Higher-Order States in the Multiple-Series, Fractional, Quantum Hall Effect

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(Received 15 June 1984)

We report results on the fractional quantum Hall effect in GaAs-$Al_xGa_{1-x}As$ heterostructures at fractional Landau-level filling factor $\nu = p/q$ obtained with the combination of a dilution refrigerator and the National Magnet Laboratory hybrid magnet. We establish conclusively the quantiztation of the higher-order states p in the series q. The Hall resistance is accurately quantized to 2.3 parts in 10^4 for the $\frac{2}{5}$ state and 1.3 in 10^3 for the $\frac{3}{5}$ state. New structures are observed near $\nu = \frac{3}{7}, \frac{4}{7}, \frac{4}{9}$, and $\frac{5}{9}$.

PACS numbers: 72.20.My, 73.40.Lq, 73.60.Fw

The fractional quantum Hall effect (FQHE) is characterized by the presence of quantized plateaus in the Hall resistance, ρ_{xy}, and the vanishing of the diagonal resistivity, ρ_{xx}, in two-dimensional (2D) carrier systems at fractional occupation $\nu = p/q$ of their Landau levels,[1,2] with p = integers and q = odd integers. Apart from the well established $\nu = \frac{1}{3}$ and $\nu = \frac{2}{3}$ states,[2,3] minima in the diagonal resistivity are also observed in the vicinity of $\nu = \frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, and $\frac{2}{7}$. At $\nu = \frac{2}{5}$ and $\frac{3}{5}$ distinct Hall plateaus develop in ρ_{xy}. These observations cannot be explained in terms of an independent-electron model used to describe the integral quantum Hall effect at integer filling factor ν. The electronic state giving rise to the experimentally observed features must be of a many-particle origin. These results have stimulated a number of theoretical attempts[4-9] to understand this new quantum phenomenon. While some theories have already been able to account for the existence of a new ground state at $\nu = 1/q$ [and, by electron-hole symmetry, also at $\nu = (q-1)/q$], much interest is presently focused on the higher-order progression $\nu = p/q$.

In this Letter, we report results obtained from magnetotransport measurements at dilution refrigerator temperatures (85 mK) in high magnetic fields (280 kG). We report data which establish conclusively the quantization of the Hall resistance in the $\frac{2}{5}$ and $\frac{3}{5}$ effects. We have found an accuracy of 2.3×10^{-4} for the quantization of the $\frac{2}{5}$ Hall resistance to $\rho_{xy} = 5h/2e^2$ and 1.3×10^{-3} for the $\frac{3}{5}$ state to $\rho_{xy} = 5h/3e^2$. The temperature dependence of ρ_{xx} is found to be activated in a manner similar to the $\frac{1}{3}$ and $\frac{2}{3}$ effects,[2,3] with typical activation energies of 1 K (0.086 meV) around 200 kG. Furthermore, experiments at high magnetic fields and low temperatures have revealed the $\frac{3}{7}$ and $\frac{4}{7}$ effects, and weaker structures in ρ_{xx} attributable to the $\frac{4}{9}$ and $\frac{5}{9}$ effects. These observations firmly establish the higher-order (p) effects in the multiple series (q) reported in Ref. 2.

Our samples are high quality GaAs-$Al_{0.3}Ga_{0.7}As$ heterostructures prepared in the same way as those used in previous experiments.[1-3] They have typical mobilities of 5×10^5 $cm^2/V \cdot s$ at 4.2 K at a density of 1.5×10^{11} cm^{-2}. The density can be varied from 1×10^{11} cm^{-2} to 2.3×10^{11} cm^{-2} using a backside gate bias.[10]

Figure 1 shows the magnetotransport data we have obtained at 90 mK. A background field of 70 kG is supplied by a superconducting magnet. The sample is gated at 700 V to a density of 2.1×10^{11} cm^{-2}. The depth of the $\frac{1}{3}$ and $\frac{2}{3}$ minima in ρ_{xx} and the flatness of the Hall plateaus are indicative of the high quality of the data. Several new results

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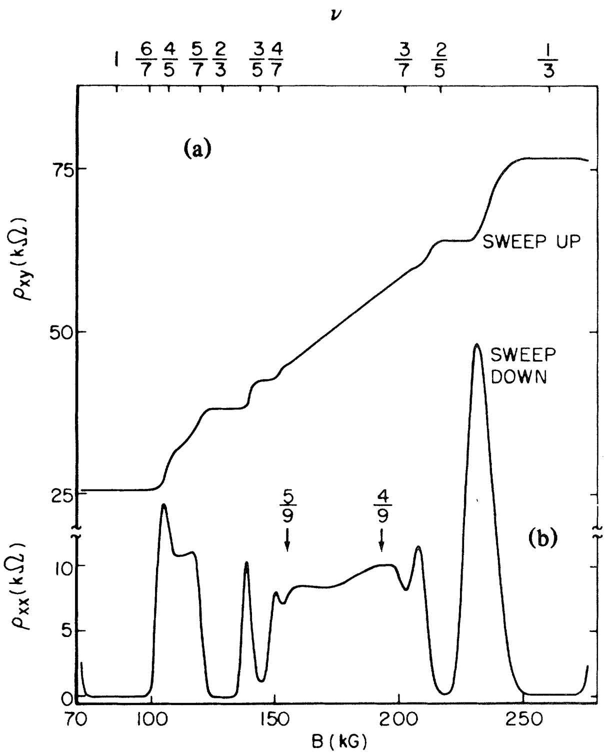

FIG. 1. Magnetic field traces of transport measurements at 90 mK: (a) the Hall resistivity and (b) the diagonal resistivity, ρ_{xx}. The sample is gated at 700 V to a density of 2.13×10^{11} cm^{-2}. The $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{3}{7}$, and $\frac{4}{7}$ fractional quantum Hall effects are clearly visible. Weak structures near $\frac{4}{9}$ and $\frac{5}{9}$ also appear in ρ_{xx}.

are observed. In ρ_{xx}, the $\frac{2}{5}$ and $\frac{3}{5}$ minima have reached low values of 280 Ω/□ and 900 Ω/□, respectively, from their background of about 7000 Ω/□. The minima are accompanied by flat plateaus in ρ_{xy}. These results represent a significant improvement over data from earlier work.[1] Pronounced new structures are observed at $\nu=\frac{3}{7}$ and $\frac{4}{7}$ as clear minima in ρ_{xx}, and weaker structures in ρ_{xy}. We also have indication for the existence of a new series. In ρ_{xy}, near the positions marked $\frac{4}{9}$ and $\frac{5}{9}$, flat tops appear. They are characterized by an abrupt change in the slope of ρ_{xx}. This behavior is typical for the development into a minimum as can be concluded from the features at other fractions at higher temperatures. We have verified the presence of both effects by increasing the measurement resolution; the subtle slope change is difficult to reproduce in the figure. In addition, a broad minimum is observed near $\nu=\frac{1}{2}$, but centered slightly above $\frac{1}{2}$. This may result from an overlap of a sequence of new unresolved features. It is interesting to point out the progression of the strong fractional effects. It starts with $\frac{1}{3}$ and $\frac{2}{3}$, moves toward the center ($\frac{1}{2}$) to $\frac{2}{5}$ and $\frac{3}{5}$, then more toward the center to $\frac{3}{7}$ and $\frac{4}{7}$, and then $\frac{4}{9}$ and $\frac{5}{9}$. It is tempting to speculate that the series continues as $m/(2m+1)$ and $1-m/(2m+1)$, where m is a positive integer.

Several observations demonstrate the exactness of the $\frac{2}{5}$ and $\frac{3}{5}$ effects. The strengths of the ρ_{xx} minima and the appearance of flat Hall plateaus provide strong evidence for their existence. Moreover, unlike earlier results,[1,11] the centers of the $\frac{2}{5}$ and $\frac{3}{5}$ plateaus are in line with the linear portion in ρ_{xy} around $\nu=\frac{1}{2}$ as expected. In other words, the line representing the classical Hall resistance given by B/ne, where n is the areal density, passes through the centers of the plateaus. This is an indication that the sample is homogeneous in density. The assignment of the quantum numbers $\frac{2}{5}$ and $\frac{3}{5}$ to these effects is made unambiguous by the accuracy of the Hall resistance quantization, $\rho_{xy}=5h/2e^2$ and $5h/3e^2$. We have measured an accuracy of 2.3 parts in 10^4 in the $\frac{2}{5}$ Hall resistance. The measured Hall resistance is larger than the theoretical value by 2.3×10^{-4}, with an experimental uncertainty of $\pm3\times10^{-5}$. This values remains constant over a magnetic field range of 2.5 kG. The measurement was carried out at 90 mK and 220 kG. The $\frac{3}{5}$ effect is accurate to 1.3 parts in 10^3 at the field position of the ρ_{xx} minimum. Table I summarizes the best experimental values for the Hall-resistance quantization of the various fractional effects. For the $\frac{1}{3}$ and $\frac{2}{3}$ effects, the values are limited by experimental resolution. For the $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{3}{7}$ effects, they are larger than the theoretical values.

The evolution of ρ_{xx} with temperature for the $\frac{2}{5}$ and $\frac{3}{5}$ effects is shown in Fig. 2. ρ_{xx} shows a roughly activated behavior with activation energies of 1 K (0.086 meV) at 243 kG for the $\frac{2}{5}$ state and 0.5 K (0.043 meV) at 167 kG for the $\frac{3}{5}$ state at a slightly different electron density established in the same sample by biasing with the gate. We need to point out that the saturation at lower temperatures may be a result of thermometry problems and probably does not represent the true behavior of ρ_{xx}. Qualitatively, these effects appear similar to the $\frac{1}{3}$ and $\frac{2}{3}$ effects. Their activation energies are also comparable to those of the $\frac{1}{3}$ and $\frac{2}{3}$ effects at similar magnetic fields.

The current theories[4-9] which predict the higher-order FQHE's can be described as heirarchical models, with the exception of Emery's theory. In these hierarchical models, the higher-order states

TABLE I. A summary of the values for the Hall-resistance quantization of various fractional quantum Hall effects.

Fractional effect	Accuracy of quantization	Sample density (cm^{-2})	B field (kG)	Range of flatness (kG)
$\frac{1}{3}$	3×10^{-5}	1.53×10^{11}	190	8.2
$\frac{2}{3}$	3×10^{-5}	2.42×10^{11}	150	2.3
$\frac{2}{5}$	2.3×10^{-4}	2.13×10^{11}	220	2.9
$\frac{3}{5}$	1.3×10^{-3}	2.13×10^{11}	147	a
$\frac{5}{3}$	1.1×10^{-3}	2.06×10^{11}	53	a
$\frac{3}{7}$	3.3×10^{-3}	2.13×10^{11}	206	a

[a]At ρ_{xx} minimum.

which give rise to the higher-order effects (e.g., $\nu=\frac{2}{5}$) are derived from lower-order states associated with the lower-order effects (e.g., $\nu=\frac{1}{3}$). The progression and the physical picture underlying the hierarchies vary among the theories. Two of the theories—Laughlin's and Tao's—are able to predict the activation energies for the $\frac{2}{5}$ and $\frac{3}{5}$ effects. Laughlin's theory is suggestive of a picture in which fractionally charged quasiparticles from a parent state recondense into a higher-order, correlated fluid ground state—a picture which also best describes Haldane's and Halperin's theories. Tao's theory is based on the idea of a discrete broken symmetry. Laughlin predicts an activation energy of about 2.5 K (0.215 meV) for the $\frac{2}{5}$ and $\frac{3}{5}$ effects at 200 kG, whereas Tao predicts an energy of 20 K

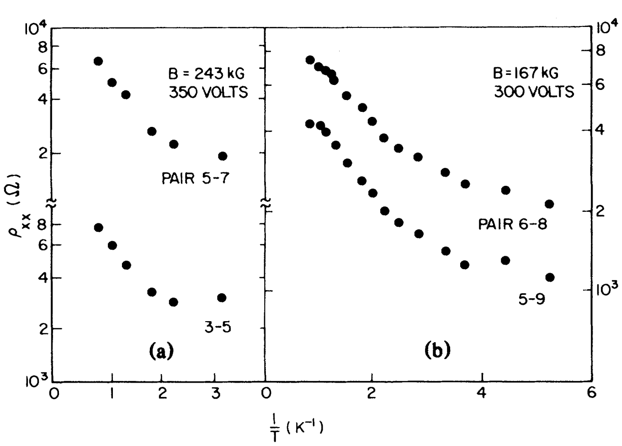

FIG. 2. Semilog plot of ρ_{xx} vs inverse temperature: (a) $\frac{2}{5}$ effect at 243 kG, (b) $\frac{3}{5}$ effect at 167 kG.

(1.72 meV). Our result of around 1 K (0.086 meV) is in much better agreement with Laughlin's theory.

The $\frac{3}{7}$ and $\frac{4}{7}$ effects are visible in both ρ_{xx} and ρ_{xy}. Weak structures in ρ_{xy} accompany the unmistakable minima in ρ_{xx}. The presence of these effects is consistent with all of the present theories.[4-9]

Mendez *et al.*[12] recently reported their observation of the $\frac{1}{5}$ effect. In Fig. 3, we present our results near $\frac{1}{5}$ from a sample with density $n=1.25\times10^{11}$ cm^{-2} at a temperature of 90 mK. The data are shown for two different contact pairs on the sample. A broad structure is visible around $\frac{1}{5}$ (250 kG), on top of a large sloping background. Since at high magnetic fields and low temperatures, sample contacts are a major concern in low-density samples, we have indicated in the figure the phase shift in our signals at 280 kG measured at a frequency of 11 Hz. The relatively small phase shifts show that the contacts are acceptable and the data trustworthy.

The weakness of the $\frac{1}{5}$ effect is likely a result of localization effects. According to theoretical predictions for an ideal system with no disorder, the gap energy of the $\frac{1}{5}$ effect should be about 35% of that of the $\frac{1}{3}$ effect at similar magnetic fields.[4] Using results from previous work,[2,11] we expect a value of about 1 K (0.086 meV) for the $\frac{1}{5}$ gap. The discrepancy with experiment may be reconciled within a phenomenological picture of an impurity-broadened lowest Landau band,[3] in which a $\frac{1}{5}$ filling corresponds to filling the band at the tail region where localization effects are strong. Significant reduction of the gap value due to disorder has been

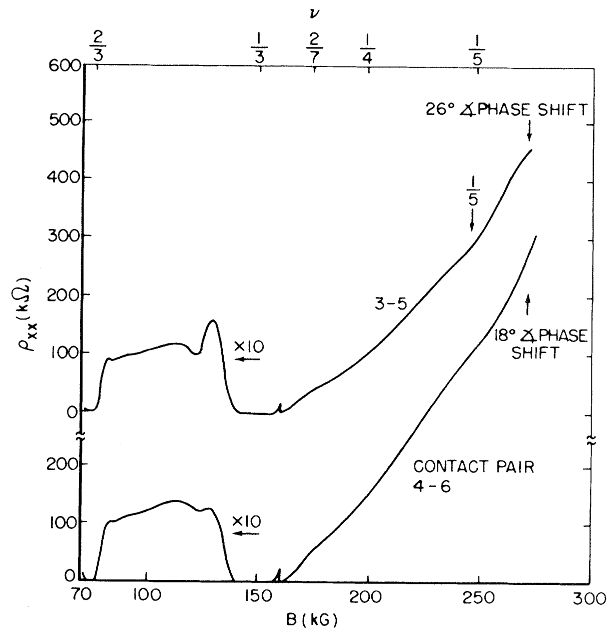

FIG. 3. Magnetic field traces of ρ_{xx} at 90 mK for a sample of density 1.25×10^{11} cm^{-2}. The two curves correspond to different contact pairs on the same sample. A broad structure is seen near $\frac{1}{5}$ filling factor on top of a large, sloping background. These results provide added evidence for the existence of the $\frac{1}{5}$ effect.

observed for the $\frac{2}{3}$ effect[3] and is likely to occur here also. This line of argument indicates that in the still higher-denominator series—$\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, etc.—the energy gap will suffer even greater reduction. This, rather than crystallization into a Wigner lattice,[13,14] may be the reason why they were not observed in a recent experiment.[12] Because of the approximations involved in the theoretical calculations and because of the presence of disorder, the filling factor below which Wigner crystallization will take place is still an open question.

In summary, we have established firmly the existence of the higher-order states, p, in the multiple series q, in the FQHE. In particular, the quantization of the $\frac{2}{5}$ Hall resistance is accurate to 2.3 parts in 10^4 and $\frac{3}{5}$ to 1.3 parts in 10^3. The development of the thermally activated transport at these fractions indicates that they are akin in nature to the $\frac{1}{3}$ and $\frac{2}{3}$ states. In agreement with Mendez *et al.*,[12] we have observed a weak, broad structure in ρ_{xx} about the $\frac{1}{5}$ filling factor. The weakness of the $\frac{1}{5}$ structure may be a consequence of disorder. Our discovery of the $\frac{3}{7}$, $\frac{4}{7}$, $\frac{4}{9}$, and $\frac{5}{9}$ effects lend strong support to the conjecture put forth in Ref. 2 that the FQHE occurs in multiple series. The new data indicate that there may be a great richness in the region of $\frac{1}{2}$ filling factor. A progression according to the formulas $m/(2m+1)$ and $1-m/(2m+1)$, where m is a positive integer, appears to be emerging.

We thank Greg Boebinger for technical assistance. This work was supported in part by the National Science Foundation through Grant No. DMR-8212167.

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