$\Upsilon(5S)$ Mass and $e^+e^- \to B\bar{B}, B\bar{B}^*, B^*\bar{B}^*$ as Sensitive Tests of the Unitarized Quark Model

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Because of hadronic mass shifts the $\Upsilon(5S)$-$\Upsilon(4S)$ mass splitting is predicted to be 80 MeV larger than in naive potential models. The behavior of ΔR is predicted to show several bumps not corresponding to resonances.

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As experimental data on the Υ resonances above the $B\bar{B}$ threshold are soon expected, it is of great interest to see what a detailed coupled-channel model predicts in this region. The importance of coupled-channel effects to $c\bar{c}$ and $b\bar{b}$ spectroscopy has been realized from the start,[1] but usually they are neglected because of their computational complexity. In two recent papers[2,3] the Υ and χ_b states below the $B\bar{B}$ threshold were studied using the unitarized quark model (UQM), which also has been successfully applied to the light mesons and baryons.[4] In this paper I extend the analysis to the $B\bar{B}$-threshold region in e^+e^- annihilation.

As shown in Ref. 2 the hadronic mass shifts due to

$$B\bar{B},\quad B\bar{B}^*+\bar{B}B^*,\quad B^*\bar{B}^*,\quad B_s\bar{B}_s,$$
$$B_s\bar{B}_s^*+\bar{B}_sB_s^*,\quad B_s^*\bar{B}_s^* \qquad (1)$$

vary from -30 MeV for $1S$ to -74 MeV for $4S$, but these mass shifts, although nonnegligible, can easily be absorbed into a conventional naive model by "renormalization" of the parameters of the potential. Therefore the Υ's below threshold do not alone provide a sensitive test of the coupled-channel effects, which of course must be present as a result of unitarity.

As one passes the most important thresholds the situation is, however, drastically changed since now the mass shifts behave very differently, becoming much smaller in magnitude and even changing sign. It is easy to see the reason for this effect, which is essentially model independent. The mass shift can be calculated from a dispersion-relation formula

$$\Delta M_{nm}(s=M^2_{\text{res}}) = -\frac{1}{\pi}\sum_{\text{thresholds}} \mathrm{P}\int \frac{\mathrm{Im}\Pi_{nm}(s')}{s-s'}ds', \qquad (2)$$

where n (m) is a resonance index. The absorptive part of the mass matrix, $\mathrm{Im}\Pi_{nm}(s)$, can be calculated for each threshold with a specific model such as the quark pair-creation model[5] for the hadronic vertices. This also provides a natural cutoff related to finite hadron size. As a result of positivity we have $-\mathrm{Im}\Pi_{nn}(s) \geq 0$. Therefore, when a resonance is below all thresholds the integrand is always >0. Consequently, when the off-diagonal terms are small, the mass shifts $\Delta M_{nn}<0$. The magnitude of the mass shift is expected to be roughly proportional to the inverse of the distance to the threshold. However, when the resonance lies above the threshold the situation changes and the continuum below the resonance contributes a positive mass shift in Eq. (2). Therefore $\Delta M(s)$ increases; cf. Figs. 1 and 2 of Ref. 2. For the Υ states this change is particularly abrupt, because all the thresholds lie relatively close to each other. In addition, the nodes in the Υ radial wave functions give rise to oscillations in $\mathrm{Im}\Pi$, which furthermore increase the rapid change from negative to positive mass shifts. Thus $\Delta M(5S)$ turns out to almost vanish (+3 MeV) compared to the -74 MeV mass shift of the $4S$. This is a large effect which cannot be absorbed into the parameters of the naive model. In Table I and in Fig. 1 we show the mass shifts for the $1S-6S$ states. Of particular interest is the anomalously high $5S$ mass, about 80 MeV heavier than in naive models. With a reasonable value for the $5S$ and $6S$ bare masses using naive models, the $5S$-$4S$ mass splitting is predicted to be anomalously large, 267

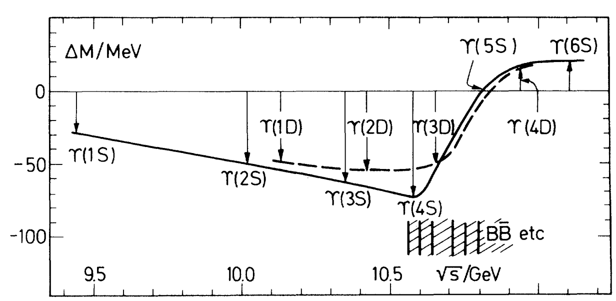

FIG. 1. The mass shifts of the S and D wave $b\bar{b}$ states as function of resonance mass. The curves are drawn to guide the eye for the NS mass shifts (solid curve) and ND mass shifts.

TABLE I. The predicted mass shifts and the predicted mass splittings for the physical and bare NS states. The bare mass splittings are fitted or predicted (in parentheses) using the potential Eq. (1.4) of Ref. 2. Note, in particular, the anomalously large value of the predicted physical $5S$-$4S$ splitting.

State	Predicted mass shift	Experimental (predicted) mass	Predicted mass splitting NS-$(N-1)S$	Bare mass splittings
$1S$	−30	9460		
$2S$	−51.9	10 023.4	563.4	585.3
$3S$	−64.0	10 355.5	332.1	344.2
$4S$	−74	10 573.0	217.5	227.5
$5S$	+3	(10 840)	(267)	(190)
$6S$	+15	(11 022)	(182)	(170)

MeV, larger than $4S$-$3S$ (217 MeV) and $6S$-$5S$ (182 MeV). Clearly the $5S$ mass provides a sensitive test of the UQM. This also confirms quantitatively the expectation[6] that resonance masses near thresholds are very sensitive to coupled-channel effects.

Preliminary data[7] indicate that the $5S$ mass is indeed anomalously high, such that the NS-$(N-1)S$ mass-splitting sequence would be 563, 332, 217, ≈ 300, ≈ 150 MeV. Any naive model would predict a monotonically decreasing sequence contradicting this observation.

The quark-loop diagrams which I have calculated should be distinguished from the perturbative quark loops which are included by having a running α_s in the potential.[8] As is well known, even $b\bar{b}$ spectroscopy is not very sensitive to the logarithmic dependence thereby introduced. Such corrections correspond to quark loops simply cut in the Q^2 variable. On the other hand, the loops $b\bar{b}$-$(b\bar{q}+\bar{b}q)$-$b\bar{b}$ give nearby singularities corresponding to nonperturbative effects, which I calculate phenomenologically using the $^3P_0$ model for hadron vertices. It is sometimes claimed that calculations of such loops are very model dependent, but in fact for $b\bar{b}$ at least, a nonrelativistic treatment should be a good approximation and the $^3P_0$ model has proven to be phenomenologically successful for hadron decays. Then, similar to the situation in nuclear physics, the unitarization is essentially unique.

The main uncertainty lies in the number of thresholds included. To include only $B\bar{B}$ is certainly not enough, since the $SU(6)_W$ weights of B^* channels are much larger. However, even then, some qualitative effects can be seen.[9] As is well known, experimentally and theoretically (e.g., dual models), multibody final states ($B\bar{B}\pi$, etc.) are saturated by two-body decays ($B\bar{B}^*$, etc.). The next important thresholds involve P-wave states, $B\bar{B}_P$. They are expected[10] to appear at ≈ 450-MeV higher energy or at 11.0 GeV. Below this energy they should contribute a smoothly increasing mass shift (cf. Fig. 1 below $B\bar{B}$), which can be absorbed into the bare potential parameters. Thus the $5S$ mass is unaffected while the $6S$ mass is expected to increase slightly if $B\bar{B}_P$ thresholds are included.

I have also calculated unitarity partial-wave amplitudes for $B\bar{B}$, $B\bar{B}^*$, . . . , including eleven resonances ($5S$ and $4D$). Previously Eichten[11] and Ono[10] have calculated some decay distributions for individual resonances without the background of the others. I show in Fig. 2 the predicted contributions to R from $B\bar{B}$, $B\bar{B}^*$, etc. Clearly these predictions are rather sensitive to the exact values of the B, B^*, B_s, and B_s^* masses and the details of the Υ wave functions used, in particular the node structure. I use the same wave functions, overlaps, and overall normalization γ_{QPC} as described in Ref. 1, and a B mass of 5272.5 MeV, $B^*-B=B_s^*-B_s=51$ MeV [estimated from $M^2(D^*)-M^2(D)=M^2(B^*)-M^2(B)$], and $B_s-B=B_s^*-B=F-D=81.5$ MeV. The couplings of the bare $b\bar{b}$ states to e^+e^- were chosen such that $\Gamma_{e^+e^-}(1S)=1.26$ keV and the ratios of the widths as predicted by data or naive models: $1S$:$2S$:$3S$:$4S$:$5S$ = 1.00:0.51:0.36:0.28:0.23. The D states up to $5D$ were also included with vanishing bare couplings to e^+e^-. The unitarization generates, however, nonvanishing couplings through mixing with the S states. This mixing turns out to be rather small. The physical ratio $\Gamma(3D)$:$\Gamma(1S)$ is only 0.02. Thus the $3D$ state (predicted at 10 650 MeV) contributes very little to ΔR. The predicted bump at 10.620 in Fig. 2 is due to the opening of $B\bar{B}^*$ and the nodes of the $4S$-$B\bar{B}$ and $4S$-$B\bar{B}^*$ overlaps. The fact that the threshold

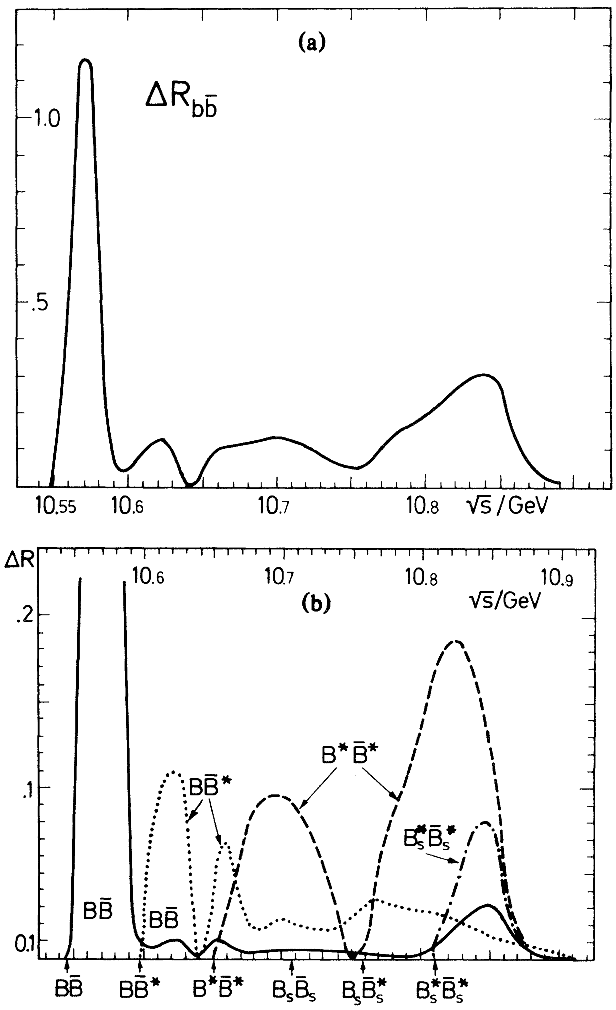

FIG. 2. (a) The contribution to R from the sum of the thresholds (1) in the energy region above the $B\bar{B}$ threshold. (b) The decomposition of ΔR into $B\bar{B}$, $B\bar{B}^*$ + c.c., $B^*\bar{B}^*$, and $B_s\bar{B}_s^*$. For values of masses and e^+e^- couplings used, see text. The $B_s\bar{B}_s$ and $B_s\bar{B}_s^*$ + c.c. contributions (not shown) are small < 0.015. Note that the bumps in the 10.6–10.75 GeV region do not correspond to resonances, but appear because of opening of the $B\bar{B}^*$ or $B^*\bar{B}^*$ thresholds and the node structure in the overlap functions. For details, see text.

positions accidentally tend to coincide with the node structure enhances the predicted bumpy structure of ΔR. Thus a third bump is predicted, especially in $B\bar{B}^* + \bar{B}B^*$ at 10.7 GeV, which does *not* correspond to a resonance. The only nearby resonance poles, in the energy interval of Fig. 2, are $\Upsilon(4S)$ at 10 570 MeV, and $\Upsilon(1D)$ at 10 650 MeV, which, however, couples very weakly as discussed above, and the $\Upsilon(5S)$ at 10 840 MeV. All the other resonances contribute to a "background" which interferes with the dominant resonance contribution, shifting the minima of Fig. 2 somewhat from the positions of the nodes in the overlap functions. It is obvious from Fig. 2 that one should be very cautious to interpret any structure in the cross section as due to non-$b\bar{b}$ states (hybrid states, gluonium). Figure 2 should also be useful for experimentalists when looking for where to find the strongest signals of $B^* \rightarrow B\gamma$ and $B_s^* \rightarrow B_s\gamma$.

Obviously, the details of Fig. 2 should not be taken too serously as they depend on the exact values of the threshold positions. However, Fig. 2 clearly demonstrates the importance of threshold effects and of the node structure. Any naive determination of resonance parameters above the $\Upsilon(4S)$ from measurements of ΔR must be considered with caution. On the other hand, experimental determination of the quantities in Fig. 2 would provide ample tests of the form factors determined by the overlap of Υ and B, B^* wave functions.

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