Proton Resonance Scattering Confirms the Gaussian Statistics of Decay Amplitudes

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(Received 24 May 1984)

High-resolution proton-scattering experiments yield sets of resonance-decay amplitudes with sometimes strong correlations between different decay channels. The joint probability distribution of the amplitudes is shown to be compatible with the Gaussian one, if all experiments are properly averaged and their finite-range-of-data error evaluated.

PACS numbers: 24.30.He, 24.60.–k, 25.40.Ep

In the statistical model of nuclear reactions,[1,2] it is assumed that the reduced partial-width amplitudes γ_c measuring the decay of a resonance into the channel c follow a Gaussian distribution centered at zero. This can be experimentally tested by extracting the partial widths γ_c^2 from reactions proceeding through isolated resonances. The partial widths should then follow a Porter-Thomas[3] distribution, a property which seems to be well established from low-energy neutron scattering.[4,5] A large amount of data are also available from high-resolution proton scattering.[6–9] Again the partial widths were found to be compatible with the Porter-Thomas distribution.

In recent years, a more subtle test on the statistics of the γ_c has become available, since a technique has been established[10] to measure relative signs of the γ's. It is now possible to study[6–9, 11–13] the joint probability distribution of pairs of amplitudes γ_c, γ_d pertaining to different decay channels c and d.

The first striking result is the appearance of large correlations between γ_c and γ_d. This is not at variance with the statistical model if nonelastic direct reactions are included that compete with the compound-nuclear–resonance reactions. Direct reactions are known to induce such correlations.[14,15] They do not contradict the assumed Gaussian distribution of the γ's either. Their existence says that the covariance matrix of the γ's is not necessarily diagonal.

However, if the joint probability distribution of the amplitudes γ_c is Gaussian, then any correlation between γ_c and γ_d determines the correlation between the widths γ_c^2, γ_d^2. The above-mentioned experiments can be used to check this relation and, hence, provide a test on the Gaussian distribution of the γ's.

To be definite, let us introduce the normalized coefficient r of correlation between the statistical variables x and y:

$$r(x,y) = \overline{(x-\bar{x})(y-\bar{y})}\,[\overline{(x-\bar{x})^2}\;\overline{(y-\bar{y})^2}]^{-1/2}. \quad (1)$$

Here, the bars denote averages over the statistical ensemble. If now the distribution of the pair is Gaussian then[16]

$$r^2(\gamma_c,\gamma_d) = r(\gamma_c^2,\gamma_d^2). \quad (2)$$

In Table I, the experimental information is listed. We indicate the measured reaction, the compound nucleus, the number N of resonances analyzed, the exit channels c and d in the channel-spin representation, and the experimental values $r_{\text{ex}}(\gamma_c,\gamma_d)$ of the amplitude correlation and $r_{\text{ex}}(\gamma_c^2,\gamma_d^2)$ of the width correlation as well as their ratio

$$v = r_{\text{ex}}^2(\gamma_c,\gamma_d)/r_{\text{ex}}(\gamma_c^2,\gamma_d^2). \quad (3)$$

From the statistical model one expects v to be unity. The experiments yield, however, important deviations from unity. That they are in fact compatible with the Gaussian distribution and even may be considered to confirm it shall be shown in the sequel.

The quantities r_{ex} are constructed according to Eq. (1) but with the ensemble averages

$$\overline{(x-\bar{x})(y-\bar{y})},$$

etc., replaced by the finite averages $\langle (x-\langle x\rangle)(y-\langle y\rangle)\rangle$, etc., over the N resonances available from experiment. The question is then this: What is the variance σ of v, if N pairs of amplitudes γ_c, γ_d are drawn from a Gaussian ensemble? The answer is given elsewhere,[18] namely

$$\sigma^2 = N^{-1}(\rho^{-4} + 4\rho^{-2} - 8 + 3\rho^4). \quad (4)$$

Here, ρ is the expectation value $r(\gamma_c,\gamma_d)$ of the amplitude correlation. We may call σ the "finite range of data" (FRD) error of the experiment.

The distribution of v has also been studied via Monte Carlo simulations.[19] The analytical result (4) agrees with them. It should be stressed, however, that Eq. (4) takes care of the FRD error only, but not of the experimental errors in the proper sense, as, e.g., accuracy of measured cross sections and detection threshold for weak resonances. Their influence on the distribution of the test quantity v

TABLE I. Experimental tests on the statistical distribution of correlated resonance-decay amplitudes γ_c, γ_d. For joint Gaussian distributions the test quantity v in column 9 is expected to be unity up to the error σ in column 10. These results can be contracted into an overall test quantity V. See text.

Experiment	Ref.	Compound Nucleus	Channel c	Channel d	N	$r_{ex}(\gamma_c,\gamma_d)$	$r_{ex}(\gamma_c^2,\gamma_d^2)$	v	σ
$^{50}Cr(p,p')^{50}Cr(2^+)$	13	$^{51}Mn, \frac{5}{2}^+$ states	$l=0, s=\frac{5}{2}$	$l=2, s=\frac{3}{2}$	38	0.62	0.90	0.43	0.24
			$l=0, s=\frac{5}{2}$	$l=2, s=\frac{5}{2}$		0.58	0.56	0.60	0.42
			$l=2, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		0.55	0.33	0.92	0.63
$^{48}Ti(p,p')^{48}Ti(2^+)$	7	$^{49}V, \frac{5}{2}^+$ states	$l=0, s=\frac{5}{2}$	$l=2, s=\frac{3}{2}$	45	-0.06	0.46	0.008	0.79
			$l=0, s=\frac{5}{2}$	$l=2, s=\frac{5}{2}$		0.88	0.71	1.1	0.14
			$l=2, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		0.01	0.28	0.0004	1.3
$^{44}Ca(p,p')^{44}Ca(2^+)$	8	$^{45}Sc, \frac{5}{2}^+$ states	$l=0, s=\frac{5}{2}$	$l=2, s=\frac{3}{2}$	53	0.22	0.67	0.072	0.46
			$l=0, s=\frac{5}{2}$	$l=2, s=\frac{5}{2}$		0.72	0.27	1.9	0.41
			$l=2, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		0.06	-0.08	-0.045	-
$^{48}Ti(p,p')^{48}Ti(2^+)$	11	$^{49}V, \frac{3}{2}^+$ states	$l=0, s=\frac{3}{2}$	$l=2, s=\frac{3}{2}$	30	0.84	0.43	1.7	0.32
			$l=0, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		-0.51	0.85	0.31	0.33
			$l=2, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		-0.65	0.15	3.0	0.79
$^{44}Ca(p,p')^{44}Ca(2^+)$	9	$^{45}Sc, \frac{3}{2}^-$ states	$l=1, s=\frac{3}{2}$	$l=1, s=\frac{5}{2}$	37	-0.66	0.23	1.9	0.60
$^{50}Cr(p,p')^{50}Cr(2^+)$	9	$^{51}Mn, \frac{3}{2}^-$ states	$l=1, s=\frac{3}{2}$	$l=1, s=\frac{5}{2}$	24	0.34	0.04	2.9	2.9
$^{48}Ti(p,p')^{48}Ti(2^+)$	6	$^{49}V, \frac{3}{2}^-$ states	$l=1, s=\frac{3}{2}$	$l=1, s=\frac{5}{2}$	19	-0.65	0.17	2.5	0.95
			$l=1, s=\frac{3}{2}$	$l=1, s=\frac{5}{2}$	24	-0.58	0.51	0.66	0.56
$^{56}Fe(p,p')^{56}Fe(2^+)$	17	$^{57}Co, \frac{5}{2}^+$ states	$l=0, s=\frac{5}{2}$	$l=2, s=\frac{3}{2}$	83	0.36	0.79	0.16	0.33
			$l=0, s=\frac{5}{2}$	$l=2, s=\frac{5}{2}$		0.60	0.51	0.71	0.29
			$l=2, s=\frac{3}{2}$	$l=2, s=\frac{5}{2}$		0.36	0.28	0.46	0.68

has been investigated in Ref. 19, too. They were not found very important for the relatively small number of resonances ($N \approx 50$) investigated in a typical experiment. Nevertheless, Eq. (4) gives only the minimum error with which the results of Refs. 6–9 and 11–13 are beset.

Note that σ grows beyond all limits if ρ approaches zero. This means that it is increasingly difficult to verify relation (2) for decreasing correlation.

We have estimated ρ from the average of $r_{ex}(\gamma_c, \gamma_d)$ and $r_{ex}(\gamma_c^2, \gamma_d^2)^{1/2}$ and listed σ in Table I. The FRD errors are such that a significant deviation from the expected value of $v = 1$ is not visible.

As a result of formula (4) one can express this statement in a precise form, since one can lump all the experiments together and find an average value V of the test quantity v together with an average error Σ. Let us weight the different v_i with the inverse errors σ_i^{-1} and define

$$V = \left(\sum_{i=1}^{M} \frac{v_i}{\sigma_i}\right)\left(\sum_{i=1}^{M} \frac{1}{\sigma_i}\right)^{-1}. \tag{5}$$

Here, the index i runs over the M available experiments. The variance of V is then estimated by

$$\Sigma^2 = M\left(\sum_{i=1}^{M} \frac{1}{\sigma_i}\right)^{-2}. \tag{6}$$

From Table I, $M = 18$ values for v can be included in expressions (5) and (6). We exclude the ninth line with a negative result for $r_{ex}(\gamma_c^2, \gamma_d^2)$ showing that one has practically no estimate for the very small correlation ρ in this case. Out of the eighteen experiments, only fourteen are independent. Consider, e.g., the first three lines of the Table. Three exit channels $(l,s)=(0,\frac{5}{2})$, $(2,\frac{3}{2})$, and $(2,\frac{5}{2})$ were studied, from which two independent correlation coefficients can be extracted. All three possible combinations are, however, listed in Table I. We schematically take care of this by multiplying the error Σ of Eq. (6) by $(18/14)^{1/2}$. The result is

$$V=0.93\pm0.11. \quad (7)$$

It is in agreement with the assumption that the resonance-decay amplitudes have joint Gaussian distributions which requires $V=1$.

The author benefitted from stimulating conversations with Professor G. E. Mitchell and Professor T. H. Seligman. He is indebted to Professor G. E. Mitchell for the communication of yet unpublished results.

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