Resonance Localization and Poloidal Electric Field due to Cyclotron Wave Heating in Tokamak Plasmas

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(Received 1 March 1984)

The perpendicular heating in cyclotron waves tends to pile up the resonant particles toward the low magnetic field side with their banana tips localized to the resonant surface. A poloidal electric field with an $\vec{E}\times\vec{B}$ drift comparable to the ion vertical drift in a toroidal magnetic field may result. With the assumption of anomalous electron and neoclassical ion transport, density variations due to wave heating are discussed.

PACS numbers: 52.55.Gb, 52.50.Gj

With application of the electron cyclotron resonance heating (ECRH) to tokamak plasmas, the plasma density has been observed to drop.[1] By contrast, with application of the ion cyclotron resonance frequency (ICRF) heating, an increase in the plasma density occurs.[2] The very nature of the cyclotron wave heating is to increase the perpendicular kinetic energy of the resonant particles. This tends to pile up the resonant species toward the weak-field side in a magnetic well. A poloidally varying electrostatic potential is expected to rise and to saturate at a level that balances the trapping effect due to the magnetic well and the rf heating. The electrostatic potential should have strength large enough to expel particles out of the magnetic well, which means that a $\cos\theta$ component of $e\phi/T$ could be on the order of the well depth r/R, and consequently an $\vec{E}\times\vec{B}$ drift of order $v^2/R\,\Omega$ comparable to the ∇B drift could result. Depending on the sign of the resonant charge, this may increase or decrease the neoclassical ion transport rate. If one argues that electrons move across the magnetic field lines more freely because of the magnetic stochasticities or other turbulent diffusions, and the particle confinement is primarily determined by the neoclassical transport at the ion rate, then the ECRH can enhance the ion vertical drift and cause the density to drop, while the ICRF heating can reduce the ion vertical drift and improve the density confinement.

We start our analysis by examining the single-particle trajectory in a tokamak of circular cross section. Without the rf effect, the energy W and the magnetic moment μ are constants of the motion, where $W = mv_\parallel^2/2+\mu B$, and $B=B_0(1-r\cos\theta/R)$. Trapped particles with $\mu B_0(1+r/R) > W$ will be reflected at turning points where $v_\parallel$ vanishes. If it is assumed that the resonance occurs at $\theta=\theta_0$, where the parallel velocity $v_\parallel$ remains unchanged at the value $v_{\parallel 0}$, but μ and W are increased to μ' and W', the tips of the banana trajectories can be obtained from

$$\frac{mv_{\parallel 0}^2}{2}+\begin{pmatrix}\mu\\ \mu'\end{pmatrix}B_0\left(1-\frac{r}{R}\cos\theta_0\right) =\begin{pmatrix}W\\ W'\end{pmatrix}=\begin{pmatrix}\mu\\ \mu'\end{pmatrix}B_0\left[1-\frac{r}{R}\cos\begin{pmatrix}\theta_t\\ \theta_t'\end{pmatrix}\right], \quad (1)$$

by simply subtracting the two equations, $(\cos\theta_t' - \cos\theta_0)/(\cos\theta_t-\cos\theta_0)=\mu/\mu'$, where θ_t and θ_t' are the poloidal angles of the banana tips at energies W and W', respectively. If μ' continues to increase, the banana tip will move to the resonance location, viz., $\theta_t'\to\theta_0$, and a stationary orbit results. This resonance localization has also been shown by a Monte Carlo code.[3]

The piling up of the charge density at the low-field side is limited by the poloidally varying electrostatic potential, which pushes the same species to the high-field side. The guiding-center drift kinetic equation[4] in terms of the variables (W,μ,θ,t) can be written as

$$\frac{\partial f}{\partial t}+v_\parallel\frac{\Theta}{r}\frac{\partial f}{\partial\theta}+eE_\theta v_\parallel\Theta\frac{\partial f}{\partial W} =\mathscr{L}D\mathscr{L}f+cf, \quad (2)$$

where $v_\parallel=\pm(W-\mu B)^{1/2}(2/m)^{1/2}$ and $\Theta=B_\theta/B_0$. The left-hand side has the energy $W_\phi=mv_\parallel^2/2+\mu B_0[1-r/R(\cos\theta)]+e\phi$ as the constant of the motion, which is invalid only inside the rf resonance region where the rf term $\mathscr{L}D\mathscr{L}f$ is important. By assuming $\phi=0$ initially when the particle has the energy E and the magnetic moment μ, and taking $\theta_0=\pi/2$ and $\phi(\theta_0)=0$, we have $[e\phi(\theta_t')/\mu B_0]R/r = (\mu'/\mu)\cos\theta_t' - \cos\theta_t$. To maintain the charge neutrality, we demand that $\theta_t'\simeq\theta_t$ so that the charge distribution is relatively unshifted. This gives $e\phi(\theta_t)/\mu B_0\simeq(r/R)\times\cos\theta_t(\mu'-\mu)/\mu$. Therefore, $e\phi/T\simeq(r/R)(\Delta\mu/\mu)\cos\theta$. Since $\Delta\mu/\mu$ is roughly the temperature anisotropy $(T_\perp-T_\parallel)/T_\parallel$, which can be destroyed by the collisions from the term cf while the rf tries to maintain it, it is conceivable that $\Delta\mu/\mu \sim O(\nu_{\rm rf}/\nu_c)$, where $\nu_{\rm rf}$ is the effective rf heating rate.

The theory proceeds with the assumption that the bounce frequency ω_B is greater than both the collisional frequency ν_c and $\nu_{\rm rf}$. If we further assume that $\omega_B >> \epsilon\omega_B \sim \nu_c \sim \nu_{\rm rf}$, $\partial f^{(0)}/\partial\theta = 0$ to lowest order. To the next order,

$$\frac{\partial f^{(0)}}{\partial t} + \sigma q \Theta \frac{1}{r}\frac{\partial f^{(1)}}{\partial \theta} + eE_\theta q\sigma\Theta\frac{\partial f^{(0)}}{\partial W} = \mathscr{L} D \mathscr{L} f^{(0)} + cf^{(0)}. \tag{3}$$

Here, $q = |v_\parallel|$, $\sigma = v_\parallel/q$, and $\epsilon = r/R$. Taking the bounce average of Eq. (3) gives

$$\langle q^{-1}\rangle \frac{\partial f^{(0)}}{\partial t} = \langle q^{-1}\mathscr{L} D \mathscr{L} f^{(0)}\rangle + \langle q^{-1} cf^{(0)}\rangle, \tag{4}$$

with the angular bracketing defined as $\langle x\rangle = \int x\, d\theta/\int d\theta$. Subtracting Eq. (4) from Eq. (3) gives

$$\sigma q\Theta\left(\frac{1}{r}\frac{\partial f^{(1)}}{\partial\theta} + eE_\theta\frac{\partial f^{(0)}}{\partial W}\right) = \mathscr{L} D \mathscr{L} f^{(0)} + cf^{(0)} - \langle q^{-1}\rangle^{-1}(\langle q^{-1}\mathscr{L} D \mathscr{L} f^{(0)}\rangle + \langle q^{-1}cf^{(0)}\rangle). \tag{5}$$

With division of the right-hand side by $\sigma q\Theta$ and integration over the velocity space, the summation over σ nullifies the collisional terms, since $cf^{(0)}$ is independent of the direction of $v_\parallel$. Similarly, we may take the parallel wave number $k_\parallel$ to be zero to make the $\mathscr{L} D \mathscr{L}$ term, which is symmetrical in $v_\parallel$ except in the resonance condition $\omega - n\Omega = k_\parallel v_\parallel$, independent of σ and its contribution vanish. It will be shown that the dominant effect in creating the electrostatic potential comes from $f^{(0)}$; the contribution to $f^{(1)}$ from finite $k_\parallel$ is therefore ignored. Equation (5) gives

$$n^{(1)} = \int\int f^{(1)} d^3v = \sum_\sigma \int\int e\phi\frac{\partial f^{(0)}}{\partial W}\frac{2\pi\, dE\, d\mu B}{m^2 q}, \tag{6}$$

which is nothing more than the adiabatic response.

We will now restrict ourselves to ECRH. The general conclusions are expected to be valid for ICRF heating. Since the rf effect is unimportant to ions during the ECRH, the ion response is determined by Eq. (6), which gives $n_i = n_0(1 - e\phi/T_i)$ by taking $f^{(0)}$ as Maxwellian. The analytical treatment will proceed in the weak rf limit, namely, $\delta = \nu_{\rm rf}/\nu_c << 1$. This regime was studied for current drive[5] by calculating the first moment of $f^{(0)}$. Here the zeroth moment is needed for the charge density. The first-order charge perturbation of electrons can be obtained by keeping accuracy to the lowest order in δ in Eq. (6), so that $n^{(1)} = n_0 e\phi/T_e$. Imposing charge neutrality gives

$$e\phi\left(\frac{1}{T_i} + \frac{1}{T_e}\right) = 1 - \frac{1}{n_0}\int\int 4\pi\, dW\, d\mu B\frac{f^{(0)}}{qm^2}.$$

Note that the θ variation of ϕ comes from B and q. If $f^{(0)}$ is a function of W alone, then the charge density will be independent of θ. The perpendicular heating by cyclotron waves results in a poloidally varying charge density by making $f^{(0)}$ μ dependent.

To solve for $f^{(0)}$, the collisional term is taken to be the pitch-angle scattering operator as in the Lorenz model, and the rf term is taken to be the quasilinear diffusion operator with the fundamental cyclotron harmonic resonance at the plasma center. The appropriate forms for Eq. (4) are given by

$$\langle q^{-1}cf^{(0)}\rangle = \frac{m\nu_{\rm ei}}{2}\frac{Z_{\rm eff}}{v^3}\frac{\partial}{\partial\mu}\langle qB^{-1}\rangle\mu\frac{\partial}{\partial\mu}f^{(0)},$$

and

$$\langle q^{-1}\mathscr{L} D \mathscr{L} f^{(0)}\rangle = \mathscr{L}_0\frac{e^2|E_-|^2\mu B_0 N}{4\Omega_0\epsilon[2m(W-\mu B_0)]^{1/2}\pi}\mathscr{L}_0 f^{(0)},$$

where $\nu_{\rm ei} = 4\pi ne^4\ln\Lambda/m^2$, $Z_{\rm eff}$ is the effective Z, $\mathscr{L}_0 = \partial/\partial W + \partial/\partial\mu B_0$, E_- is the right-hand polarized electric field, and N is the number of passages through the rf region in one bounce. Here $N=0$ for deeply trapped particles with $\mu B_0 > W$ since they do not come to the rf region, $N=4$ for trapped particles with $\mu B_0(1+\epsilon) > W > \mu B_0$ by approximating the turning points to be $\theta_t = \pm\pi$, and $N=2$ for passing particles with $W > \mu B_0(1+\epsilon)$.

Equation (4) may be solved by the multiple-time expansion by writing $\partial/\partial t = \nu_c(\partial/\partial t_0 + \delta\partial/\partial t_1 + \ldots)$, where $\nu_c = \frac{1}{2}\nu_{\rm ei}Z_{\rm eff}(m/2T)^{3/2}$, and $\nu_{\rm rf} = e^2E_-^2/2^{3/2}m\Omega_0\epsilon T$. With expansion of $f^{(0)}$ in a power series of δ, viz., $f^{(0)} = f_0^{(0)} + \delta f_1^{(0)} + \ldots$, the lowest-order equation is satisfied by taking $f_0^{(0)}$ Maxwellian, namely, $f_0^{(0)} = n(t_1)/[2\pi T(t_1)/m]^{3/2} \times \exp[-W/T(t_1)]$ with the density and the temperature slowly varying. We generally omit the subscript e from n and T for electrons to be brief.

To next order,

$$\left\langle\frac{1}{q}\right\rangle\left(\frac{\partial f_0^{(0)}}{\partial t_1}+\frac{\partial f_1^{(0)}}{\partial t_0}\right)=\left(\frac{T}{W}\right)^{3/2}\frac{\partial}{\partial\mu}\mu\left\langle\frac{q}{B}\right\rangle m\frac{\partial f_1^{(0)}}{\partial\mu}+\mathscr{L}_0\frac{N}{2\pi}\frac{m^{1/2}T\mu B_0}{(W-\mu B_0)^{1/2}}\mathscr{L}_0 f_0^{(0)}. \quad (7)$$

Since $f_0^{(0)}$ has no t_0 dependence, the t_0 dependence of $f_1^{(0)}$ has to come from the homogeneous solution, which can be absorbed into $f_0^{(0)}$ in the lowest-order equation; one can therefore set $\partial f_1^{(0)}/\partial t_0=0$. Integrating Eq. (7) over μ once, we have

$$\left(\frac{n'}{n}+\frac{WT'}{T^2}-\frac{3T'}{2T}\right)f_0^{(0)}\int_0^\mu\langle q^{-1}\rangle d\mu=\left(\frac{T}{W}\right)^{3/2}\mu m\langle qB^{-1}\rangle\frac{\partial f_1^{(0)}}{\partial\mu}+\int_0^\mu\frac{m^{1/2}N}{2\pi}\left(\frac{\mu B_0}{T}-1\right)(W-\mu B_0)^{-1/2}f_0^{(0)}d\mu,$$

where the primes over n and T refer to the time derivative $\partial/\partial t_1$. To obtain n'/n, we divide Eq. (7) by $\langle q^{-1}\rangle$ and integrate over the velocity space. It is clear that n'/n is of order ϵ since both the rf and the collisions conserve the number of particles before the bounce average, and makes a contribution of order $\delta\epsilon^2$ to the electrostatic potential. We therefore neglect n'/n. Similarly, by multiplying Eq. (7) by $W/\langle q^{-1}\rangle$ and integrating over the velocity space, we can find T'/T. Again, the collisional term is of order ϵ since it conserves energy before the bounce average. The rf term gives to lowest order in ϵ, $T'/T\simeq 2^{3/2}/3\pi$.

The charge-neutrality condition can be simplified by taking the μ integration by parts to give

$$e\phi\left(\frac{1}{T_i}+\frac{1}{T_e}\right)=1-\frac{n(t_1)}{n_0}-\frac{\delta}{n_0}\int\int\frac{4\pi\, dW\, d\mu B_0}{m}q\frac{\partial f_1^{(0)}}{\partial\mu B_0}. \quad (8)$$

Subtracting the θ-average of Eq. (8) from itself allows us to approximate n by n_0. Defining $x=W/T$, $y=\mu B_0/T$, $T\,\partial f_1^{(0)}/\partial\mu B_0=Hf_0^{(0)}$, and taking $T_e=T_i=T$, we have that the $\cos\theta$ component of $e\phi$ is given by

$$\frac{e\phi_1}{T}\simeq-\frac{2\delta}{\pi^{1/2}}\int\int dx\,dy\,e^{-x}H\langle[x-y(1-\epsilon\cos\theta)]^{1/2}\cos\theta\rangle. \quad (9)$$

With use of the identity

$$\int_0^\mu\langle q^{-1}\rangle d\mu=(2Wm)^{1/2}(1-\epsilon^2)^{-1/2}/B_0-\langle mq/B\rangle,$$

$\partial f_1^{(0)}/\partial\mu B_0$ obtained from Eq. (7) gives

$$H=\left(x-\frac{3}{2}\right)\frac{(2x)^{3/2}}{3y\pi}\left\{x^{1/2}\left\langle\frac{[x-y(1-\epsilon\cos\theta)]^{1/2}}{1-\epsilon\cos\theta}\right\rangle^{-1}-1\right\}$$
$$-\frac{Nx^{3/2}}{2^{3/2}y\pi}\left\langle\frac{[x-y(1-\epsilon\cos\theta)]^{1/2}}{1-\epsilon\cos\theta}\right\rangle^{-1}\int_0^y dy'\frac{y'-1}{(x-y')^{1/2}}. \quad (10)$$

For the trapped particles with $\epsilon y>x-y>-\epsilon y$, since the phase volume is reduced to $O(\epsilon)$, the integrand is kept to lowest-order accuracy. With use of the approximation

$$\frac{\langle[x-y(1-\epsilon\cos\theta)]^{1/2}\cos\theta\rangle}{\langle[x-y(1-\epsilon\cos\theta)]^{1/2}\rangle}\approx\frac{\int_0^{\pi/2}\cos^{3/2}\theta\,d\theta}{\int_0^{\pi/2}\cos^{1/2}\theta\,d\theta}=\frac{\Gamma^2(\frac{1}{4})}{12\Gamma^2(\frac{3}{4})}\approx 0.73, \quad (11)$$

and neglect of the second term in the braces of Eq. (10), the contribution from the trapped population is given by

$$\frac{e\phi_t}{T}=-\frac{1.46\delta}{\pi^{3/2}}\int_0^\infty dy\left[\int_{y(1-\epsilon)}^{y(1+\epsilon)}\frac{2^{3/2}}{3}dx\,e^{-x}\frac{x^2}{y}(x-\tfrac{3}{2})-\int_y^{y(1+\epsilon)}\frac{2^{1/2}x^{3/2}}{y}e^{-x}dx\int_0^y dy'\frac{y'-1}{(x-y')^{1/2}}\right],$$
$$\simeq O(\delta\epsilon^{3/2}). \quad (12)$$

Here, we have made use of the identities $\int_0^\infty e^{-y}y^{n-1}dy=\Gamma(n)$ and

$$\int_0^y dy'(y'-1)/(x-y')^{1/2}=\tfrac{4}{3}x^{3/2}-2x^{1/2}-\tfrac{4}{3}(x-y)^{3/2}-2(y-1)(x-y)^{1/2}.$$

The contribution from the trapped population is negligible to the leading order $O(\delta\epsilon)$. For the passing particles, we make the following approximations:

$$\langle [x-y(1-\epsilon\cos\theta)]^{1/2}\cos\theta\rangle \simeq \tfrac{1}{4}y\epsilon/(x-y)^{1/2}, \quad \langle [x-y(1-\epsilon\cos\theta)]^{1/2}/(1-\epsilon\cos\theta)\rangle \simeq (x-y)^{1/2}.$$

Their contribution to the electrostatic potential is given by

$$\frac{e\phi_p}{T} \simeq -\frac{\delta\epsilon}{(2\pi)^{3/2}}\int_0^\infty dx\, e^{-x}x^{3/2}\int_0^{x/(1+\epsilon)} dy \left\{\frac{4}{3}\left(x-\frac{3}{2}\right)\left[\frac{x^{1/2}}{x-y}-\frac{1}{(x-y)^{1/2}}\right]-\frac{1}{x-y}\int_0^y dy' \frac{y'-1}{(x-y')^{1/2}}\right\}, \simeq 0.2\delta\epsilon. \quad (13)$$

Taking typical DIII parameters, $R=140$ cm, $r=a/2=20$ cm, 1 keV temperature, 60-GHz cyclotron wave, we have $\nu_c \sim 2\times 10^4 Z_{eff}$/sec and δ is around unity when the right-hand polarized electric field $E_- \sim 50$ V/cm with $Z_{eff} \sim 2$ to account for the electron-electron collisions. It should be emphasized that if electron transport is anomalous and exceeds the ion transport, one might expect that the diffusion coefficient will be proportional to the square of the vertical drift velocity. For $e\phi_1/T \sim 0.4\epsilon$, the ion vertical drift is enhanced by a factor of 1.4, thus doubling the diffusion coefficient, and the density drop should be significant. The electric field at the waveguide mouth from a 200-kW gyrotron is typically $E_0 \sim 10$ kV/cm in an area of 100 cm^2. At the plasma center, the rf area could be $2\pi Ra \simeq 3.6\times 10^4$ cm^2, so that the electric field is reduced to $E_0 \sim 500$ V/cm. Typically, $E_- \sim k_\perp \rho_e E_0 \sim \frac{1}{10}E_0 \sim 50$ V/cm. There is an uncertainty about the effecive rf volume. The assumption of $k_\parallel \simeq 0$ gives a weaker effect because of the smaller resonance width and smaller $v_\parallel$ in resonance. Nonetheless, the above estimate allows us to conclude that the present mechanism is relevant to explain the density drop. The present mechanism is enhanced with the second-harmonic heating. Replacing the effective rf frequency by $(k_\perp^2 T/2\Omega_0^2)\nu_{rf}$ with use of the finite Larmor radius expansion, we find the potential to be $e\phi_1/T \simeq 0.6\delta\epsilon$.

Turning to ICRF, since the collisional effect may also come from the slowing down, the numerical coefficient of Eq. (13) may be different. Nevertheless, to make the present calculation valid, we may consider a relatively dirty plasma so that the ion-impurity pitch-angle scattering dominates. The collisional frequency is reduced by $m^{1/2}$, and therefore δ is unity when the left-hand polarized electric field $E_+ \simeq 10$ V/cm at the fundamental and $E_+ \simeq E_0$ has to be $(k_\perp \rho_i)^{-1}$ larger at the second harmonic. From the experimentally observed strong tail temperature anisotropy it indicates that the present mechanism could also be operative. The possible existence of the poloidal electric field during the neutral-beam injection has been discussed.[6] The density change and the poloidal electric field, however, have not been correlated because the neutral-beam injection also provides the particle source. The lower-hybrid heating tends to increase the electron parallel energy and/or the ion perpendicular energy; the density increase can therefore be expected, and has been noted in Versator.[7]

To correlate the electrostatic potential with the density change during ECRH or ICRF heating, it is most instructive to measure the poloidally varying potential directly. Other indirect observations such as impurity transport or $\Delta n/n$ as functions of the resonance location may shed light on the mechanism. Also, to gas puff up-down asymmetrically may help reveal the flow pattern of the enhanced particle transport. The dependence of the density drop on $\nu^* \equiv \nu_c/\epsilon^{3/2}\omega_b$ will identify the importance of the particle-trapping effect. Early indication from PDX appears to be favorable in this regard.[8] The combination of both ECRH and ICRF heating is an obvious way to stabilize the density variation occurring when they are applied separately.

We thank F. Hinton for pointing out the importance of Eq. (6) to the ion response. This work was supported by the U.S. Department of Energy under Contract No. DE-AT03-84ER53158.

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