Power-Spectrum Analysis of Fluctuations of Pseudorapidity Distributions in Nucleus-Nucleus Collisions at Very High Energies

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Two events of ultrarelativistic nucleus-nucleus collisions observed in the Japanese-American cooperative emulsion experiment show large fluctuations of the pseudorapidity distributions of the charged secondaries. The fluctuations are analyzed by use of the power spectrum. A simulation is used to estimate the statistical noise. Fairly strong signals are found in the Si-AgBr event, while the Ca-C event exhibits only weak signals.

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Quark-gluon plasma (QGP) is one of the biggest topics in the latest hadron physics. The relevant main subjects are (i) the properties of the phase transition, (ii) the possibility of formation of QGP in, for example, nucleus-nucleus collisions at high energies, and (iii) the signals of QGP.[1] There have been many proposals about the signals; for example, lepton pairs, direct photons, strange particles, and so on.[1-5] Furthermore, fluctuations of various quantities, in particular, those of the rapidity density distribution on an event-by-event basis have been suggested recently as new, useful signatures.[1b, 5-7] It is then very important to establish a quantitative method to analyze the observed fluctuations and to extract true signals from them. In this paper, we propose the power-spectrum analysis as a useful method to study the fluctuations of the rapidity (or pseudorapidity) distributions. The method is applied to the two events (Si-AgBr and CaC interactions) observed in the Japanese-American cooperative emulsion experiment (JACEE).[8] The power spectra of both events (in particular, Si-AgBr) show some prominent peaks with heights significantly higher than the level of the statistical noise estimated from a simulation, thus indicating possible existence of fluctuations of nonstatistical origin. In this paper, we present only the most essential result of our analysis. Further details of our power-spectrum analysis as well as the results of analyses using quantities other than the power spectrum will be given in a forthcoming paper.

Let $dN/d\eta \equiv f(\eta)$ be the pseudorapidity distribution of charged particles produced in *one event* of hadronic collisions. The multiplicity of the charged particles is then fixed to a definite value. Then, one can suppose the existence of a statistical ensemble of equivalent events with the same incident particles, the same incident energy, the same multiplicity, and the same nonstatistical fluctuation, if any, with generally different $f(\eta)$. The ensemble average of $f(\eta)$ gives a smooth distribution $f_S(\eta)$. The rapidity density fluctuation of nonstatistical origin may manifest itself as an oscillatory pattern of $f_S(\eta)$.

We begin with fitting $f(\eta)$ by a smooth function $f_0(\eta)$ with least oscillation because we are looking for signals of oscillations of nonstatistical origin. If there were experimental data on $f(\eta)$ for a sufficient number of equivalent events, the average over those events of $f(\eta)$ may be used as $f_0(\eta)$. Unfortunately, data on only one event are available for both Si-AgBr and Ca-C interactions at some incident energies and some multiplicities. Therefore we assume rather arbitrarily a function of the form

$$f_0(\eta)=A\{(1-e^{-Y-\eta})(1-e^{-Y+\eta})\}^B \quad \text{for } -Y\leq\eta\leq Y, \tag{1}$$

and have carried out a least-squares fit to $f(\eta)$ by changing A and B with Y fixed. The parameter Y corresponds approximately to the value of the rapidity at the kinematical boundary. There is an uncertainty in the choice of Y because the estimated energies of the JACEE events have some uncertainties. Here, we tentatively take $Y=5.5$ for Si-AgBr and $Y=7.0$ for Ca-C. These choices correspond to incident energies about 4 TeV/nucleon for Si-AgBr and 100 TeV/nucleon for Ca-C. The least-squares fit then gives

$$\begin{aligned} A&=184, \quad B=8.1 \quad \text{for Si-AgBr;} \\ A&=81, \quad B=5.4 \quad \text{for Ca-C.} \end{aligned} \tag{2}$$

The smooth fits given by Eqs. (1) and (2) are shown in Figs. 1 and 2.

If a given number of particles are produced randomly in the η space with the probability distribution $f_0(\eta)$ either by a simulation or in a real event, the fluctuation of the obtained $f(\eta)$ around $f_0(\eta)$ may be regarded as a kind of noise. It is well known that the power spectrum is useful to extract

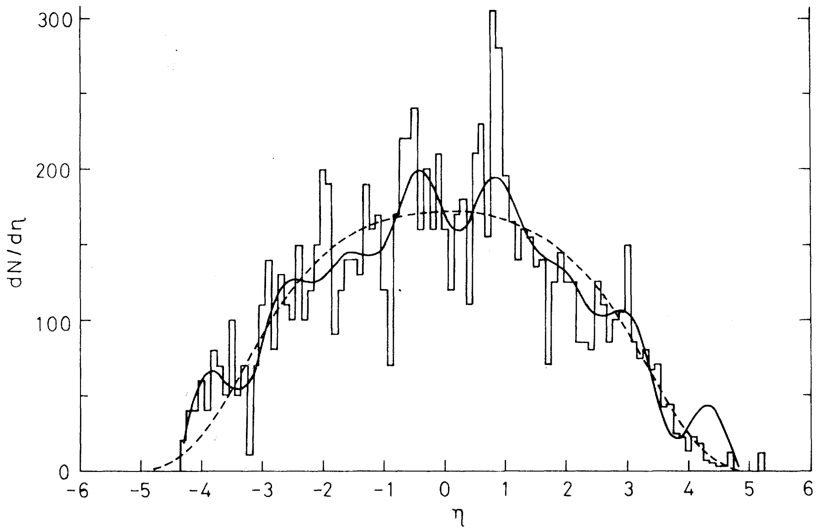

FIG. 1. Pseudorapidity distribution in Si-AgBr interaction. The histogram is the distribution observed by JACEE (Ref. 8). The dashed curve is the smooth fit $f_0(\eta)$ given by Eqs. (1) and (2), while the solid curve is the possible fluctuation $f_S(\eta)$ suggested from the power-spectrum analysis.

a signal from noise. When $f(\eta)$ is continuous, the power spectrum is defined as

$$\phi(\omega)=\frac{1}{2Y}\left|\int_{-Y}^{Y}d\eta e^{2\pi i\omega\eta}\{f(\eta)-f_0(\eta)\}\right|^2.$$

In the present case, $f(\eta)$ is given only at discrete points $\eta=\eta_k$ ($k=1,2,\ldots$), where η_k is the center of the k th bin of the histogram. Therefore, the power spectrum is defined as

$$\phi(\omega)=\frac{\Delta^2}{2Y}\left|\sum_k \exp(2\pi i\omega\eta_k)\times\{f(\eta_k)-f_0(\eta_k)\}\right|^2, \quad (3)$$

where Δ is the bin width. The bin used in the two JACEE events is 0.1. So we fix the bin width to 0.1. The power spectra of the two events are shown in Figs. 3 and 4. On the other hand, 1000 events were produced for each interaction by simulation with the probability distribution $f_0(\eta)$ in order to estimate the magnitude of the statistical fluctuation. The multiplicity assumed in the simulation is 1032 for Si-AgBr and 760 for Ca-C. The mean power spectrum $\langle\phi(\omega)\rangle$ of the 1000 events and the corresponding dispersion $D_\phi(\omega)=\{\langle\phi^2(\omega)\rangle-\langle\phi(\omega)\rangle^2\}^{1/2}$ were calculated and the results are shown also in Figs. 3 and 4. Both $\langle\phi(\omega)\rangle$ and $D_\phi(\omega)$ are essentially independent of ω except for the neighborhood of $\omega=0$. This is characteristic of white noise and hence is a proof that the quality of the pseudorandom numbers used in our simulation is very good.

The power spectra of both events exhibit some prominent peaks with heights exceeding $\langle\phi(\omega)\rangle$ by more than $3D_\phi(\omega)$. The characteristics of those peaks are summarized in Table I. The peaks listed in Table I are so prominent that they may be re-

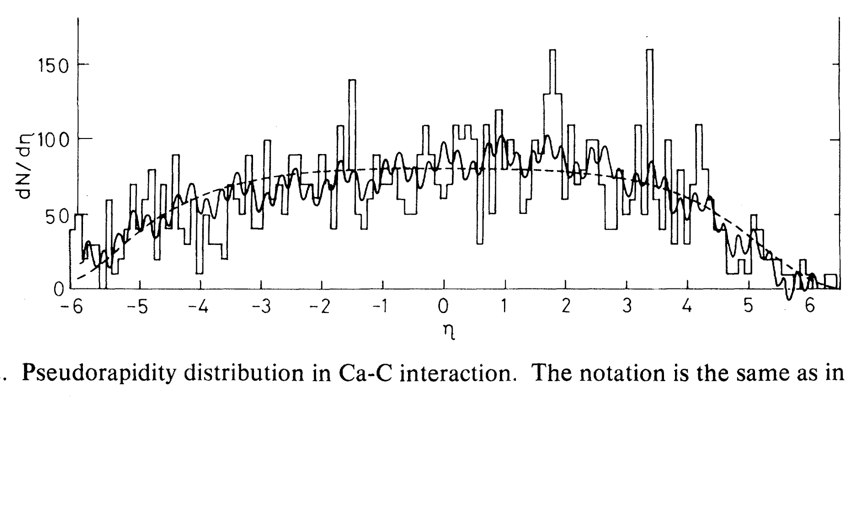

FIG. 2. Pseudorapidity distribution in Ca-C interaction. The notation is the same as in Fig. 1.

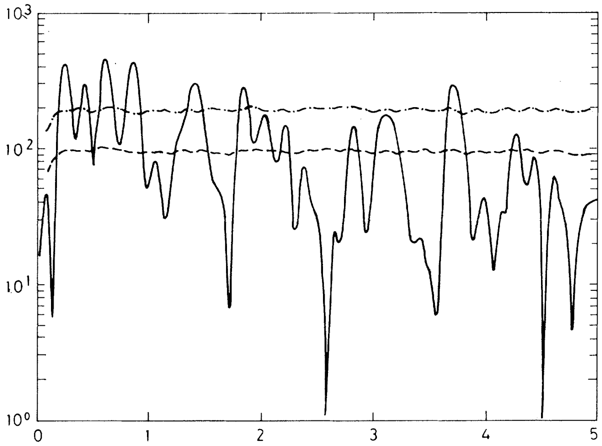

FIG. 3. The solid curve is the power spectrum $\phi(\omega)$ of the Si-AgBr event observed by JACEE. The dashed and the dash-dotted curves are, respectively, the mean power spectrum $\langle\phi(\omega)\rangle$ and $\langle\phi(\omega)\rangle + D_\phi(\omega)$ obtained from the simulation.

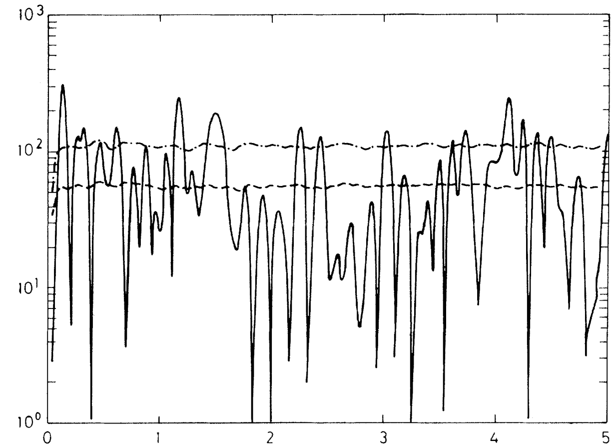

FIG. 4. The power spectrum of the Ca-C event. The notation is the same as in Fig. 3.

garded as true signals of nonstatistical fluctuations. The fluctuation in the η space will then have the following form:

$$f_{\rm FL}(\eta) = \sum_i a_i \sin(2\pi\omega_i\eta + b_i), \tag{4}$$

where ω_i is the position of the i th peak. The constant b_i can be determined approximately by maximizing the Fourier-sin transform $\sum_k \sin(2\pi\omega_i\eta_k + b_i)\{f(n_k) - f_0(\eta_k)\}$ for varying b_i. The amplitude a_i can be approximately given as

$$a_i \simeq [2\{\phi(\omega_i) - \langle\phi(\omega_i)\rangle\}/Y]^{1/2}, \tag{5}$$

where the background $\langle\phi(\omega_i)\rangle$ has been subtracted from $\phi(\omega_i)$ to give the most probable magnitude of the nonstatistical fluctuation. Addition of $f_{\rm FL}(\eta)$ to $f_0(\eta)$ yields the net probability distribution $f_S(\eta)$ which underlies the observed distribution $f(\eta)$

$$f_S(\eta) = f_0(\eta) + f_{\rm FL}(\eta). \tag{6}$$

The results are shown in Figs. 1 and 2. It should be noted here that the highest peak of $\phi(\omega)$ of the Ca-C event at $\omega = 0.125$ is most probably a fake signal. In fact, the corresponding wave length $\omega^{-1} = 8$ is very long and the contribution to $f_{\rm FL}(\eta)$ gives simply a very smooth left-right asymmetric component in $f_S(\eta)$. This implies that the left-right symmetric $f_0(\eta)$ given by Eq. (1) is not necessarily a good choice in the Ca-C case and an asymmetric component must be added to have a better $f_0(\eta)$. In this sense, the form of $f_0(\eta)$ can be corrected easily and systematically, thus making the final conclusion about the signals rather independent of the choice of the starting $f_0(\eta)$. So, rigorously speaking, one should recalculate the power spectrum of the Ca-C event by use of the modified $f_0(\eta)$. Such calculations will be done soon and the result will be given in a forthcomign paper. Here, we present only the result without such a correction.

Shown in Figs. 1 and 2 are the pseudorapidity density fluctuations of nonstatistical origin indicated from the power-spectrum analysis. However, one cannot rule out the possibility that the observed prominent peaks of $\phi(\omega)$ (three peaks in the Si-AgBr and two peaks in Ca-C with the additional one fake signal in the latter) are due to purely statistical fluctuations. The probability for such a case can be estimated by analyzing the statistics of the peak structure of $\phi(\omega)$ of all the events produced by the simulation. There are various methods to estimate the probability. Here the most direct method is

TABLE I. Characteristics of prominent peaks in power spectra.

Processes	Peak position ω	Peak height $\phi(\omega)$	$\dfrac{\{\phi(\omega) - \langle\phi(\omega)\rangle\}}{D_\phi(\omega)}$
Si-AgBr	0.244	429	3.63
	0.588	471	3.96
	0.850	431	3.60
Ca-C	0.125	324	5.02
	1.17	254	3.70
	4.11	257	3.69

presented. Since $\phi(\omega)$ of the Si-AgBr event has three prominent peaks with heights exceeding 400 ($\simeq \langle\phi\rangle + 3D_\phi$), we counted the number of events which have three or more peaks with heights exceeding 400 among the 1000 events produced by the simulation. It was found to be 84. Therefore, if all the fluctuations are entirely statistical, the probability per event to have such fluctuations as found in $\phi(\omega)$ of the Si-AgBr event is $(8.4 \pm 0.9)\%$. If the reference height is set at 420 instead of 400, the probability reduces to about 5% to 6%. Similarly, in the Ca-C case, the number of events which have two or more peaks with heights exceeding 200 (250) was found to be 700 (298) among the 1000 events generated by the simulation. From these results, one can conclude that the signals found in the Si-AgBr case are likely to be true signals while those seen in the Ca-C case are probably due to mere statistical fluctuations.

In our Monte Carlo simulation, particles are produced independently with the probability distribution $f_0(\eta)$. There is no difficulty in calculating the background $\langle\phi(\omega)\rangle$. The situation changes, however, if one wants to take into account many-particle correlations or the clustering effect. If the first particle in the first cluster is produced randomly at $\eta = \eta_c$ with the probability distribution $f_0(\eta)$ while the residual particles from the same cluster are produced around the first particle with, say, the Gaussian distribution $\exp\{-(\eta-\eta_c)^2/\delta^2\}$ and if the same procedure is repeated for the remaining clusters, then the statistical limit of the particle distribution deviates from $f_0(\eta)$ by a smearing effect. The smearing effect is the largest at the end regions. For Si-AgBr case, we have carefully estimated the pure effect from the clustering on $\langle\phi(\omega)\rangle$ by eliminating suitably the end-region effect. The result for the charged cluster of size 2 with $\delta = 1.2$ is that $\langle\phi(\omega)\rangle$ shows about 84% enhancement around $\omega = 0.1$ in comparison with $\langle\phi(\omega)\rangle$ for no clustering. The enhancement remains at some 35% level at $\omega = 0.24$ and vanishes for $\omega \gtrsim 0.4$. Therefore, the signal-to-noise ratio does not change for the second and the third peaks while it is reduced by about 25% for the first peak at $\omega = 0.244$.

Interpretation of the possible signals found in the Si-AgBr event is not easy. However, it is highly improbable that they are entirely due to the known clustering effect. If they are true signals, they may be due to an unknown collective effect such as large clustering, excited multiquark states or, more excitingly, the decay of quark-gluon plasma.

A general remark: Experimentally, events with extremely high multiplicities and very wide rapidity ranges are difficult to measure. Theoretically, however, such events are most suitable to study event-by-event fluctuations of various quantities. This is because, if the process is described by an incoherent sum of many "elementary" processes, a single event should already show a very statistical distribution with small fluctuations. In this sense, nucleus-nucleus collisions may be more appropriate than hadron-hadron or hadron-nucleus collisions to study fluctuations of nonstatistical origin.

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[1a]For the recent situation on this issue, see for example M. Jacob and J. Tran Thanh Van (editors), Phys. Rep. **88**, 325 (1982).

[1b]M. Jacob, CERN Report No. TH.3728-CERN, 1983 (to be published).

[1c]L. Van Hove, CERN Report No. TH.3725-CERN, 1983 (unpublished).

[2]K. Kajantie and H. I. Mietinnen, Z. Phys. C **9**, 341 (1981).

[3]J. Cleymans, M. Dechantsreiter, and F. Halzen, Z. Phys. C **17**, 341 (1983).

[4]J. Rafelski, CERN Report No. TH.3745-CERN, 1983 (unpublished).

[5]M. Gyulassy, Lawrence Berkeley Laboratory Report No. LBL-16800, 1983 (unpublished).

[6]M. Gyulassy, Lawrence Berkeley Laboratory Report No. LBL-15175, 1982 (unpublished).

[7]M. Gyulassy, K. Kajantie, H. Kurki-Suonio, and L. McLerran, Lawrence Berkeley Laboratory Report No. LBL-16277, 1983 (unpublished).

[8]T. H. Burnett *et al.*, Phys. Rev. Lett. **50**, 2062 (1983).