eq. (4). However, here there is the distinct advantage that the families are nontrivially unified, i.e., on expansion into irreducible representations of SU(7) the coefficients have no common factor; this requires that $m,n \geqslant 1$ in Eq. (9) and $m,n$ are relatively prime.[8]

So far, we have ignored the isometry gauge group of $N_6$. This isometry is too small[9] to contain all of $SU(3)_c \otimes [SU(2) \otimes U(1)]_{\text{ew}}$, where the subscript "ew" means "electroweak," because that would need at least seven compact dimensions. If we were to put only $SU(3)_c$ in the isometry group, we would necessarily obtain mirror fermions because of the direct product structure. Hence, we may consider putting $[SU(2) \otimes U(1)]_{\text{ew}}$ in the isometry group and leaving $SU(3)_c$ in $G$. This requires an isometry group with complex representations, hence excluding, e.g., $N_6 = S_6$, $S_4 \times S_2$, $S_2 \times S_2 \times S_2$, $K_3 \times S_2$, $G_2/SU(3)$, $SO(5)/SO(3) \otimes SO(2)$. The cases with a complex isometry, e.g., $CP_3$ or $CP_2 \times S_2$, would allow us to consider such a non-grand-unified view.

The absence of triangle anomalies in $M_4$ requires further discussion. For O(32) this uses extra conditions on the compactification, analogous to the Dirac quantization condition, as discussed in Ref. 7; in our present brief discussion of O(32) we did not use this explicitly. For $E(8) \otimes E(8)'$ the absence of triangle anomalies is guaranteed already by the algebraic features of E(8). Because there are no irreducible tensors between second and eighth rank we have the peculiar property of the hexagon anomaly that[5]

$$S\,\mathrm{Tr}(\Lambda^{a_1}\Lambda^{a_2}\Lambda^{a_3}\Lambda^{a_4}\Lambda^{a_5}\Lambda^{a_6}) = \tfrac{1}{7200} S[\mathrm{Tr}(\Lambda^{a_1}\Lambda^{a_2})]^3 \,, \quad (12a)$$

$$S\,\mathrm{Tr}(\Lambda^{a_1}\Lambda^{a_2}\Lambda^{a_3}\Lambda^{a_4}) = \tfrac{1}{100} S[\mathrm{Tr}(\Lambda^{a_1}\Lambda^{a_2})]^2 \,, \quad (12b)$$

where $\Lambda^{a_i}$ are generators of E(8) in the **248** representation. Equations (12) have dramatic consequences. Let us take $E(8) \to G \times H$ where $G$ is the unbroken gauge group on $M_4$. Suppose that

$$\mathbf{248} = \sum_i g_i \times h_i \quad (13)$$

where $g_i$, $h_i$ are irreducible representations of $G$, $H$ respectively. The triangle anomaly $(A)$ will be proportional to[7]

$$A \sim \int_{N_6} \sum_i \mathrm{Tr}_{g_i} J^3 (\mathrm{Tr}_{h_i} K^3 - \tfrac{1}{8} \mathrm{Tr}_{h_i} K \,\mathrm{tr} R^2) \quad (14)$$

where $J$, $K$ are Lie-algebra–valued field strength two-forms corresponding to $G$, $H$, respectively, and $R$ is the curvature two-form. Equation (14 ) is equivalent to evaluation of $\mathrm{Tr}(J^3 K^3)$ and $\mathrm{Tr}(J^3 K)$ in the adjoint of E(8). With use of Eqs. (12) these are all proporal to $\mathrm{Tr}(JK)$ which vanishes since one may always choose an orthogonal basis where $\mathrm{Tr}(\Lambda^{a_i}\Lambda^{a_j}) \sim \delta_{a_i a_j}$. Hence no subsidiary conditions need ever be explicitly considered, even though such conditions are actually being met by compensation with the second $E(8)'$ in $E(8) \otimes E(8)'$.

On the question of "shadow matter,"[7,10] i.e., light chiral fermions transforming under the second $E(8)'$, this can be avoided in some cases. For example, in an $N_6 = S_4 \times S_2$ compactification the instanton on $S_4$ must be compensated by an anti-instanton of $E(8)'$ but the monopole on $S_2$ need not have a shadow monopole, which allows no light $E(8)'$ matter. Of course, $E(8)'$ matter may or may not be desirable.

In our analysis we have approximated the string theory by a low-energy truncation where the field equations parallel the classical Einstein-Yang-Mills field equations. Our compactification schemes solve these classical equations and hence the complete string equations in the same approximation. We should point out that the supergravity equations in ten dimensions[11] involve also a scalar field which in the classical field equations disallows[12] any compactification which preserves four-dimensional Lorentz invariance. However, naively giving a nonzero vacuum value for the Yang-Mills field strength leads, classically, to an ever increasing and arbitrarily large vacuum value for this scalar field. We may therefore suppose that when this large vacuum value approaches the Planck mass, the classical equations considered in Ref. 12 became unreliable and the associated no-go theorem ceases to apply in the complete string theory.

The string theory with gauge group O(32) or $E(8) \otimes E(8)$ has several outstanding problems. The first is whether or not it is truly a finite quantum theory. This would already be a remarkable achievement since we know no other consistent quantum gravity. Second, we may hope that the string underlies a complete unification of gravity with other particle forces. This requires a much more detailed knowledge of string dynamics than is available at present, and is necessarily less probable. Nevertheless, it is this most optimistic philosophy that we have adopted.

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*Note added.*—We have received a related paper by Candelas, Horowitz, Strominger, and Witten[13] who discuss the additional assumption that unbroken supersymmetry survives in four dimensions.

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