

Electromagnetic Interactions in the σ - ω Model

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The predictions of the σ - ω model for electromagnetic current are explored and found to be quite different from those given by the impulse approximation. In particular, large variations in the quasielastic electron scattering cross section are found depending upon the choice of the operator used for the current.

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In nuclear physics the nucleons are usually treated as elementary particles. That is, one does not attempt to solve the more fundamental problem of understanding the origins of the structure of the nucleon, but rather uses its experimentally measured properties. In employing such a procedure, one is implicitly assuming that the nuclear medium does not affect the properties of the nucleon. This impulse approximation also leads to an ambiguity: How one extrapolates from the nucleon structure measured on shell to the off-shell values required in the nucleus is not unique.

Generally for loosely bound systems, like the nucleus, one expects these difficulties to be minor since the nucleons are kinematically almost on shell. Indeed, in a recent study¹ it was found that for the electron-nucleon cross section this off-shell ambiguity is quite small provided one employs the restrictions which follow from QED and gauge invariance. These considerations reinforce the notion that the electromagnetic interaction is very well known and thus provides an extremely clean method of probing nuclear structure.

The validity of the impulse approximation itself depends upon how the nucleons interact. In particular, in the σ - ω model (which gives a schematic representation of the more general Dirac-based relativistic theories) of nuclear dynamics, which has become quite popular in the last few years,^{2,3} one can expect that the nucleon's structure in the nucleus will deviate strongly from that in free space. In this Letter the consequences of this model for the nucleon current are discussed. Preliminary results from this investigation have been presented elsewhere.^{4,5}

In the σ - ω model the average force on the nucleon is assumed to arise from a strong (several hundred megaelectronvolts) attractive scalar field, V_s , and a repulsive vector field, V_0 , which is almost as strong. The effect of these fields can be included by modifying the mass and energy terms in the

Dirac equation,

$$m \rightarrow m^* = m + V_s, \quad (1)$$

$$E \rightarrow E^* = E - V_0. \quad (2)$$

In the nonrelativistic reduction of the Dirac to the Schrödinger equation the mass and energy appear in the combination $(m - E) \rightarrow (m^* - E^*) = (m - E) + (V_s + V_0)$, and the relatively weak central optical potential one observes experimentally is explained as being due to a strong cancellation between these fields. In the small components of the Dirac spinors, by contrast, the mass and energy occur in the combination $m + E$. The potentials V_s and V_0 thus enter in the combination $V_s - V_0$, and contribute constructively. For spatially constant potentials one has

$$u(\vec{p}) \rightarrow u^*(\vec{p}) = \left(\frac{E^* + m^*}{2E^*} \right)^{1/2} \left(\frac{\vec{\sigma}^1 \cdot \vec{p}}{E^* + m^*} \right) \chi. \quad (3)$$

The resulting effect is very large: The small components attain roughly twice their free-space values. Among other things, this leads to a strong increase in the spin-orbit potential, which was one of the main reasons for considering the model in the first place.⁶

From the nonrelativistic point of view, where the fields only enter in the combination $V_s + V_0$, this result is quite puzzling, and this is one of the aspects of this model which makes it so intriguing. One can now have weak binding, i.e., nucleons that are kinematically almost on shell, but their properties are clearly very different from those for free particles. In short one cannot expect the impulse approximation to be very good, even though kinematically it would appear that it should be. We note that these effects are not a consequence of the relativistic nature of the model. The Dirac equation does, however, provide a convenient way to represent them. In the "nonrelativistic," i.e., two-component spinor, approach, the effects of the

small components are also included, but by using the free-space relationship to the large components. In the σ - ω model this is obviously a poor first approximation. As a result one obtains very large, $\sim (V_s - V_0)$, higher-order corrections in the coupling to the negative-energy states⁷ and the normal nonrelativistic expansions are quite poor.⁸

In the present investigation we are primarily interested in the qualitative question of how strongly such a model would modify the impulse approximation and thus our standard "well-known" picture of the electromagnetic interactions of nucleons. We therefore do not consider the effect of the model on the nuclear structure, i.e., the nuclear wave function itself. It is considered that these aspects have already been included in the phenomenology developed to describe nuclear structure. This is in contrast to other treatments of electromagnetic interactions in this model.⁹⁻¹⁴ For similar reasons, and also in order to display how the scalar and vector potentials affect the nucleon current analytically, we take the potentials to be spatially constant (we use the values $V_s = -420$ MeV and $V_0 = 328$ MeV¹⁵). The nucleon current can then be obtained simply by replacing the appropriate masses and energies by their effective values in the expression for the free current.

The σ - ω model itself provides only one of the ingredients necessary to obtain the current. In addition to the spinors, one must know the operator that is sandwiched between them. In treating the nucleons as elementary particles, one takes the operator to be the same as that for free nucleons. This, however, gives rise to an ambiguity: Although the form of the free-nucleon current is completely determined by invariance principles, the corresponding operator is not. By use of the Gordon decomposition,¹⁶ i.e., applying the Dirac equation, the operator can be expressed in different forms. The Dirac structure of these forms, i.e., the coupling between the large and small components, however, is quite different. Thus one can expect large ambiguities in the predictions for the current when one uses the spinors of the σ - ω model. The two most commonly encountered forms for the nucleon current are^{1, 16, 17}

$$j_\mu = j_\mu^D + j_\mu^a, \quad (4)$$

and

$$j_\mu = j_\mu^D (1 + 2mF_2/F_1) - j_\mu^c 2mF_2/F_1, \quad (5)$$

where j_μ^D , j_μ^c , and j_μ^a are the Dirac, convection, and anomalous magnetic currents, respectively. These are given, upon extrapolation to the σ - ω model, by

$$[p_\mu^* = (\vec{p}, iE^*), \quad q_\mu = p'_\mu - p_\mu = p_\mu^{*'} - p_\mu^*]$$

$$j_\mu^D = i\bar{u}^*(\vec{p}')\gamma_\mu F_1 u^*(\vec{p}), \quad (6)$$

$$j_\mu^c = \bar{u}^*(\vec{p}') (p_\mu^{*'} + p_\mu^*)/2m^* F_1 u^*(\vec{p}), \quad (7)$$

$$j_\mu^a = -i\bar{u}^*(\vec{p}')\sigma_{\mu\nu} q_\nu F_2 u^*(\vec{p}). \quad (8)$$

All these currents are conserved. We note, in particular, that the use of effective energies and masses in j_μ^c is required in order that the current be conserved and the charge properly normalized. By use of the Gordon decomposition the term proportional to F_2 in the coefficient of the Dirac current in (5) can be reexpressed as the sum of a convection and a magnetic current. The former cancels the last term on the right-hand side, and the latter gives the anomalous-magnetic-moment contribution which now, in comparison to (4), is enhanced by a factor of m/m^* , i.e., $j_\mu = j_\mu^D + (m/m^*)j_\mu^a$. This prescription can also be obtained in another way. The more commonly used definition of F_2 differs from the present one by a factor $\kappa/2m$. If one assumes that this factor m should become m^* ,^{10, 12} one obtains the same result but starting from (4). However, since the presence of such a factor m is quite arbitrary, this procedure is not usually followed.

In order to compare the different prescriptions for the nucleon current we consider the quasielastic electron-scattering cross section. The advantage of this process is that it is almost independent of any details of the nuclear structure and thus the magnitude of the cross section directly reflects that of the nucleon current. The model used for the response function has been applied previously and is described in de Forest¹⁸ and Zimmerman *et al.*¹⁹

The results for quasielastic scattering on ¹²C at two different scattering angles, but approximately the same momentum transfer, are shown in Figs. 1 and 2. It can be seen that the two prescriptions (4) and (5) give very different predictions, and that those given by (4) lie much closer to that of the impulse approximation. Though the last two are very similar, it is interesting to note that the predictions within the framework of the σ - ω model agree better with experiment. This corresponds to a decrease in the Coulomb part and an increase in the transverse part of the electromagnetic interaction.

Empirically it is clear that (5) does not provide a satisfactory current. The same conclusion could have also been drawn from the calculation of nuclear magnetic moments. One could therefore reject prescription (5) on empirical grounds if one had to choose between (4) and (5). But many different operators are possible, for example, linear

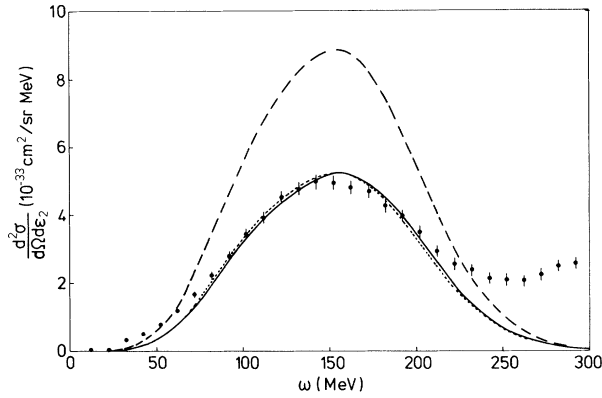


FIG. 1. The cross section for electron scattering on ^{12}C at 560 MeV and 60° (Ref. 20) compared with various theoretical predictions. The solid line is the result of the impulse approximation. The dotted and dashed lines are calculated with the σ - ω model using the currents (4) and (5), respectively.

combinations of (4) and (5). We therefore still consider (5) in order to provide an indication of the range of results that are possible.

The origin of the difference in these predictions can be seen by examining the expressions for the Coulomb and transverse currents. For prescription (4) one need only replace all masses and energies in the free case (see, e.g., Ref. 1) by their effective values, and one finds

$$|\rho|^2 = [(E^* + E^*)^2 (F_1^2 + q_\mu^2 F_2^2) - \vec{q}^2 (F_1 + 2m^* F_2)^2] / 4E^* E^*, \quad (9)$$

$$|\vec{j}_T|^2 = [\vec{p}_T^2 (F_1^2 + q_\mu^2 F_2^2) + q_\mu^2 (F_1 + 2m^* F_2)^2] / 2E^* E^*. \quad (10)$$

For prescription (5), in addition, F_2 is multiplied by a factor m/m^* . To first approximation the change in the (effective) mass and in the energy in the σ - ω model are about equal. Although the magnitude of V_s is larger than that of V_0 , for the latter one has the additional effect that in the σ - ω model one should use the single-particle, rather than the free, energy. Thus in (9) all terms in the denominator and numerator, except that proportional to F_1 in the last term, decrease by about the same amount. The net result is a decrease in the Coulomb interaction. Similar arguments show that the transverse interaction (10) increases. Compared with (4), prescription (5) predicts a much larger effective anomalous magnetic moment. This primarily increases the transverse interaction and accounts for the very large cross sections. Since the spinors in the σ - ω model differ strongly from free ones, the

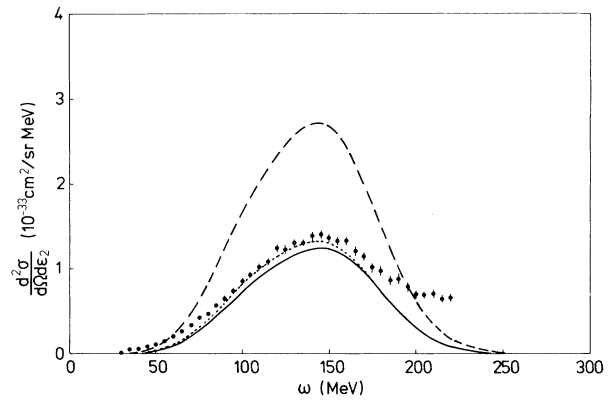


FIG. 2. Same as Fig. 1, but for 320-MeV electrons and a scattering angle of 145° . The momentum transfer is approximately the same as in Fig. 1.

choice of which operator to use is crucial. Similar remarks have been made recently by Cooper, Gattone, and Macfarlane²¹ concerning the difference between pseudoscalar and pseudovector pion-nucleon coupling in this model.

In conclusion, we note that the σ - ω model gives one the latitude for explaining the anomalously small observed Coulomb cross sections.^{11, 22, 23} Further investigations of the structure of the nucleon within the nuclear medium, however, will be required to provide a truly satisfactory solution to this problem.

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