Collective-Interaction Klystron

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It is shown that by a mere bending of the drift tube of a two-cavity klystron into a circular arc, the current bunching near the output cavity is significantly enhanced. This increase in the bunching results from the "negative mass" behavior of a rotating electron beam. Operations with both low- and high-voltage beams are suggested.

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The klystron may be regarded as the most highly developed among conventional microwave tubes. It has a wide range of applications, from communication transmission at low power level to high-energy particle acceleration where tens of megawatts are required. The simplest (though not necessarily the most practical) klystron configuration consists of two cavities separated by a linear drift region.[1] The input signal is injected into the first cavity to provide a velocity modulation of the electron beam. This velocity modulation, after being carried through the drift region, becomes a density modulation near the second cavity where the output power is extracted. Because of the mutual Coulombic repulsion among the ac space charges, the charge bunching near the output cavity cannot reach the level expected from kinematic (ballistic) considerations. In fact, the efficiency of the klystron depends sensitively on the grouping of electrons near the output cavity.[2]

In this paper, I show that by a bending of the drift tube into a circular arc, the grouping of the space charges is greatly enhanced. This is possible because of the rather powerful "negative mass" effects in rotating electron beams, where the ac space charges tend to accumulate instead of self-dispersing.[3] The present device is, therefore, distinguished from the conventional klystron in that the *collective* motions strengthen rather than weaken the density modulation.[1] Of course, for the electron beam to follow a circular arc in the drift section, a radial force must be externally supplied. This can be achieved via a vertical magnetic field, or preferably in the case of low beam energy, via a radial dc electric field. By way of examples, I show below that this principle may be applied to electron beams with energy ranging from a few kiloelectronvolts to hundreds of kiloelectronvolts or higher.

To illustrate the idea, the simple two-cavity klystron suffices. In such a configuration (Fig. 1), the input and output cavities and the drift region may be analyzed separately. Since the novel aspects of the present device lie mainly with the drift region, we shall focus on the charge bunching processes there. The elementary analysis is analogous to that used in standard textbooks,[1] but with emphasis on

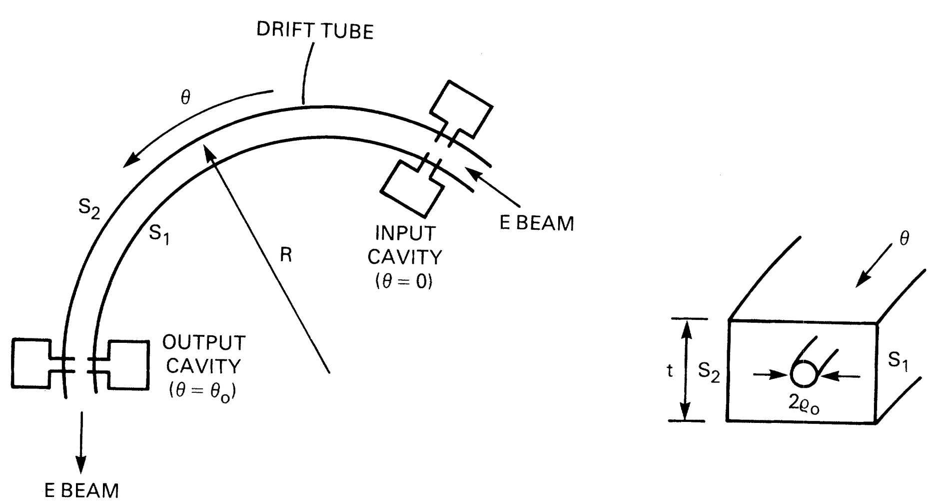

FIG. 1. A schematic drawing of the klystron (left) and the cross-sectional view of its drift tube (right).

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the negative-mass mechanism. However, the result obtained agrees with the classical limit of low current, in which case the bunching is essentially kinematic (ballistic).

In the absence of the rf modulation, the electron beam is assumed to move along the circular arc of the drift tube at linear velocity $\vec{v}_0 = \hat{\theta} v_0(r) \equiv \hat{\theta} r\omega_0(r)$, where r is the radial distance from the center of curvature of the circular arc, and θ is the angular variable along the drift tube (Fig. 1). For the time being, we shall leave unspecified the relative strength of the radial electric field E_0 and the vertical magnetic field B_0 which are needed to provide the circular motion of the beam. Thus, v_0 is governed by

$$\gamma_0 v_0^2/r = -(e/m_0)(E_0 + v_0 B_0), \tag{1}$$

where e and m_0 are, respectively, the electron charge and rest mass, and $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$ is the relativistic mass factor with c being the speed of light. In writing (1), we have for simplicity ignored the dc self-fields of the electrons. The beam is assumed to be monoenergetic, have a small beam cross section, and be located at mean radius $r = R$ with number density N per unit arc length along the drift tube.

We shall use a small-signal analysis. Associated with the velocity modulation is a longitudinal ac displacement η of an electron from its unperturbed orbit. Similar to plasma oscillation, this displacement leads to a charge perturbation which then generates an ac self-electric-field $E_{1\theta}$. In response to this ac electric field $E_{1\theta}$, the angular displacement η of an electron obeys the following linearized force law[3,4]

$$\ddot{\eta} = r v_0 (\partial\omega_0/\partial\epsilon) e E_{1\theta}. \tag{2}$$

Here a dot denotes the substantial derivative, ϵ is the total energy of the electron, and $\partial\omega_0/\partial\epsilon$ is the negative-mass factor. For an equilibrium governed by Eq. (1), $\partial\omega_0/\partial\epsilon$ is given by

$$\frac{\partial\omega_0}{\partial\epsilon} = -\frac{1}{r m_0 \gamma_0 v_0}\left(\frac{\beta_0^2 + 2h}{1 + \gamma_0^2 h^2}\right), \tag{3}$$

where

$$h = -erE_0/m_0\gamma_0^3 v_0^2, \tag{4}$$

and $\beta_0 \equiv v_0/c \equiv r\omega_0/c$.

A few words on the negative-mass effect are in order. Recall that the negative-mass behavior arises if the frequency of rotation ω_0 is a decreasing function of energy ($\partial\omega_0/\partial\epsilon < 0$). In this case, the angular acceleration is *opposite* to the applied force $eE_{1\theta}$, as if the inertia of the electron is negative.[3,4] There would then be an intrinsic tendency of beam bunching. If the rotation of the electron is supported solely by a magnetic field, $E_0 = 0$ and therefore $h = 0$ by (4). Equations (2) and (3) then yield $\ddot{\eta} = -\beta_0^2 e E_{1\theta}/\gamma_0 m_0$ which clearly shows the negative-mass nature of a rotating electron, under a uniform magnetic field, as a result of the relativistic mass correction ($c \neq \infty$). A new class of microwave generation devices, known as the gyrotron, has been developed[5] based on this principle.[6] Another limiting case is when $h = 1/\gamma_0^2$, i.e., when the rotation of the electron is supported *solely* by an outward radial electric field [cf. Eqs. (1) and (4)]. In this case, $\partial\omega_0/\partial\epsilon < 0$ also. This is precisely the equilibrium condition in the microwave generation experiment of Alexeff and Dyer.[7] In fact, one may easily demonstrate from (3) that $\partial\omega_0/\partial\epsilon$ is maximized with respect to h when $h = 1/\gamma_0^2$. Thus, among all possible combinations of E_0 and B_0, the negative-mass effect is most pronounced[8] when the rotation of the electron is supported just by a radial electric field [cf. Eqs. (3) and (4)] for a given beam energy and a given beam radius R. One may compare the negative-mass effects for $h = 0$ and for $h = 1/\gamma_0^2$ through the relation

$$\left.\frac{\partial\omega_0}{\partial\epsilon}\right|_{h=1/\gamma_0^2} = \frac{1}{\beta_0^2}\left.\frac{\partial\omega_0}{\partial\epsilon}\right|_{h=0}, \tag{5}$$

which is readily deduced from Eq. (3). Equation (5) clearly suggests the advantage of using only a radial dc electric field ($h = 1/\gamma_0^2$) at low beam voltage (low β_0). At a high beam voltage, a vertical magnetic field suffices.

A signal of frequency ω impressed upon the rotating electron beam yields an angular variation proportional to $\exp(-il\theta)$, where $l = \omega/\omega_0$. Note that l may be regarded as a propagation constant and that it is not necessarily an integer in the present case. Such an angular variation in the displacement η produces ac space charges whose number density N_1 per unit arc length is

$$N_1 = \frac{N}{R}\frac{\partial\eta}{\partial\theta} = i\frac{Nl\eta}{R}. \tag{6}$$

This ac line charge yields a self-electric-field $E_{1\theta}$ at the beam:

$$E_{1\theta} = \frac{il}{R}\frac{geN_1}{4\pi\epsilon_0} = -\frac{l^2}{R^2}\frac{geN\eta}{4\pi\epsilon_0}, \tag{7}$$

where ϵ_0 is the free-space permittivity (MKS units) and g is essentially the dimensionless impedance experienced by the beam and is given by[3,4]

$$g \cong [1 + 2\ln(2t/\pi\rho_0)]/\gamma_0^2, \tag{8}$$

for the dimension specified in Fig. 1.

Upon substituting Eq. (7) into Eq. (2), we obtain

$$\ddot{\eta}-\Gamma^2\eta=0, \tag{9}$$

where

$$\Gamma^2=g\frac{l^2c^2}{R^2}\frac{\nu}{\gamma_0}\left(\frac{\beta_0^2+2h}{1+\gamma_0^2h^2}\right), \tag{10}$$

with $\nu\equiv Ne^2/4\pi\epsilon_0 m_0c^2$ being the dimensionless Budker parameter. Note that Γ is essentially the rate of growth of the negative-mass instability in a rotating relativistic electron beam. Equation (9) has its counterpart in the conventional klystron theory in which $-\Gamma^2$ is replaced by ω_p^2, ω_p being the electron plasma frequency.

We may now calculate the fundamental harmonic of the ac current at the output gap.[1] Consider an electron leaving the input gap at $\theta=0$ (Fig. 1) at time $t=t_1$, with a velocity modulation $\dot{\eta}(t_1)=(\alpha v_0/2)\sin(\omega t_1)$, where α is the modulation depth. Then Eq. (9) gives

$$\eta(t)=(\alpha v_0/2\Gamma)\sin(\omega t_1)\sinh[\Gamma(t-t_1)] \tag{11}$$

if we assume that $\eta(t_1)=0$. Equation (11) may be used to transform from the Lagrangian to the Eulerian variables at $\theta=\theta_0$, the angular position of the output cavity (Fig. 1). From the definition of η, the electron arrives at $\theta=\theta_0$ at time $t=T$, where θ_0 and T are related by

$$\theta(T)\equiv\theta_0=\omega_0(T-t_1)+\eta(T)/R$$
$$=\omega_0(T-t_1)+\frac{\alpha\omega_0}{2\Gamma}\sin(\omega t_1)\sinh[\Gamma(T-t_1)].$$

This relation may be inverted to yield

$$T\simeq t_1+\frac{\theta_0}{\omega_0}-\frac{\alpha}{2\Gamma}\sin(\omega t_1)\sinh\left(\frac{\Gamma\theta_0}{\omega_0}\right), \tag{12}$$

in the small-signal theory (small α). The rf current $I_2(\theta_0,T)$ at the output gap contains all harmonic frequencies and may be represented in Fourier series as

$$I_2(\theta_0,T)=\sum_{n=-\infty}^{\infty}[a_n(\theta_0)\cos(n\omega T)+b_n(\theta_0)\sin(n\omega T)].$$

The Fourier coefficient is given by

$$a_n(\theta_0)=\frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}dT\,I_2(\theta_0,T)\cos(n\omega T)$$
$$=\frac{\omega I_0}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}dt_1\cos(n\omega T), \tag{13}$$

where I_0 is the dc current carried by the beam at the input gap. In writing the last expression, we have used the charge-conservation relation $I_2\,dT=I_0\,dt_1$. Upon substituting (12) into (13), we obtain $a_n(\theta_0)$ [and a similar expression for $b_n(\theta_0)$]. The total current I_2 at the output gap is

$$I_2(\theta_0,T)=I_0+\sum_{n=1}^{\infty}2I_0J_n(X_n)\times\cos[n\omega(T-\theta_0/\omega_0)], \tag{14}$$

where $X_n=(n\alpha\omega/2\Gamma)\sinh(\Gamma\theta_0/\omega_0)$ and J_n is the Bessel function of the first kind, of order n.

Equation (14) has a similar structure as the corresponding one for conventional klystron. As a check, we note that in the limit of zero density, $\Gamma\to 0$ by (10) and the *collective* effect disappears. In this limit, (14) indeed agrees with the classical result where only ballistic bunching is present. The negative-mass effect may be examined by considering just the fundamental harmonic $2I_0J_1(X_1)$ in the output current I_2. The peak value of this quantity is $1.16I_0$, occurring at $X_1=1.841$. Since $X_1=(\alpha\omega/2\Gamma)\sinh(\Gamma\theta_0/\omega_0)$, a very small velocity modulation at the input gap may yield the maximum achievable rf current $1.16I_0$ at the output gap if $\Gamma\theta_0/\omega_0$ is sufficiently large. This is quite different from the conventional klystron, and is due to the enhanced charge bunching resulting from the negative-mass effect associated with a bent drift tube. In contrast, a weak modulation at the input gap of a conventional two-cavity klystron is unable to achieve this peak value of current bunching because of the Coulombic repulsion among the ac space charges.[9]

As a proof-of-principle experiment, take $\theta_0=2\pi/3$, $R=4$ cm, $\beta_0=0.1389$, corresponding to a beam energy of 5 keV. Then $\omega_0/2\pi=\beta_0c/2\pi R=0.1658$ GHz. Let $l=20$, say, so that the tube may operate at 3.32 GHz. Then Eq. (10) yields $\Gamma/\omega_0=1.86$ for $g=4$, $h=1/\gamma_0^2$, and a beam current of 0.1 A. For these parameters, the maximum achievable rf current (according to the present linear theory) may be attained at the output cavity if the modulation factor α is as low as 0.014. In this example, a radial electric field $E_0=2.5$ keV/cm is used to provide the circular motion in the drift tube.

For high-voltage operation, consider, for example, a beam at a voltage of 300 keV and a current of 160 A. For $R=22.36$ cm, $\theta_0=100°$, $h=0$, $g\simeq 1.59$, $\omega/\omega_0=20$ (i.e., operation at 3.32 GHz), saturation occurs when $\alpha=0.0174$. Here, only a vertical magnetic field $B_0=94$ G is used to provide the circular motion in the drift tube. Examples with other beam energy and frequency ranges may similarly be constructed.

In this paper, several well-known principles are synthesized to yield a novel klystron which promises high gain. The device is efficient and compact. It has the simplicity of a conventional two-cavity klystron. It also uses the very mechanism which makes gyrotrons efficient, as bunching along the rotational orbits indeed takes place.

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[1]See, e.g., M. Chodorow and C. Susskind, *Fundamentals of Microwave Electronics* (McGraw-Hill, New York, 1964); J. W. Gewartowski and H. A. Watson, *Principles of Electron Tubes* (Van Nostrand, New York, 1965).

[2]A. Staprans, E. W. McCume, and J. A. Ruetz, Proc. IEEE **61**, 299 (1973), and references therein.

[3]C. E. Nielson, A. M. Sessler, and K. R. Symon, in *Proceedings of the International Conference on Accelerators* (CERN, Geneva, 1959), p. 239; also A. A. Kolomenskii and A. N. Lebedev, *ibid.*, p. 115.

[4]V. K. Neil and W. Heckrotte, J. Appl. Phys. **36**, 2761 (1965); R. W. Landau and V. K. Neil, Phys. Fluids **9**, 2412 (1966); R. J. Briggs and V. K. Neil, J. Nucl. Energy **C9**, 209 (1967).

[5]See, e.g., V. L. Granatstein, M. E. Read, and L. R. Barnett, in *Infrared and Millimeter Waves,* edited by K. J. Button (Academic, New York, 1982), Vol. 5, p. 267, and references therein.

[6]Y. Y. Lau, IEEE Trans. Electron Devices **29**, 320 (1982), and **31**, 329 (1984).

[7]I. Alexeff and F. Dyer, Phys. Rev. Lett. **43**, 351 (1980).

[8]Y. Y. Lau and D. Chernin, Phys. Rev. Lett. **52**, 1425 (1984); D. Chernin and Y. Y. Lau, to be published.

[9]See, for example, Fig. 9.3–5 on p. 335 of Gewartowski and Watson (Ref. 1). It is shown in this figure that the ac-space-charge effects, in a conventional two-cavity klystron, limit the rf current to $0.25 I_0$ (or less) if the input signal is sufficiently weak. This rf current is significantly less than $1.16 I_0$. The latter value is quoted in the main text. It is also the limiting value according to the ballistic theory.