Comment on "ϵ Expansion for the Conductivity of a Random Resistor Network"

Recently, Harris, Kim, and Lubensky[1] have shown that an infinity of relevant crossover exponents arises in the $s \to 0$ limit of the diluted s-states Potts model near six dimensions ($d = 6 - \epsilon$). They show that this implies new exponents for the cumulants of the resistance in the percolation problem. More specifically, the resistance between connected points separated by a distance L is a random variable whose cumulants $[R^m(L)]_c$ are described by the new crossover exponents: $[R^m(L)]_c \sim L^{(-2\beta_p + \phi_m)/\nu}$. β_p and ν are the usual percolation exponents and ϕ_m are new exponents close to unity below six dimensions and equal to one above six. In this notation, the conductivity exponent is $t = (d-2)\nu + \phi_1$.

On the other hand, the usual assumption on which most finite-size scaling calculations of the conductivity exponent[2] are implicitly based is that the whole probability distribution for the resistance scales with a single exponent $L^{t/\nu}$. Hence, either the Harris-Kim-Lubensky exponents are irrelevant below six dimensions or previous calculations of the exponent t/ν are doubtful. We show that the former statement is the correct one, at least in two dimensions.

Figure 1 displays normalized histograms for the probability law of the conductivity σ of square blocks of square lattices of size 20×20 and 10×10 at the percolation threshold $p_c = 0.5$. Top and bottom sides of each square are equipotentials. We have divided the conductivity of the 10×10 squares by $2^{0.979}$ to illustrate that the whole probability distribution scales as $\sigma / L^{-t/\nu}$ with $t/\nu = 0.979 \pm 0.006$. We have also performed more quantitative calculations[3] by evaluating the first three cumulants of the conductivity and resistivity distributions or squares of sizes 7×7 to 60×60. The scaling of these cumulants, exhibited in Fig. 2, is consistent with the whole probability law scaling as $\sigma / L^{-t/\nu}$ with the previous value of t/ν. This result also implies that conductivity fluctuations, normalized to the mean, are size independent.

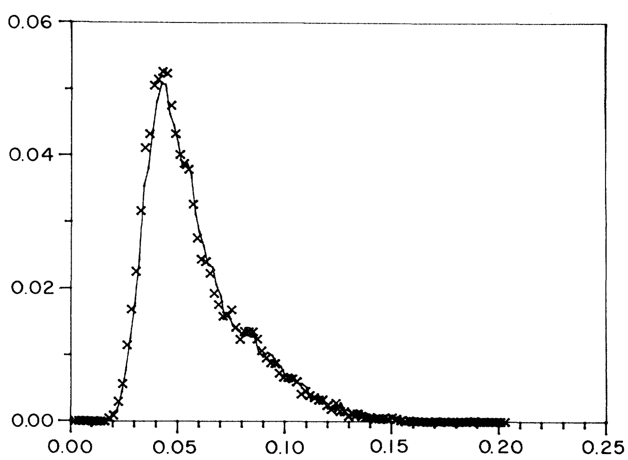

FIG. 1. The solid line is the normalized histogram $P(\sigma)$ for the conductivity of 2.9×10^4 samples of 20×20 squares at the percolation threshold $p_c = 0.5$. The conductivity of an elementary occupied bond is 1 and in these units the conductivity intervals on the abscissa are 2×10^{-3}. About half of the samples have $\sigma = 0$ and are not shown here. The crosses show the corresponding histogram for 4×10^4 samples of 10×10 squares whose conductivity has been divided by $2^{0.979}$.

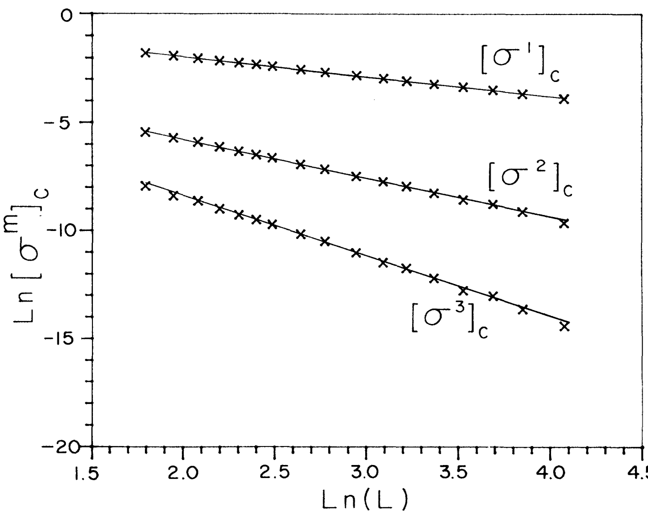

FIG. 2. Scaling behavior of the first three cumulants of the conductivity $[\sigma^m]_c$ vs L for $7 \leq L \leq 60$. The corresponding exponents are, starting from the top curve, -0.976 ($\simeq -t/\nu$), -1.93 ($\simeq -2t/\nu$), and -2.97 ($\simeq -3t/\nu$). Similar results have been obtained for σ^{-1} and for the first cumulant of $\ln\sigma$.

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[3] M.-A. Lemieux, P. Breton, and A.-M. S. Tremblay, unpublished.

