Undulator and Čerenkov Free-Electron Lasers: A Preliminary Comparison

J. Walsh and B. Johnson
Department of Physics & Astronomy, Dartmouth College, Hanover, New Hampshire 03755

and

G. Dattoli and A. Renieri
Comitato Nazionale per la Ricerca e per lo Sviluppo dell' Energia Nucleare e delle Energie Alternative, Dipartimento Technologie Intersettoriali di Base, Divisione Fisica Applicata, I-00044 Frascati, Italy
(Received 14 May 1984)

A preliminary comparison of the fundamental characteristics of undulator and Čerenkov free-electron lasers is presented. It is assumed that both devices are operating in the Compton regime and that they are driven by a short-pulse relativistic electron beam.

PACS numbers: 42.55.-f

The purpose of this work is to compare general characteristics of undulator[1] and Čerenkov[2] free-electron lasers (FEL). The former class of device has been operated over a wavelength span which covers the millimeter through visible regions, while, to date, the latter has been primarily used as a near-millimeter–wavelength source.[3] Furthermore, there exists a very detailed[4–7] body of theory covering many aspects of undulator FEL's. However, operation of Čerenkov devices at far-infrared wavelengths has been discussed only briefly. The purpose of this note is thus to compare the gain, the beam energy, and the beam quality requirements of a Čerenkov FEL to those of an undulator-based source.

Highly simplified versions of the two devices are shown in Fig. 1. In Fig. 1(a), a relativistic electron beam moves along the axis of the magnetic undulator and produces radiation at the characteristic wavelength

$$\lambda = \lambda_p (1+\kappa^2)/2\gamma^2, \qquad (1)$$

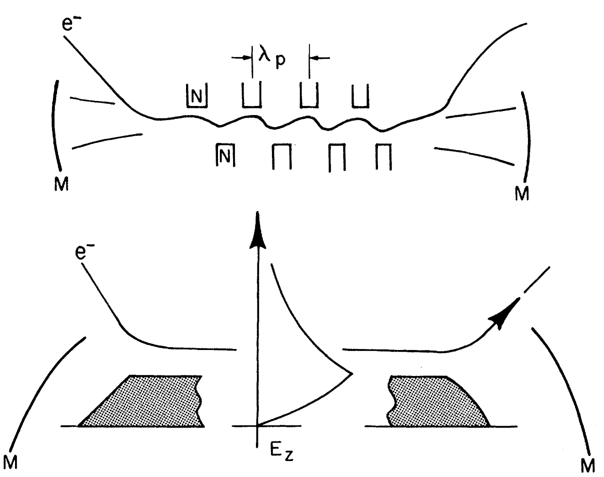

FIG. 1. Schematic form of two free electron lasers. (a) undulator form; (b) Čerenkov form.

where λ_p is the wavelength of the undulator, $\kappa = e \times \lambda_p \bar{B}_p / mc^2$ is the undulator parameter, $\bar{B}_p$ is the pump magnetic field strength, and γmc^2 is the beam energy. Stimulated emission causes bunching and the addition of mirrors forms an oscillator.

In Fig. 1(b), an electron beam moves near and parallel to the surface of a thin-film dielectric waveguide. The beam couples to the axial component of a transverse-magnetic mode of the guide and thereby emits spontaneous Čerenkov radiation in the bounded structure. Again, with the addition of mirrors, it is possible to form an oscillator. The characteristic wavelength of the emitted radiation is determined by a velocity synchronism of the beam and the guided mode. This condition is illustrated schematically in Fig. 2. The exact dispersion curve

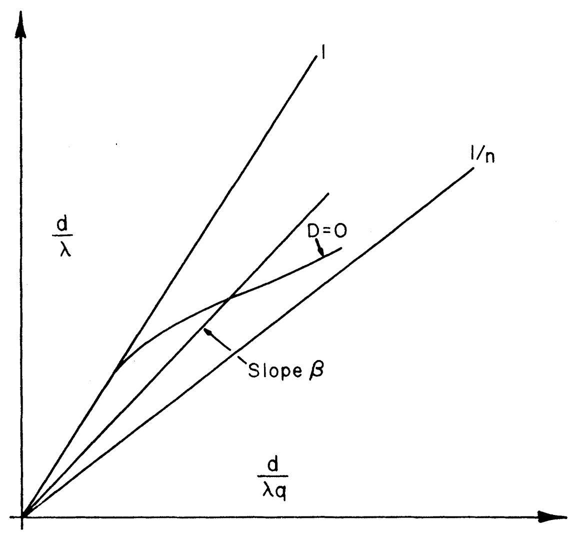

FIG. 2. Typical dispersion curve (inverse wavelength λ vs inverse guide wavelength λ_g) and beam velocity line for a thin-film guide (n is index of refraction of the film material, d is film thickness). The curve D represents the solution to Eq. (2) and β is line with slope v/c.

for a mode with a transverse extent which is considerably larger than a wavelength is given by the well known expression[8]

$$pd \tan pd = q \epsilon d, \tag{2}$$

where $p = (\omega^2 \epsilon/c^2 - k^2)^{1/2}$, $q = (k^2 - \omega^2/c^2)^{1/2}$, k is the guide wavelength, ϵ is the relative dielectric constant of the film material, and d is its thickness.

Discussion will be facilitated if we also introduce an approximate solution of Eq. (2), which is

$$\lambda \simeq 2\pi d\gamma/\gamma_T^2, \tag{3}$$

where $\gamma_T^2 = \epsilon/(\epsilon - 1)$ is the relative energy at Čerenkov threshold. Equation (3) yields wavelengths which are extremely close to the exact solution when $\gamma >> \gamma_T$ and d is small compared to the beam thickness.

The comparative wavelength-energy curves for the two devices are shown in Fig. 3. They are quite distinct. The wavelength of the Čerenkov laser decreases with decreasing energy and depends linearly on the film thickness which can be small. It also varies inversely with γ_T^2 and hence lower ϵ implies shorter wavelength. The wavelength produced by the undulator decreases as γ^2 increases and it depends linearly on λ_p. Pump strength requirements ($\kappa \simeq 1$) generally fix λ_p in the 2–5 cm range. When $d \simeq 3$ μm, $\epsilon \simeq 2$, $\kappa \simeq 1$, and $\lambda_p = 2.5$ cm, the wavelength-energy curves cross in the vicinity of $\lambda = 140$ μm and $\gamma = 14$. The crossover point can be moved about by varying parameters, but these results are typical. Thus, provided other conditions tions can be met, a Čerenkov device has a potential advantage if both short wavelength and lower relative operating electron-beam energies are desired.

The small-signal gain per pass in the resonator of a Čerenkov device is given by the general expression[2]

$$g_0 = \frac{1}{2}\frac{1}{(\gamma)^3}\frac{I}{I_0}\frac{L^2}{A_b}\int \frac{dA\,|E_z|^2}{\mathscr{E}/L}\Gamma(\theta), \tag{4}$$

where I/I_0 is the beam current measured in units of $I_0 = ec/r_0$ ($r_0 = e^2/mc^2$), L is the length of the coupling region, and A_b is the beam area. The integral is done over the beam volume and the symbols E_z and $\mathscr{E}$ are, respectively, the axial component of the electric field and the stored energy in the resonator. The gain line shape is determined by

$$\Gamma(\theta) = \frac{\partial}{\partial\theta}\left(\frac{1-\cos\theta}{\theta^2}\right), \tag{5}$$

where $\theta = (kv_0 - \omega)L/v_0$ is the relative phase angle change experienced by an electron moving with initial velocity v_0.

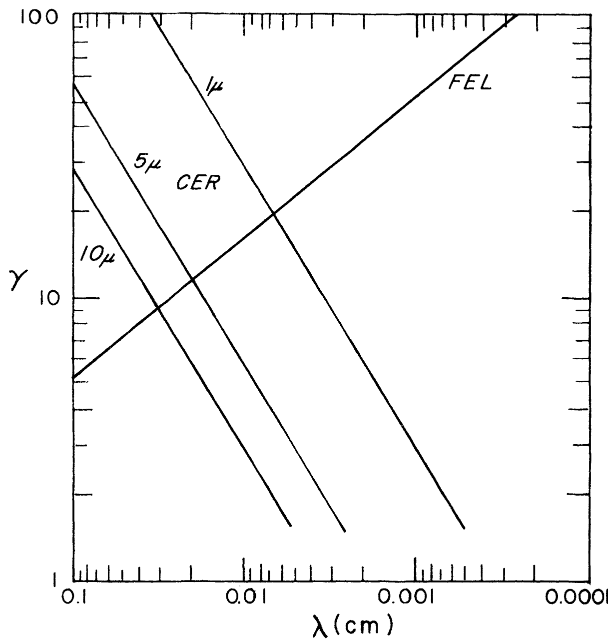

FIG. 3. Comparison of energy wavelength relations for a Čerenkov and an undulator FEL. Three film thicknesses—1, 5, 10 μm—are shown.

When evaluated at the phase-velocity–beam-velocity synchronism and at the maximum of $\Gamma(\theta)$, the general expression is accurately represented by

$$g_0^{(C)} = \frac{2\pi}{\lambda}\frac{L^3}{\sigma_x\sigma_y}\frac{I}{I_0}\frac{e^{-\alpha_0}}{\gamma^5}, \tag{6}$$

where σ_x and σ_y are the beam dimensions in the x and y directions, and

$$\alpha_0 = 4\pi\delta/\lambda\beta\gamma \tag{7}$$

is determined by the gap δ between the beam and the dielectric (β = the relative velocity v/c).

The gain of an undulator-based FEL is given by

$$g_0^{(u)} = \frac{2\pi}{\lambda_p}\frac{L^3}{\sigma_x\sigma_y}\frac{\kappa^2}{(1+\kappa^2)^{3/2}}\left(\frac{2\lambda}{\lambda_p}\right)^{3/2}\frac{I}{I_0}. \tag{8}$$

If it is also assumed for the purposes of comparison that L, σ_x, σ_y, and I/I_0 are the same, and that $\kappa \simeq 1$, the comparative magnitude and the general behavior of $g_0^{(C)}$ and $g_0^{(u)}$ can be displayed (Fig. 4).

Clearly, if λ and γ are also the same for the two devices and α_0 is not large, the gains will be comparable. In general, the gain of a Čerenkov laser will rise with decreasing wavelength, reach a maximum near $\alpha_0 = 1$, and then rapidly decrease as λ becomes still smaller. The value of the wavelength, $\lambda = \lambda_m$, at $g^{(C)}$ = max, is determined by

$$\lambda_m = 4\pi\delta/\gamma. \tag{9}$$

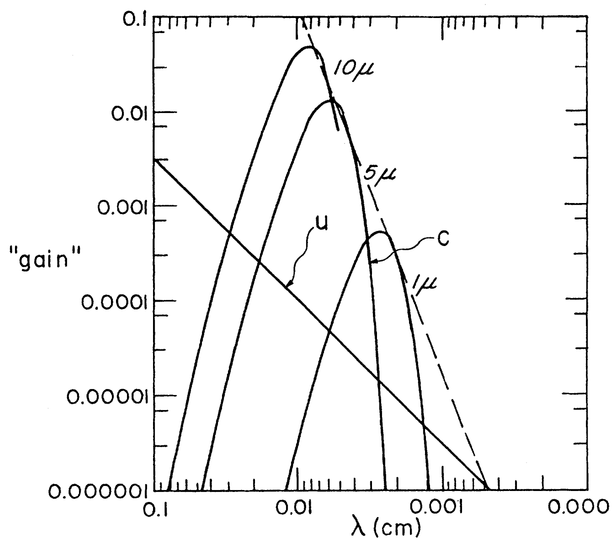

FIG. 4. Comparison of $g_0^{(C)}(\lambda^{-1})$ with $g_0^{(u)}(\lambda^{-1})$. The common factors are suppressed.

Assuming, as is the case in microwave tubes, that δ can be maintained in the 10 to 100 μm range (over a length of ten or twenty centimeters), and that γ ranges between 2 and 20, λ_m will occur in the far-infrared region of the spectrum.

In obtaining the results displayed in Fig. 4, the wavelength variation of the undulator FEL is obtained by varying γ and the curve shows the characteristic $\lambda^{-3/2}$ dependence. In the case of the Čerenkov laser, the film and beam thickness are chosen and the gap is fixed at 50 μm. The device is then tuned along the dispersion curve by again varying γ. The gain at the Čerenkov laser exceeds the undulator laser in those regions where it achieves a given λ at a smaller γ. Furthermore, since the wavelength and the film thickness are proportional, the energy dependence of the gain insures that at longer wavelength the peak $g^{(C)}$ can always be made larger than $g^{(u)}$. Examination of the approximate equation, however, also shows that

$$g_{\max}^{(C)} \sim \lambda^4 \ , \tag{10}$$

and hence also that there exists some wavelength for which $g^{(u)} > g_{\max}^{(C)}$. With the parameter choices in Fig. 4, this occurs near $\lambda = 1$ μm. The envelope of $g_{\max}^{(C)}$ is shown as a dashed line in Fig. 4.

It is also important to consider the relative efficiency of the two sources. The gain line shape is the same for both and hence in either case, consideration of the field strength needed to cause electron trapping yields an expression for efficiency which is proportional to $\gamma^2\lambda/L$. The Čerenkov laser will operate at a given λ with a lower value of γ and thus would tend to have a lower efficiency. However, $g_0^{(C)} > g_0^{(f)}$ in this region, and therefore L can be made smaller, and thus a part of the difference of η can be recovered.

The gain expressions used assume that the beam is perfectly collimated and hence another important point of comparison is a consideration of the effect of beam inhomogeneities on gain. The principal inhomogeneities listed in Table I have been defined previously.[4] The effects of angular spread and energy spread will have the same form for either laser. The parameters $\mu_{x,y,\epsilon}$ are dimensionless measures of relative dephasing due to angular divergence in the transverse directions or energy spread. When these parameters exceed unity, the beamwidth in frequency space is larger than the gain linewidth and the gain is reduced.

It is anticipated that a Čerenkov laser could be operated with a short-pulse rf-accelerated electron beam, and hence the relative slippage or "lethargy" may also play a significant role. In an undulator FEL the optical pulse slips forward,[4] while in the Čerenkov laser, it slips back. The exact expression for the lethargy of the Čerenkov laser depends on the relative size of the group velocity and the phase velocity. If, however, the operating wavelength is

TABLE I. Beam homogeneity parameters. σ_E is the relative energy spread, $\epsilon_{x,y}$ is the relative emittance (mm · mrad), and σ_z is beam pulse length.

Inhomogeneity parameter	Čerenkov	Undulator
μ_ϵ	$2L\sigma_E/\gamma^2\lambda$	Same
$\mu_{x,y}$	$\sqrt{2}L\epsilon_{x,y}^2/4\pi^2\lambda\sigma_{x,y}^2$	Same
μ_z	$(L/\sigma_z)(1-\beta_g/\beta_0)$	$\lambda_p N/2\gamma^2\sigma_z$

very much smaller than the length of the beam pulse, the gain is relatively unaffected by this slippage. This is the anticipated operating range, and thus a detailed examination of this effect can be deferred.

In conclusion, although many practical details must be examined, a Čerenkov laser is a promising, moderately compact, far-infrared source. The beam intensity and quality parameters (Table I) required for operation in the 50–500 μm range are within the capability[4] of a small (1–5 MeV) microtron accelerator. Support of U.S. Army Research Office through Grant No. DAAG29-83-K-0018 is acknowledged. We would also like to acknowledge the suggestions of U. Bizzarri, W. B. Colson, T. Letardi, A. Marino, and A. Vignati.

[1]D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. **38**, 892 (1977); J. Benson, D. A. G. Deacon, J. N. Eckstein, J. M. J. Madey, K. Robinson, T. I. Smith, and R. Taber, J. Phys. (Paris), Colloq. **44**, C1-353 (1983).

[2]J. B. Murphy and J. E. Walsh, IEEE J. Quantum Electron. **18**, 8, 1259 (1982); B. Johnson and J. Walsh, in *Lasers in Fluid Mechanics & Plasmadynamics,* edited by Charles P. Wang (AIAA, New York, 1983), p. 154.

[3]S. Von Laven, J. Branscum, J. Golub, R. Layman, and J. Walsh, Appl. Phys. Lett. **41**, 408 (1982).

[4]G. Dattoli and A. Renieri, in "Laser Handbook, Volume 4," edited by M. L. Stitch and M. S. Bass (North-Holland, Amsterdam, to be published).

[5]W. B. Colson and A. Renieri, J. Phys. (Paris), Colloq. C1-**44**, 11 (1983).

[6]W. B. Colson, IEEE J. Quantum Electron **17**, 173 (1981).

[7]G. Dattoli, T. Letardi, J. M. J. Madey and A. Renieri, to be published.

[8]D. Marcuse, *Light Transmission Optics,* Bell Laboratory Series (Van Nostrand Reinhold, New York, 1972).