Foucault Pendulum at the South Pole: Proposal For an Experiment to Detect the Earth's General Relativistic Gravitomagnetic Field

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An experiment is proposed for measuring the earth's gravitomagnetic field by monitoring its effect on the plane of swing of a Foucault pendulum at the south pole ("dragging of inertial frames by earth's rotation"). With great effort a 10% experiment in a measurement time of several months might be achieved.

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When four-dimensional space-time is split into space plus time, the electromagnetic field $F^{\mu\nu}$ breaks up into two parts, the electric field $\vec{E}$ and the magnetic field $\vec{B}$. Similarly the general relativistic gravitational field (space-time metric $g^{\mu\nu}$) breaks up into three parts: (i) an electriclike part, g^{00}, whose gradient for weak gravity is the Newtonian acceleration $\vec{g}$; (ii) a magneticlike part, g^{0j}, whose curl for weak gravity is the "gravitomagnetic" or "GM" field $\vec{H}$; and (iii) a spatial metric, g^{ij}, whose curvature tensor is the "curvature of space."[1]

Experiments have probed g^{00} and g^{ij} with $\lesssim 0.1\%$ accuracy; but the GM potential g^{0j} is so weak in the solar system that it has never been detected. This is sad for astrophysics as well as fundamental physics, since some theories of quasars and galactic nuclei rely on the GM field of a supermassive black hole for energy storage, power generation, jet formation, and jet alignment.[1,2]

Although many experiments to detect GM fields have been proposed,[3,4] only one has seemed sufficiently feasible to be pursued vigorously: the GM-induced precession, relative to the stars, of a superconducting gyroscope in an earth-orbiting satellite (gravity probe B, "GPB," which may fly in ~ 1990).[3,5]

This paper proposes an earth-based variant of GPB. An earth-based rotating-sphere gyroscope cannot possibly reach the required sensitivity of (GM-precession rate $\Omega_{\rm GM}) \simeq 5\times 10^{-10}\times$ (earth rotation rate $\Omega_\oplus$): Precessions due to errors in the support system and to Newtonian gravity $\vec{g}$ acting on gyroscope inhomogeneities are orders of magnitude too large.[5] That is why GPB with its rotating-sphere gyros must fly in space. Our proposed experiment circumvents these problems using as its earth-based gyroscope a Foucault pendulum.

One price of using a Foucault pendulum is the necessity to operate within a few kilometers of the north or south pole: If α is the angle between local gravity $\vec{g}$ and the earth's angular velocity $\vec{\Omega}_\oplus$ ($\alpha \simeq$ colatitude of laboratory), then a nonzero α produces a precession of the pendulum relative to distant stars with $\Omega_\alpha = \Omega_\oplus(1-\cos\alpha)$. Since one cannot possibly monitor α to a precision $\delta\alpha \sim 10^{-10} \sim 0.00001''$, this precession will mask the GM effect unless $\alpha \ll 1$ and $\delta\cos\alpha \simeq \alpha\delta\alpha \lesssim 10^{-10}$. For a pendulum one kilometer from a pole, $\alpha \simeq 1.6\times 10^{-4}$ so α must be known to $\delta\alpha \lesssim 6\times 10^{-7} \sim 0.1''$. Since there is a scientific station at the south pole, and since weather and seeing conditions are reasonably good there,[6] the south pole is the natural location for the experiment.

The experimental apparatus would consist of a Foucault pendulum and an astrometric telescope in an underground vacuum chamber. Many experimental setups are conceivable. In one the telescope might be mounted on a rotating platform, with its optic axis locked to the azimuth of a reference star (e.g., Canopus). The pendulum's swinging fiber might be used as a light pipe and part of its mass as a lens to focus a swinging light beam onto an optical system that monitors the angle ϕ between the principal axis of pendulum swing and the reference star's azimuth, and also monitors the ellipticity of swing $\epsilon \ll 1$. As the pendulum's mass m swings with velocity $\vec{v}$, the earth's gravitomagnetic field $\vec{H}=4(G/c)\times$ (earth angular momentum) / (earth radius)^3 produces a force $m d\vec{v}/dt = m(\vec{v}/c)\times\vec{H}$, which causes a precession of the pendulum's principal axis with respect to the star's azimuth, $d\phi/dt = \Omega_{GM} = H/2c = 0.281''/\text{yr}$;

$$\delta\phi_{GM} = \Omega_{GM}\hat{\tau} = 0.036''[\hat{\tau}/(60\text{ d})]. \tag{1}$$

Here $\hat{\tau}$ is the duration of the experiment. This signal at the south pole is 5 times larger than in GPB's polar orbit. An experiment with 10% accuracy requires a measurement precision $\delta\phi = 0.004''[\hat{\tau}/(60\text{ d})]$.

The next few paragraphs will describe the most dangerous sources of experimental error and methods for circumventing them.

Velocity-dependent forces compared with position-dependent forces.—A Foucault pendulum is an excellent gyroscope because position-dependent forces produce at first order a change in the ellipticity ϵ of swing but no change in the principal-axis direction ϕ; see below. By contrast, forces linear in the velocity produce a first-order change of ϕ but no change of ϵ; they include the GM force, magnetic forces, frictional damping, and Pippard precession.

Magnetic forces.—As a result of the earth's magnetic field interacting with a charge q on the pendulum's ~ 100-g mass, magnetic forces are negligible if $q \lesssim 0.03$ esu; q could easily be kept this low by coating the mass and fiber with a thin layer of metal.

Frictional damping.—Frictional damping of the pendulum, if isotropic, would leave ϕ unchanged. However, frictional anisotropies are unavoidable. If we model the friction as linear in the velocity $\vec{v}$ of swing with a slightly anisotropic damping coefficient, $dv_i/dt = -\kappa_{ij}v_j$, then the anisotropy produces a precession $d\phi/dt = \Omega_{FA} = \frac{1}{2}(\Delta\tau_*/\tau_*^2)\sin 2\psi$. Here τ_* is the amplitude damping time, $\Delta\tau_*$ is the difference in τ_* along the principal and minor axes of the damping coefficient, and ψ is the angle between the principal axis of damping and the principal axis of swing. If the pendulum's support rotates with the telescope and the stars, ψ will be constant and Ω_{FA} will produce a huge secularly growing precession $\delta\phi_{FA} = \Omega_{FA}\hat{\tau}$ which is likely to vary so much as a result of "aging" that it cannot be subtracted from the data to yield a 10% experiment. (We thank Francis Everitt for pointing this out.) Thus, to control frictional anisotropy, the pendulum support probably must be held fixed relative to the earth, thereby making $\delta\phi_{FA}$ sinusoidal in time t

$$\delta\phi_{FA} = \frac{\Delta\tau_*}{\tau_*}\frac{\sin 2\Omega_\oplus t}{4\Omega_\oplus \tau_*} = 0.04''\frac{\Delta\tau_*/\tau_*}{10^{-2}}\frac{\sin 2\Omega_\oplus t}{\tau_*/(5\text{ yr})}. \tag{2}$$

($\tau_* \simeq 1$ y was achieved by one of the authors, V.B.B., several years ago, using a fused quartz fiber with diameter $d \simeq 100$ μm and mass $m \simeq 100$ g.) The sinusoidal precession (2), with amplitude of order of the GM signal and period 24 h, can easily be removed from the data for a 10% experiment.

Pippard precession.—Brian Pippard has pointed out to us that with the pendulum support fixed relative to earth, the pendulum's mass m has a spin angular momentum $mk^2\Omega_\oplus$ whose direction changes as the pendulum swings, $d\vec{S}/dt = mk^2\Omega_\oplus \vec{v}/l$. Here k is the mass's radius of gyration, $\vec{v}$ its velocity, and l the length of its fiber. Since no torques act along $\vec{v}$, there must be an equal and opposite change in the mass's orbital angular momentum—i.e., a gyroscopic force must act proportional to $\vec{v}$. This causes a precession

$$\delta\phi_P = \frac{\Omega_\oplus}{2}\frac{k^2}{l^2}\hat{\tau} = 40''\frac{[k/(0.2\text{ cm})]^2}{[l/(2\text{ m})]^2}\frac{\hat{\tau}}{60\text{ d}}, \tag{3}$$

which is 10^3 times larger than the GM signal. (To achieve $k \simeq 0.2$ cm with $m \simeq 100$ g requires that the mass be long, thin, and dense; e.g., tungsten.) Fortunately, one can probably measure $(k/l)^2$ with precision 10^{-4} and subtract $\delta\phi_P$ from the data to achieve a 10% experiment. However, this requires an optical readout system with dynamic range 10^4. (Elsewhere we will describe possible designs for such a system.) This 10^{-4} precision and 10^4 dynamic range are reduced for larger r; but seismic noise may require $l \lesssim 2$ m (see below). If the mass were supported gently near its center, thereby acquiring the freedom to swing with frequency $\omega_1 << \omega = (g/l)^{1/2}$, then it could remain nearly vertical as the pendulum swings, and $\delta\phi_P$ would be reduced below (3) by a factor $\simeq (\omega_1/\omega)^2$.

Position-dependent forces.—If a Foucault pendulum is swinging in the x direction, $x = a\cos\omega_1 t$, any position-dependent force in the orthogonal y direction $\vec{F} = F(x)\vec{e}_y$ (e.g., as a result of gravity gradients or pressure from a static light beam) produces a growing ellipticity of swing, $d\epsilon/dt = F_0/2ma\omega$ with $F_0 = (2/\pi)\int_0^\pi F(a\cos\xi)\cos\xi\, d\xi$. Only secondarily, as a result of a frequency difference $\Delta\omega = \omega_y - \omega_x$ between infinitesimal-amplitude y motions and finite-amplitude x motions, does the principal axis precess, $d\phi/dt = \epsilon\Delta\omega$. If all forces are kept below $|f_0/ma\omega^2| \sim 10^{-8}$ (which should be possible), one can monitor ϵ optically and apply a gravitational feedback force (by adjusting the positions of nearby gravitating masses[7]) to keep $|\epsilon| < 10^{-7}$. The precession will then be

$$\delta\phi_{\rm PDF} \simeq \langle\epsilon\Delta\omega\rangle\hat{\tau} \simeq 0.004'' \left\langle \frac{\epsilon}{10^{-7}} \frac{\Delta\omega/\omega}{2\times10^{-8}} \right\rangle \left(\frac{2\ {\rm m}}{l}\right)^{1/2} \frac{\hat{\tau}}{60\ {\rm d}}, \tag{4}$$

where $\langle\ \rangle$ denotes a time average. $\Delta\omega/\omega$ can be determined before and after the experiment by measurements of $\delta\phi_{\rm PDF}$ with $\epsilon \simeq +10^{-4}$ and -10^{-4}. ($\delta\phi_{\rm PDF}$, unlike other precessions, changes sign when ϵ changes sign.)

Frequency anisotropy.—The most serious source of frequency difference $\Delta\omega = \omega_y - \omega_z$ is finiteness of the amplitude a of swing: $\Delta\omega/\omega \simeq \frac{3}{8}(a/l)^2$. (For this $\Delta\omega/\omega$, the precession $d\phi/dt = \epsilon\Delta\omega$ produces the well-known nonclosure of the orbit of a finite-amplitude pendulum.) Accuracy of readout requires a large amplitude, e.g., $a \sim 5$ cm, whereas seismic noise (see below) requires a small fiber length, e.g., $l \sim 1$ to 10 m; thus $\Delta\omega/\omega \sim 10^{-3}$ to 10^{-5}. For a 10% experiment this $\Delta\omega/\omega$ must be reduced to $\lesssim 10^{-8}$ [Eq. (4)]. For $l \simeq 10$ m ($\Delta\omega/\omega \simeq 10^{-5}$) the reduction can be made by the gravitational pulls of large masses placed on each side of the swinging pendulum. For $l < 10$ m gravity is too weak to make the reduction, but electrostatic forces might work. For example, parallel plates with fixed voltages $+V_0 \sim 1000$ V and $-V_0$ might be placed on each side of the grounded ($V=0$) swinging pendulum, though jitter in the rotation of the plates might be an insurmountable problem. If achieving $\Delta\omega/\omega \lesssim 10^{-8}$ with a fiber-and-mass pendulum turns out to be harder than we expect, one might try a pendulum made of a magnetically levitated mass sliding over a superconducting surface, with height z as a function of radius ρ chosen to avoid the finite-amplitude $\Delta\omega/\omega$: $Z/l = \frac{1}{2}[(\rho/l)^2 + (\rho/l)^4 + 2(\rho/l)^6 + \ldots]$.

In addition to the finite-amplitude $\Delta\omega/\omega \simeq \frac{3}{8}(a/l)^2$, there is a $\Delta\omega/\omega$ due to anisotropy of the top of the pendulum fiber, $\Delta\omega/\omega \simeq (\beta/8)(d/l)(l/\Delta l)^{1/2}$. Here $\Delta l/l$ is the strain in the fiber due to gravitational loading, d is the fiber's diameter, and β is the fractional anisotropy of the fiber's diameter or half the anisotropy of its Young's modulus. For $\beta \simeq 0.01$, $d \simeq 100$ μm, $\Delta l/l \simeq 2\times10^{-3}$, and $l \simeq 2$ m, $\Delta\omega/\omega$ is 10^{-6}. This anisotropy is fixed relative to the earth and can be reduced to $\lesssim 10^{-8}$ by the gravitational pulls of fixed masses.

Seismic noise.—If the pendulum swings in the x direction with amplitude a, then seismically-induced y displacements of its support produce a y swing and associated precession $\delta\phi_{\rm seismic}$. If S is the spectral density of the y component of support displacement, then after an integration time $\hat{\tau}$

$$\delta\phi_{\rm seismic} = (1/4a)(S\omega^2\hat{\tau})^{1/2} \simeq 0.004'' \frac{S^{1/2}}{8\times10^{-11}\ {\rm cm/Hz^{1/2}}} \frac{5\ {\rm cm}}{a} \left(\frac{2\ {\rm m}}{l} \frac{\hat{\tau}}{60\ {\rm d}}\right)^{1/2}. \tag{5}$$

At pendulum frequencies $f = \omega/2\pi \sim 0.2$ to 1 Hz and in the Antarctic winter when human activity is at a minimum, the south pole is among the quietest sites in the World-Wide Standard Seismograph Network (WWSSN); but nevertheless, $S^{1/2}$ is so large that substantial antiseismic isolation will be needed for a 10% experiment. $S^{1/2}$ and the amount I it must be reduced by isolation, as functions of pendulum length l and frequency f are[8]

l (m)	f (Hz)	$S^{1/2}$ (cm/Hz$^{1/2}$)	I
1.0	0.5	1.8×10^{-7}	3×10^3
2.3	0.33	1.6×10^{-6}	2×10^4
10	0.16	1.6×10^{-5}	9×10^4

The shorter the pendulum, the easier the isolation. The required isolation might be achieved by active antiseismic devices,[9] plus removal of remaining seismic effects from the data by comparing two pendula, attached to the same support and with the same plane of swing but opposite phases, for which seismic accelerations produce opposite precessions. (This trick, suggested by Ron Drever, might require a sapphire fiber rather than fused quartz to keep stretching small enough for frequency compensation to hold the pendula out of phase.)

Atmospheric refraction.—Variations in the measured position of the reference star will be caused by changes in azimuthal atmospheric refraction, both near the telescope and far. Experience in astrometry indicates the seriousness of the problem: The best observatories contributing data to the

Bureau International de L'Heure show semiannual residuals in their photographic-zenith-tube measurements as large as 0.03″,[10] and the combined data from all the observatories, averaged over thirty days, has an accuracy of 0.006″.[11] These numbers are reasonable, since a 1% atmospheric-density variation over a horizontal distance of 1000 km, due to a large weather pattern, will deflect a star's position by ~0.01″. At the south pole the telescope's 24 h rotation may reduce refraction effects somewhat, and they might be reduced further by tracking two stars on opposite sides of the sky. If this and careful design of the telescope-atmosphere interface are not adequate for 0.004″ in sixty days, correction for refraction by two-color refractometry[12] may do the job.

Distortion of the telescope.—For 10% accuracy the telescope and associated optics must remain stable azimuthally during two months to 0.004″, despite gravitational stresses, temperature fluctuations, and aging effects. Experience with the GPB prototype telescope suggests that this may be feasible, though difficult. To control index-of-refraction changes and thermal expansion, the thermal stability across the telescope may need to be ~0.01 K.

Tilt of the telescope.—If, for maximum stability, the telescope rests horizontally or vertically and light is brought into it by a mirror, tilts of the mirror will change the apparent azimuthal position of the star. To avoid this, one must monitor the tilt relative to gravity $\vec{g}$ and apply feedback to hold it steady.

Conclusions.—While the proposed experiment is very difficult it might be doable. To determine just how difficult (or impossible) it is requires laboratory development in several areas: frictional anisotropy, Pippard precession, frequency compensation, and antiseismic isolation for the pendulum; atmospheric refraction and physical distortion for the telescope; and dynamic range for the readout system. Because of the great importance of GM fields for astrophysics and fundamental physics, such laboratory development should be pursued.

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