Neutrinoless Double Beta Decay between Pairs of Single-Beta Emitters

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The rate of neutrinoless double beta decay between pairs of single-beta emitters is computed. The ejection of electrons with a kinetic energy higher than Q_β is the experimental signature for this phenomenon. The probability of $\beta\beta_{0\nu}$ occurrence is proportional to the $\frac{4}{3}$ power of the volume of the radioactive source.

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It is widely accepted that the phenomenon of double beta decay ($\beta\beta$) is the best-suited laboratory for testing the principle of conservation of the lepton number[1] (L): Should the neutrino be a Dirac fermion ($\nu_e \neq \bar{\nu}_e$), the mode with two neutrinos ($\beta\beta_{2\nu}$) is the only way of disintegration. But, if the neutrinos are Majorana particles ($\nu_e = \bar{\nu}_e$), i.e., L is not conserved, the neutrinoless mode ($\beta\beta_{0\nu}$) is also possible, and in these cases the two ejected electrons would carry all the energy released in the process.

Double beta decay has been experimentally checked by geochemical analysis of certain primigenic rocks,[2] and a direct cloud-chamber experiment also exists with positive results,[3] although in all of them, unfortunately, a universal consensus about their validity or quantitative interpretation is missing. In fact, the possibility of extracting strong conclusions about the existence, or not, of the $\beta\beta_{0\nu}$ process is quite small; because even in performing analysis *à la* Pontecorvo of the tellurium ratio, the uncertainties both experimental and theoretical (especially in connection with the computing of the nuclear matrix elements)[4,5] are too great, and therefore it is plausible that the situation will not settle soon.

On the other hand, the putative existence of the neutrinoless mode, apart from the Majorana nature of the electronic neutrino, would imply the existence of explicit impurities in the chirality of the weak leptonic current, and/or a nonvanishing value for the neutrino mass[6] ($\mu \neq 0$). This second consequence provokes, even more, the general curiosity about the $\beta\beta_{0\nu}$ phenomenon, because in the case that it exists, it could unravel one of the great and urgent questions of modern physics.[7]

The basic difficulty that lies in any $\beta\beta$ experiment comes from the largeness of their half-lives: for $\beta^-\beta^-$ processes, in a $0^+ \rightarrow 0^+$ transition, $T_{1/2} \geq 10^{20}$ yr,[8] and in a $0^+ \rightarrow 2^+$ transition, $T_{1/2}$ is probably larger than 10^{24} yr.[9] And in the analogous $\beta^+\beta^+$, $K\beta^+$, and KK, the advantage of having positrons in the final state is unfortunately compensated by an enlargement of half-lives up to $T_{1/2} \geq 10^{25}$ yr.[8] These time scales are so long for three reasons: (a) $\beta\beta$ decays are a second-order weak process. (b) There exist often dynamic inhibitions[10] (for example, with Ca^{40}). (c) The $\beta\beta$ emitters are not specially favored with respect to phase space.

From the theoretical point of view, it is risky to rely too heavily on the accuracy of the present values of the nuclear matrix elements,[4] because as A and Z correspond to nuclides located far from the comfortable areas—in the sense of nuclear physics computation—their computation implies the solution of a rather complicated many-body problem. In this sense, the discrepancy is remarkable between the theoretical results, obtained by using the state of the art shell model, and the geochemical results.[5]

In view of the situation it is, I think, justified to look for experimental or theoretical alternatives which may clarify the whole body—or at least a part—of the present difficulties. In that spirit suppose we have a radioactive β^- sample, of a given Q. Our question is: What is the probability of emission of electrons with a kinetic energy higher than Q? These events would come only from the occurrence of neutrinoless double beta decays between the different pairs of nuclei in the source. This idea has never been considered, and our purpose here is to analyze its possibilities as a competitor with the usual case, i.e., when single nuclei are the emitters.

One realizes that proceeding in that way, the $\beta\beta$ process may happen only in the neutrinoless mode. That is an advantageous difference with the conventional case. Should, on the contrary, the radioactive source correspond to a two-step single β^- emission,

$$\begin{aligned} X_Z^A \rightarrow\ & X_{Z+1}^A + e^- + \bar{\nu}_e \\ & \hookrightarrow X_{Z+2}^A + e^- + \bar{\nu}_e, \end{aligned} \qquad (1)$$

the occurrence of a double beta process could be $(\beta^-\beta^-)_{2\nu}$ or $(\beta^-\beta^-)_{0\nu}$. So let us suppose that we have a couple of nuclei A, which are single-β^- emitters, with an energy release equal to Q:

$$A \to A' + e^- + \bar{\nu}_e, \quad E_A = E_{A'} + m + Q,$$
$$\Delta = E_A - E_{A'} = Q + m. \tag{2}$$

The process $(\beta^-\beta^-)_{0\nu}$ which we are referring to consists, in the emission by one of them, of a real electron and a virtual Majorana neutrino which reaches the second nucleus, whereupon this one emits the second electron, the decay proceeding in a covariant Feynman diagram:

$$(A_1, A_2) \to (A_1', A_2') + e^- + e^-,$$
$$T_0 = 2Q, \quad \tilde{T}_0 = T_0/m. \tag{3}$$

If we denote by $P_1 = (E_1, \vec{P}_1)$ and $P_2 = (E_2, \vec{P}_2)$ the four-momenta of both electrons, and set $E_1 + E_2 = T_0 + 2m$ and $G_\beta = G_F \cos\theta_c$, the S-matrix element is

$$S_{if} = \int d^4x\, d^4y\, \tfrac{1}{2} G_\beta^2\, {}_1\langle A'|J^\mu(x)|A\rangle_{1\,2}\langle A'|J^\nu(y)|A\rangle_2 \langle \phi_{P_1}, \phi_{P_2}|T[j_\mu(x) j_\nu(y)]|0\rangle - (P_1 \leftrightarrow P_2). \tag{4}$$

$|A\rangle$ and $|A'\rangle$ are nuclear states spatially anchored at positions $\vec{x}$ and $\vec{y}$. For simplicity we assume they are $J^\pi = 0^+$. $|\phi_{P_1}$ and $|\phi_{P_2}\rangle$ are electronic states, and J^μ and j^μ are the hadronic and leptonic weak current operators, respectively. j^μ is assumed to be a pure $V-A$ current. Setting $q = (q^0, \vec{q})$ the four-momentum carried by the virtual neutrino, we would have

$$S_{if} = \int dx^0\, dy^0 \frac{d^4q}{(2\pi)^4} \frac{G_\beta^2}{2} |\langle A'|\textstyle\sum_n \tau_n^t|A\rangle|^2 [(2\pi)^6 4E_1E_2]^{-1/2}$$
$$\times \exp[ix^0(E_{A'} + q^0 + E_1 - E_A)] \exp[iy^0(E_{A'} + E_2 - q^0 - E_A)] \exp i\vec{q}\cdot(\vec{y} - \vec{x})$$
$$\times [\mathcal{F}(E_1,Z)\mathcal{F}(E_2,Z)]^{1/2} \frac{\bar{u}_1\gamma_0(1-\gamma_5)(\not{q} + \mu)\gamma_0 C\bar{u}_2^T}{q_0^2 - \vec{q}^2 - \mu^2} - (P_1 \leftrightarrow P_2). \tag{5}$$

$\mathcal{F}(E,Z)$ are the usual Coulomb distortion factors. With the notation

$$\langle A'|\textstyle\sum_n \tau_n^+|A\rangle = \int 1, \quad \bar{u}_1\gamma_0(1-\gamma_5)(\not{q}+\mu)\gamma_0 C\bar{u}_2^T = \mathcal{L}(1,2), \quad \mathcal{F}_i(E_i,Z) = \mathcal{F}_i, \tag{6}$$

and denoting the total energies by E_i and E_f we have,

$$S_{if} = \frac{G_\beta^2}{2}\left(\int 1\right)^2 \frac{\delta(E_i - E_f)}{2(2\pi)^5\sqrt{E_1E_2}} \sqrt{\mathcal{F}_1\mathcal{F}_2}\, d^3q \frac{\exp[i\vec{q}\cdot(\vec{x}-\vec{y})]}{-\vec{q}^2 - \mu^2 + q_0^2}\{\mathcal{L}(1,2) - \mathcal{L}(2,1)\}, \quad q^0 = \Delta - E_1. \tag{7}$$

As $|q^0|$ is usually much bigger than μ, because we are considering a light Majorana neutrino, μ^2 can be neglected in the denominator under the integral; and setting $D = |\vec{x} - \vec{y}|$ gives

$$S_{if} = \frac{G_\beta^2 (\int 1)^2 \delta(E_i - E_f)}{2^6\pi^3\sqrt{E_1E_2}} \sqrt{\mathcal{F}_1\mathcal{F}_2} \frac{\cos(q^0D)}{D} \{\mathcal{L}(1,2) - L\ (2,1)\}, \tag{8}$$

and therefore, in the usual approximation for $\mathcal{F}$, and averaging

$$\langle \cos^2\theta\rangle = \tfrac{1}{2}, \quad \mathcal{F}_i = (E_i/|\bar{p}_i|)H,$$
$$H = 2\pi\alpha Z(1 - e^{-2\pi\alpha Z})^{-1}, \tag{9}$$

$$\Gamma = \frac{G_\beta^4\mu^2}{2(2\pi)^5} \frac{(\int 1)^4 H^2 m^5 f_\mu}{D^2}, \tag{10}$$

$$f_\mu = \tfrac{1}{15}\tilde{T}_0[\tilde{T}_0^4 + 10\tilde{T}_0^3 + 40\tilde{T}_0^2 + 60\tilde{T}_0 + 30].$$

As commented on before, the signature of these events consists in the production of electrons with a kinetic energy higher than Q $[Q + m < E < 2(Q + m)]$. As it is not necessary to measure accurately the electron spectrum, but only to count the number of electrons fulfilling that condition, the radioactive source may be rather thick, and slight losses of energy would not affect the clarity of the event.

Additionally to the advantage referred before of not having $(\beta^-\beta^-)_{2\nu}$ events, but only neutrinoless decay, we remark that (10) is exactly known from the ft of the single β^- decay:

$$\left|\int 1\right|^2 = 2\ln 2\, \pi^3/G_\beta^2 m^5 ft, \tag{11}$$

$$\Gamma = \frac{(\ln 2)^2\mu^2\pi H^2 f_\mu}{16D^2m^5(ft)^2}, \tag{12}$$

and therefore there would be no uncertainty to extract μ from the experimental value of Γ. On the other hand, the possibility of choosing between all the β^--unstable nuclides may provide reasonable values of ft and Q to enhance Γ with respect to conventional emitters.

So far we have silenced the two difficulties of this idea. The first one is the large value of D in the denominator of (12). This fact makes Γ considerably smaller than its usual values, where $D=d$ is the distance between two nucleons (quarks) inside a nucleus (a Δ resonance). However, notice that in the usual case the number of $\beta\beta$ events is proportional to the number N of emitters,

$$n_{\beta\beta}=\Gamma_{\beta\beta}N. \tag{13}$$

Here, it is proportional to the number of pairs,

$$n=\Gamma N^2/2\propto D^6/D^2=D^4. \tag{14}$$

Assuming that Γ and $\Gamma_{\beta\beta}$ were equal in the other features, except in the value of D, we have

$$f=\frac{n}{n_{\beta\beta}}=\frac{N}{2}\left(\frac{d}{D}\right)^2; \tag{15}$$

taking $d\simeq 10$ fm gives

$$N=\frac{N_A\rho V}{A}\simeq\frac{N_A\rho}{A}(2D)^3=\frac{8N_A\rho D^3}{A}, \tag{16}$$

$$f=\frac{8N_A\rho d^2 D}{A}\simeq\frac{\rho(\mathrm{g\,cm^{-3}})D(\mathrm{cm})}{A} \tag{17}$$

Hence, in spite of the initial disadvantage with respect to I f may be bigger than 1 if D is large enough.

From the experimental point of view, the electrons coming from ordinary β^- decays are a true serious difficulty. As (17) imposes the use of large-volume sources, where most of the electrons would be self-absorbed, it is necessary to find specific signals coming only from $(\beta^-\beta^-)_{0\nu}$ events. If, for example, one is using a nuclide with 1.022 keV $> Q >$ 0.511 keV, ordinary electrons would be completely unable to make e^--e^- pairs; on the contrary a part of the fast electrons from $(\beta^-\beta^-)_{0\nu}$ decay could, by bremsstrahlung, materialize in pairs, and the two photons from the positron annihilation be detected. That would be an example of a specific $(\beta^-\beta^-)_{0\nu}$ signature.

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