Phase Shifts of the Skyrmion Breathing Mode

J. D. Breit
The Institute for Advanced Study, Princeton, New Jersey 08540

and

Chiara R. Nappi
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 7 May 1984)

Phase shifts for the skyrmion breathing mode are calculated and indications of a resonance in this channel are found. We identify this resonance with the $N(1440)$ and $\Delta(1600)$.

PACS numbers: 11.10.Lm, 11.10.Ef, 14.20.Gk

The identification of the solitons of the Skyrme model[1] with baryons in the large-N limit of QCD[2] has enjoyed some success in describing the nucleon and delta.[3–5] One is naturally led to question just how far the Skyrme effective Lagrangian can be pushed; in particular, whether it can be used to predict the properties of other baryon resonances. To study this question we look at the simplest vibrational excitation of the skyrmion, the breathing mode. This excitation has been studied by a number of authors,[6] all of whom use essentially variational methods. Because there is no variational principle for finding the masses of unstable resonances, we prefer to use a different method. We expand the Lagrangian to leading order in the semiclassical approximation to find the phase shifts of the vibrational modes. The energy at which the phase shift passes through 90° is then identified with the mass of a resonance.

We realize that this procedure is sensitive to the form of the effective Lagrangian we use. Because resonances occur at fairly large pion energies, one could include terms involving arbitrary numbers of derivatives of the pion field with arbitrary coefficients, thereby obtaining whatever phase shifts are desired. There is, however, the possibility that a simple effective Lagrangian could reproduce the spectrum of baryon resonances, and it is this possibility that we wish to investigate.

We use the Lagrangian

$$L=\tfrac{1}{16}F_\pi^2\,\mathrm{tr}(\partial_\mu U\partial^\mu U^\dagger)+\tfrac{1}{32}e^{-2}\,\mathrm{tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\}+\tfrac{1}{8}m_\pi^2F_\pi^2(\mathrm{tr}\,U-2),$$

where U is an SU(2) matrix and in the large-N limit F_π^2 and $1/e^2$ are both of order N.

To find the breathing mode we let

$$U=\exp[iF(r)\hat{x}\cdot\vec{\tau}+ie\delta F(r,t)\hat{x}\cdot\vec{\tau}],$$

where F is the classical Skyrme solution and the τ_i are the usual Pauli matrices. Note that these fluctuations in F about the classical solution are orthogonal to the collective rotations and translations to leading order. Furthermore, because the classical solution is invariant under rotations in spin plus isospin the vibrational modes can be divided into multiplets of $\vec{J}+\vec{I}$. This mode transforms as a singlet and is orthogonal to the higher vibrational multiplets. So use of this *Ansatz* guarantees that we find fluctuations that diagonalize the small-oscillation Hamiltonian. We expand the Lagrangian up to second order in δF; high powers of δF are suppressed by powers of $N^{-1/2}$. Note that this procedure is simply the usual semiclassical approximation with $1/N$ the expansion parameter instead of $\hbar$. We ignore the rotational degrees of freedom since they are $1/N$ effects. The equation of motion that results is

$$0=(\tfrac{1}{4}r^2+2s^2)\delta F''+(\tfrac{1}{2}r+4scF')\delta F'+[\tfrac{1}{4}\omega^2r^2+2s^2\omega^2-\tfrac{1}{2}c^2-s^2+4scF''+2(c^2-s^2)(F')^2$$
$$+2(c^2-s^2)s^2/r^2-4s^2c^2/r^2-\tfrac{1}{4}\beta^2r^2c]\delta F,$$

where $s=\sin F$, $c=\cos F$, $\beta=m_\pi/eF_\pi$, and we have scaled r by eF_π and given δF the time dependence $e^{-ieF_\pi\omega t}$.

We solve this equation subject to the boundary conditions $\delta F=0$ at $r=0$ and $r=\infty$ in order to ensure that the baryon number is unchanged. Near the origin $\delta F\sim r$, while as $r\to\infty$, $\delta F\to aj_1\times(kr)+bn_1(kr)$, where j_1 and n_1 are the usual spherical Bessel functions of order 1 and $k=(\omega^2-m_\pi^2)^{1/2}$. We solve the equation by

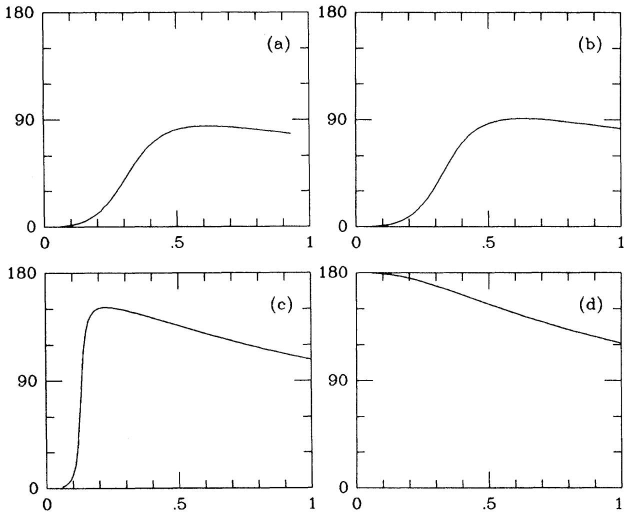

FIG. 1. Phase shift δ (in degrees) vs pion momentum k (in units of eF_π) for (a) $\beta = m_\pi/eF_\pi = 0$, (b) $\beta = 0.263$, (c) $\beta = 0.75$, and (d) $\beta = 1.0$.

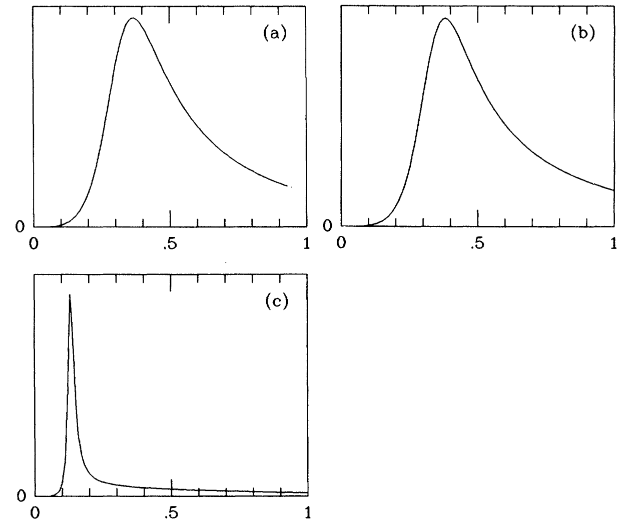

FIG. 2. Cross section σ (in arbitrary units) vs pion momentum k (in units of eF_π) for (a) $\beta = m_\pi/eF_\pi = 0$, (b) $\beta = 0.263$, and (c) $\beta = 0.75$.

integrating numerically from the origin out to $r = 50$ for $\beta = 0$, $r = 25$ for $\beta = 0.263$, $r = 15$ for $\beta = 0.75$, and $r = 15$ for $\beta = 1.0$. At those values of r, $F(r)$ is negligible and $\delta F(r)$ approaches its asymptotic form. We then fit δF for r large to $a(k)j_1(kr) + b(k)n_1(kr)$ and find the phase shift

$$\delta(k) = \tan^{-1}[-b(k)/a(k)].$$

The results are plotted in Fig. 1 for all four values of β. Note that for $\beta = 0.263$ we have $m_\pi = 139$ MeV if we take the values of e and F_π used in Ref. 5 to fit the nucleon and delta masses. We include the other values for β because this fit is not necessarily the best overall when resonances are included and we wish to investigate at what value of β a bound state forms.

As we can see from the graphs, for $\beta = 0$ the phase shift rises rapidly from zero around $k = 0.3$, levels off at about 85°, and then slowly declines as k increases. For $\beta = 0.263$, the phase shift rises rapidly around $k = 0.35$ and reaches a maximum value of 91°. For $\beta = 0.75$ the phase shift rises quite rapidly through 90°. For these values of β, $\delta(0) = 0$, and so, by Levinson's theorem, there is no bound state. For $\beta = 1.0$, however, the phase shift starts at 180° and gradually declines, indicating that for very massive pions a bound state forms in this channel. In the discussion that follows, we concentrate on $\beta = 0.263$ since that value corresponds to the physical pion mass. In this case δ passes through 90°, but it does not look like a classical narrow resonance, in which δ rises rapidly through 90° and levels off at about 180°. Nevertheless, if we take δ's passing through 90° as the signal of a resonance, one occurs at $\omega \approx 0.63 eF_\pi$. Using the values for e and F_π given in Ref. 5, we expect a resonance at a pion energy of 330 MeV. Because this excitation has the same symmetries as the ground-state skyrmion and we are neglecting rotational effects, we identify this resonance with a nucleon breathing mode at 1270 MeV and a Δ breathing mode at 1560 MeV. Experimentally we have the Roper resonance at about 1440 MeV and a Δ at 1600.

We should point out that because the phase shift does not rise rapidly through 90°, the identification of the mass of the resonance is somewhat ambiguous. The contribution of this channel to the cross section is proportional to $\sin^2\delta/k^2$, which is plotted in Fig. 2 for $\beta = 0$, 0.263, and 0.75. If we take the resonance mass to be the energy at which the cross section peaks, using the values of e and F_π of Refs. 3 and 5, we have a nucleon at about 1200 MeV and a Δ at about 1500 for both $\beta = 0$ and $\beta = 0.263$. The width of the peak at half maximum is $\sim$100–200 MeV. For $\beta = 0.75$ the resonance becomes quite sharp.

The masses we obtain are rather too small, but this is not unexpected since they scale with F_π which is also too small in the fits to the nucleon and Δ of Refs. 3 and 5. (The variational calculations of Ref. 6 also give too small a mass splitting.) More disturbing is the failure of the phase shift to rise above 91° for $m_\pi = 139$ MeV since the experimental phase shifts for the P_{11} resonance rise quite rapidly up to about 180°. This, however, may be an artifact of the low value of β forced on us by the fit of Ref. 5. When other resonances are found, we may find that a higher value of β gives the best overall fit, which would certainly improve this resonance. Note, however, that one cannot allow β to become too large, since eventually a stable bound state forms. Our choice of Lagrangian is also somewhat arbitrary, and more realistic models of pion physics, as for instance introduction of the vector mesons,[7] may improve matters. We are investigating this possibility as well as looking for resonances in other channels.

This work was supported in part by the Department of Energy through Grant No. DE-AC02-76ER022220 and in part by the National Science Foundation through Grant No. PHY80-19754.

[1]T. H. R. Skyrme, Proc. Roy. Soc. London, Ser. A **260**, 127 (1961); N. K. Pak and H. C. Tze, Ann. Phys. (N.Y.) **117**, 164 (1979); J. Gibson and H. C. Tze, Nucl. Phys. B **183**, 524 (1981); A. P. Balachandran, V. P. Nair, S. G. Rajeev, and A. Stern, Phys. Rev. Lett. **49**, 1124 (1982), and Phys. Rev. D **27**, 1153 (1983).

[2]E. Witten, Nucl. Phys. **B223**, 422 (1983), and **B223**, 433 (1983).

[3]G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[4]M. Rho, A. S. Goldhaber, and G. E. Brown, Phys. Rev. Lett. **51**, 747 (1983); A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).

[5]G. Adkins and C. Nappi, Nucl. Phys. **B223**, 109 (1984).

[6]Ch. Hajduk and B. Schwesinger, to be published; J. Dey and J. LeTourneux, to be published; A. Hayashi and G. Holzwarth, to be published.

[7]G. Adkins and C. Nappi, Phys. Lett. **137B**, 251 (1984).