Propagation of Shear in a Two-Dimensional Electron Solid

G. Deville, A. Valdes, E. Y. Andrei, and F. I. B. Williams[a]
Service de Physique du Solide et de Resonance Magnetique, Centre d'Etudes Nucléaires de Saclay, F-91191 Gif-sur-Yvette Cedex, France
(Received 6 February 1984)

Shear waves are shown to propagate in the two-dimensional electron solid on liquid helium. Their velocity and damping are measured and used to deduce the shear components of the viscoelastic tensor up to melting. A linear temperature variation at low temperatures contrasts with the premelting region, where the sharper renormalization of the elastic component is accompanied by a very rapid increase in the viscous component.

PACS numbers: 68.30.+z, 63.20.Dj, 64.70.Dv

A material is solid if it returns to its original shape upon removal of an applied shear stress. At finite frequency this property combines with inertia to give rise to shear waves. Shear waves are thus a fundamental attribute of a solid, but no experimental demonstration has been given that they propagate in a two-dimensional (2D) system.[1]

The experiment described here on a classical 2D electron solid shows explicitly that shear (transverse phonons) does propagate and the manner in which it ceases to do so as melting is approached.

These observations are presented quantitatively in terms of the generalized shear modulus for an isotropic solid,

$$\mu(k,\omega)=\mu'+i\mu''=\mu'+i\omega\eta. \quad (1)$$

μ' and η describe the elastic and viscous responses to shear at wave vector k and frequency ω. The inertia of the particles, of mass m and areal number density n, combines with the restoring force from (1) to give rise to shear waves $\exp[i(\vec{k}\cdot\vec{r}-\omega t)]$ which propagate at velocity $v_t=\omega/k$ given by

$$v_t^2=\mu/mn. \quad (2)$$

We observe these waves by looking at the response of the system to a transverse excitation of imposed k. A propagating mode is then seen as a well-defined resonance whose peak position $\omega_k \sim k\mu'^{1/2}$ and whose width $\Delta\omega_k \sim k^2\eta$. We find that both the elastic response, μ', and the viscosity, η, show important modifications due to thermal excitations as melting is approached.

Electrons on helium form a 2D system of (Coulomb) interacting scalar particles subjected to no periodicity, no anisotropy, and only very small, calculable, random potentials. It is probably the simplest and most controllable solid system so far realized in 2D. Furthermore it shows signs of melting by dissociation of the dislocation pairs which are present as a dilute gas of thermal excitations in the solid. This model[2] has proved unusually tractable and considerable theoretical results exist on both static[3] and dynamic[4] properties. A striking qualitative prediction is that the solid should melt, at T_m, to an oriented liquid (hexatic) phase with free dislocations and only when these decompose into free disclinations should the liquid become isotropic, at T_i. The T_m transition is now well documented[5-9] but no experiment has been devised to detect T_i. The quantitative characterization of melting, however, most naturally hinges on the response to shear, even in the hexatic phase. The most direct way to this is to make mechanical shear measurements on a variety of length and time scales. The first attempt in this direction was only able to evaluate the shear modulus averaged over the lengths and times of the thermally excited vibrations.[9] Despite this, it showed renormalization for $T\rightarrow T_m^-$ and a "jump" to zero at $T=T_m^+$ from a value 10% above the Kosterlitz-Thouless stability limit (for pair dissociation). But the nature of the measurement, being based on the mean square fluctuation of an electron in the solid, precluded getting information on damping or propagation. Another experiment where an excess longitudinal sound damping was attributed to diffusion of dislocations[10] also pointed to dislocation-mediated melting, but again the conclusions were very dependent on being able to correct for large substrate effects. The present experiment provides values for both the elastic shear modulus and the shear viscosity at specific (k,ω) to permit a more meaningful and detailed comparison with the predicted critical behavior. The information is direct and intrinsic: Substrate effects are small and corrected for, whereas in all previous experiments they have been dominant. This demands choosing the wave vector for the measurement high enough that the intrinsic propagation dominates, but low compared to zone-boundary values. In practice $k=520$ cm^{-1} and $k(\text{Debye})\approx 3\times10^4$ cm^{-1} $(n=6\times10^7$ cm$^{-2})$ re-

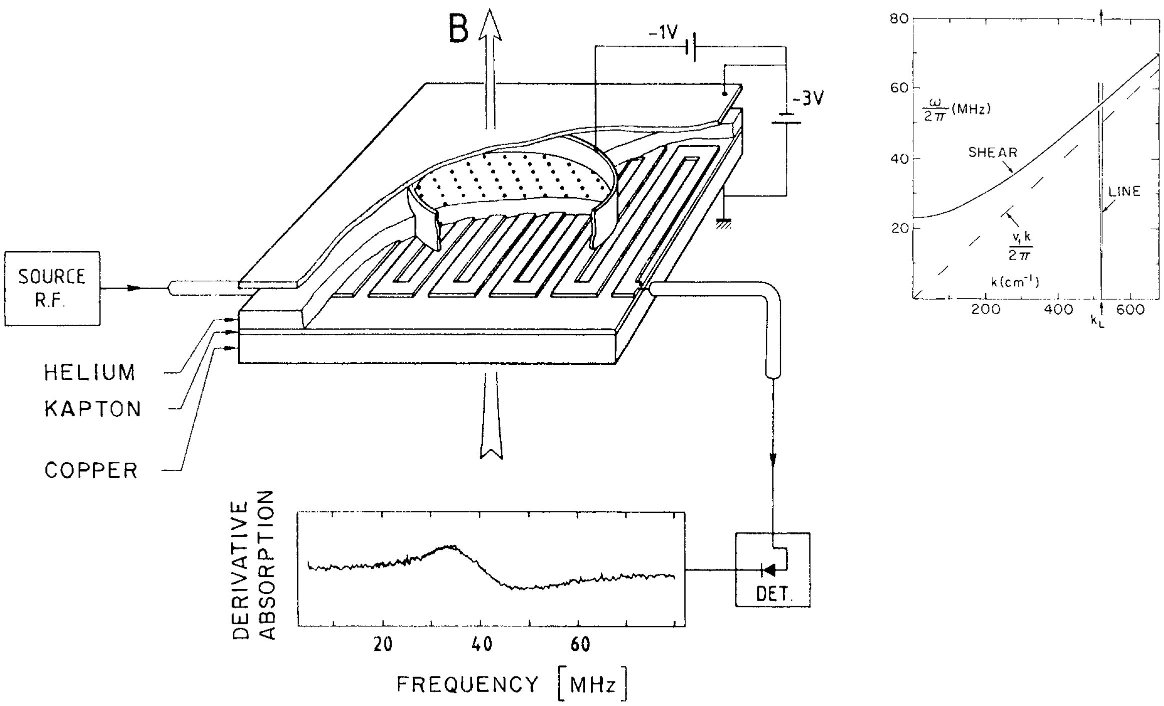


FIG. 1. Experimental disposition. Inset at right shows the relevant portion of dispersion relations of the electron solid (transverse branch) and the fundamental electrostatic mode of the meander line. Simultaneous matching of k and ω at the crossing manifests itself as a resonance giving rise to the signal illustrated. Temperature = 70 mK, holding field $E_\perp = 115$ V cm^{-1}, and density $n = 6\times10^7$ cm^{-2}. The electron longitudinal mode frequency of $k = k_L$ is 1.1 GHz.

sulting in a shear-mode frequency $\omega_t/2\pi \approx 50$ MHz.

As illustrated in Fig. 1, the electrons are confined laterally by an 18-mm-diam guard ring and vertically by two plane parallel horizontal electrodes separated by $h+d=2$ mm; they move on the 2D plane of a liquid helium surface established $d \approx 40$ μm above the lower electrode. They are subjected to a time-varying body force in their plane by the electric field from a strip line on the lower electrode which transmits about 1 nW of radio-frequency (rf) power from source to detector. This field is shaped by the meander configuration of the line to have $k = k_L = 520$ cm^{-1} with longitudinal polarization. It is coupled to the transverse motion on application of a uniform vertical ($\hat{z}$) magnetic field whose amplitude H is henceforth represented by the electron cyclotron frequency $\omega_c = eH/mc$. By virtue of the rf longitudinal electron velocity induced by the electric field $E\hat{x}$ the electrons are subject to a transverse Lorentz force $\hat{y}(\omega\omega_c/\omega_l 2)eE$. ω is the source frequency and ω_l denotes the longitudinal (plasmon) mode frequency given by

$$m\omega_l^2(k) = e^2 nk \times 4\pi/(\coth kh + \coth kd). \qquad (3)$$

As ω sweeps through the crossing of the strip-line and the shear-mode dispersion curves, the resonance is detected by a loss in transmitted rf power. The peak position and width of the resonance give μ' and η. A series of experimental traces is shown in Fig. 2.

We eliminate extrinsic effects due to the magnetic field and to the coupling to the substrate by an extrapolation of the data points. The shift arising from the Lorentz-force coupling between the transverse mode of amplitude $u_t(\vec{k})$ and the longitudinal mode of amplitude $u_l(k)$ is described by

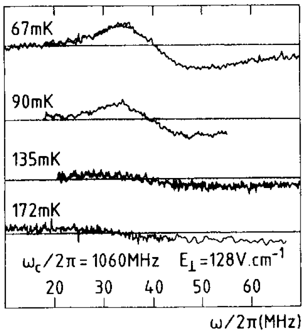
$$(\omega_l^2 - \omega^2)u_l + i\omega\omega_c u_t = (e/m)E_e$$

$$-i\omega\omega_c u_l + (\omega_t^2 - \omega^2)u_t = 0.$$

The peak of the transverse response occurs at the lower eigenfrequency, ω_-, which is given to order $(\omega_t/\omega_l)^2 \approx 2\times10^{-3}$ by

$$\omega_t^2/\omega_-^2 = 1 + \omega_c^2/\omega_l^2. \qquad (4)$$

This offers a simple extrapolation scheme to $\omega_c = 0$. The effect of the coupling to the substrate can be appreciated from the dispersion curve $\omega_t(k)$ shown in Fig. 1. The finite frequency, ω_0, at $k=0$ is a



a

FIG. 2. Experimental traces of shear-mode resonance observed at fixed magnetic field $\omega_c/2\pi = 1060$ MHz and pressing electric field $E_\perp = 128$ V cm^{-1} for a series of temperatures. Density $n = 6\times 10^7$ cm^{-2}. The ordinate is the derivative of the absorption with respect to the pressing field $E_\perp$.

result of the interaction with the soft substrate.[9,11] It has the functional dependence $\omega_0^2 = E_\perp^2 \times nF(T/T_m)$, where $E_\perp$ is the vertical field felt by the electrons, including the polarization contribution, and $F(T/T_m)$ arises from electron spread due to thermal fluctuations. The transverse-mode frequency is then modified from $v_t k$ into

$$\omega_t^2(k) = v_t^2 k^2 + E_\perp^2 nF(T/T_m)(1-\Omega^2/\omega_t^2)^{-1}, \tag{5}$$

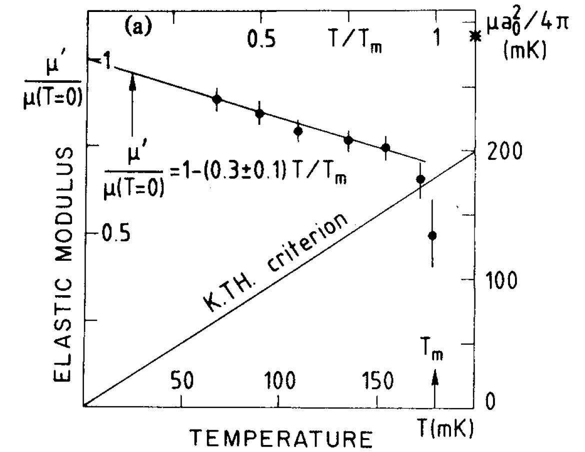
where Ω is the capillary-wave frequency at the basic reciprocal lattice wave number. This affords a simple extrapolation scheme to $E_\perp = 0$. Typically ω_0 introduces a 10% shift in ω_t before correction.

Once the electrons have been deposited, the shear-mode signal is observed for a series of ω_c at some fixed temperature: Its resonant frequency $\omega_-(k_L)$ is used to construct a plot of ω_-^{-2} vs ω_c^2. From Eq. (4), the $\omega_c = 0$ intercept gives $\omega_t(k_L)$ and the slope gives $\omega_l\omega_t$. The $\omega_l(k_L)$ so deduced is used to evaluate n from Eq. (3). But this plot also allows us to reduce all ω_- data (at the same n) to ω_t, for the ratio ω_t/ω_- depends only on ω_c/ω_l and is independent of both T and $E_\perp$, a feature we have verified experimentally. At each temperature, observations are made for a series of values of the upper electrode potential which varies the vertical field $E_\perp$ at the electrons. By plotting $\omega_t^2(k_L)$ vs $E_\perp^2$ and extrapolating to $E_\perp^2 = 0$ we get the pure $v_t k_L$ frequency we seek by virtue of Eq. (5). As a check on this procedure, we have also observed the frequency ω_0 directly at low k vector as in Ref. 9. Finally a run at a given density is completed by remeasuring the magnetic field dependence to check that the density has not changed in the course of experiment.

Once the data are extrapolated in this way to zero substrate interaction and zero magnetic field, we use (2) to evaluate μ' as a function of temperature as presented in Fig. 3(a). The viscosity η is deduced from the resonance widths and is presented as the kinematic viscosity $\nu = \eta/mn$ in Fig. 3(b).

Comparison of these figures reveals a correlation in behavior: For $T/T_m < 0.7$, both μ' and η vary linearly with temperature whereas for $0.7 < T/T_m < 1$ both show an accelerated variation. For $T/T_m > 1$, $\mu' = 0$ and η is unmeasurably large for the present noise performance of our detection. Bearing in mind that substrate effects have been eliminated from μ' and make a negligible contribution to ν at these densities, we can attribute the variations to processes intrinsic to the 2D solid. The linear portion of $\mu'(T) = \mu'(T=0)(1+\alpha T/T_m)$ with $\alpha = 0.3 \pm 0.1$ is close to the theoretical estimates of the renormalization due to phonon-phonon interactions[12,13] ($\alpha = 0.18$) and in accord with the molecular dynamics simulations[14] ($\alpha = 0.24$). One suspects the same mechanism to be responsible for the corresponding portion of $\eta(T)$. The $\mu(T=0)$ in the above expression is the linear extrapolation of the data to $T=0$; it is 0.93 ± 0.1 times the classical $T=0$ value calculated[15] with the measured value of the density. The more rapid variations of μ' and η in the region just prior to melting can be reasonably attributed to another type of excitation, responsible for the fusion. The facts that $B = \mu'(T_m)a_0^2/4\pi T_m = 0.9 \pm 0.15$ and that free dislocations should destabilize the solid phase when $B < 1^2$ (the Poisson ratio $\sigma \approx 1$) suggest very strongly that the excitations in question are dislocation pairs which screen shear by increasing their dipole moment; this proceeds by diffusion and is most probably also responsible for the increase in η.[4] To fit the results to the theory of Ref. 4 requires knowing the dislocation diffusion coefficients. These may have "anomalous" behavior as does that for vortices in superfluid helium films[16]; hopefully here, too, they can be measured independently, at which point a detailed fit would become very interesting, particularly if it also incorporates the results of Ref. 10 on low-wave-vector longitudinal sound damping. But even without this, evidence is building up that the 2D electron system does melt via the "classic" Kosterlitz-Thouless instability to independent dislocations.

The present experiment demonstrates clearly that shear propagates. And it has brought new and very direct information on the viscoelastic response at a



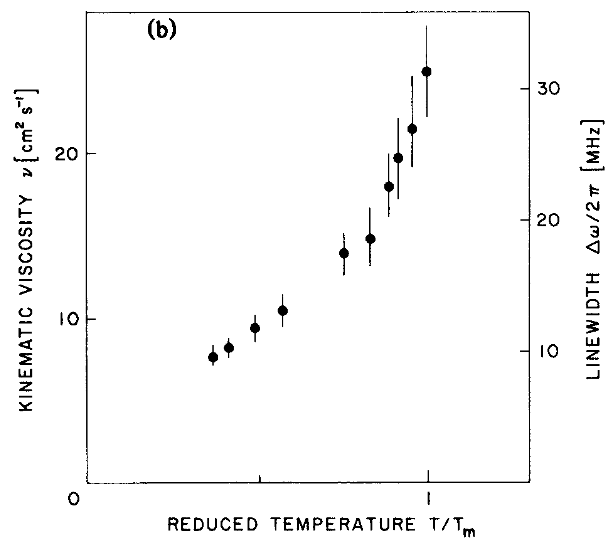


FIG. 3. (a) The elastic shear modulus μ' normalized to its extrapolated zero-temperature value $\mu(T=0)$, plotted as a function of the temperature T. The asterisk locates the calculated classical zero-temperature value (Ref. 15) for the experimentally measured density. The right-hand scale permits a direct comparison with the Kosterlitz-Thouless stability criterion $\mu a_0^2/4\pi > T$. Because the absolute temperature may be in error by $\pm 10\%$, the slope of criterion on this plot should be considered correct to $\pm 10\%$. Relative temperatures are accurate to $\pm 3\%$. (b) The kinematic viscosity $\nu = \eta/mn$ plotted against temperature. The right-hand scale is the resonance width, at $E_\perp = 128$ V cm^{-1} and $\omega_0/2\pi = 1060$ MHz, from which ν is derived. Data for $n = 6\times 10^7$ cm^{-2}.

specific value of k and ω which should help greatly in identifying the mechanism of melting. The way is open to studying μ and η as functions of (k,ω) and that would provide a very stringent test of models. The method should also be applicable to determining the shear viscosity in the liquid phase and thereby to identifying the predicted, elusive, intermediate oriented fluid (hexatic phase).[3,4]

We wish to thank Christian Glattli for much help and many useful discussions, and Patrick Pari for valued technical assistance.

---

[a]Present address: Department of Physics, Brown University, Providence, R.I. 02912.

[1]Recent low-frequency mechanical measurements of shear moduli in a freely suspended 2D liquid crystal [D. J. Bishop, W. O. Sprenger, R. Pindak, and M. E. Neubert, Phys. Rev. Lett. **49**, 1861 (1982)] and in a monomolecular film on water [B. M. Abraham, K. Miyano, J. B. Ketterson, and S. Q. Xu, Phys. Rev. Lett. **51**, 1975 (1983)] give similar information on viscoelasticity, albeit with moduli 10^9 times larger than ours, but the damping was too high for propagation. Also of possible general relevance to this point are inelastic neutron-scattering data from Ar films on Grafoil showing a broad peak from near zone-edge phonons [H. Taub, K. Carniero, J. K. Kjems, L. Passell, and J. P. McTague, Phys. Rev. B **16**, 4551 (1977)].

[2]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[3]D. R. Nelson and B. I. Halperin, Phys. Rev. B **19**, 2457 (1979).

[4]A. Zippelius, B. I. Halperin, and D. R. Nelson, Phys. Rev. B **22**, 2514 (1980).

[5]C. C. Grimes and G. Adams, Phys. Rev. Lett. **42**, 795 (1979).

[6]A. S. Rybalko, B. N. Essel'son, and Yu. Z. Kovdrya, Fiz. Nizk. Temp. **5**, 947 (1979) [Sov. J. Low Temp. Phys. **5**, 450 (1979)].

[7]D. Marty, J. Poitrenaud, and F. I. B. Williams, J. Phys. (Paris), Lett. **41**, L311 (1980).

[8]R. Mehrotra, B. M. Guenin, and A. J. Dahm, Phys. Rev. Lett. **48**, 641 (1982).

[9]F. Gallet, G. Deville, A. Valdes, and F. I. B. Williams, Phys. Rev. Lett. **49**, 212 (1982). They measured the frequency of a nonpropagating $(\partial\omega/\partial h = 0)$ mode in the limit $k \to 0$ where an electron vibrates in a substrate deformation dimple. This is sensitive to the localization of the electron in the solid, so giving $\langle u^2\rangle \sim \int T|\mu(k)|^{-1}k^{-2}d^2k$: it cannot tell if shear propagates, nor differentiate η from μ'.

[10]C. J. Guo, D. B. Mast, R. Mehrotra, Y.-Z. Ruan, M. A. Stan, and A. J. Dahm, Phys. Rev. Lett. **51**, 1461 (1983).

[11]D. Fisher, B. I. Halperin, and P. M. Platzman, Phys. Rev. Lett. **42**, 798 (1979).

[12]D. S. Fisher, Phys. Rev. B **26**, 5009 (1982).

[13]M. Chang and K. Maki, Phys. Rev. B **27**, 1646 (1983).

[14]R. Morf, Phys. Rev. Lett. **43**, 931 (1979).

[15]L. Bonsall and A. A. Maradudin, Phys. Rev. B **15**, 1959 (1977).

[16]M. Kim and W. I. Glaberson, Phys. Rev. Lett. **52**, 53 (1984).