Nuclear Equation of State from Scaling Relations for Solids

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The equation of state of nuclear matter is predicted in analogy with a universal form recently observed to hold for other self-bound systems. This approach was suggested by two findings. First, the spin-isospin–averaged realistic nucleon-nucleon interaction potential is observed to be of the universal form in the region of strong bonding. Secondly, a resulting relation between the cohesive and surface energy is shown to hold for nuclear matter as well as metals and the electron-hole liquid.

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The equation of state of nuclear matter is the subject of much current interest.[1] Since different many-body methods yield different results, to date no firm predictions are available even for zero temperatures.

In such situations the use of analogies with other areas of physics can be fruitful. It has been suggested[2,3] that the interaction potential for two nucleons is similar to the binding-energy–distance relation of diatomic molecules. Recently it has been shown that the binding-energy versus separation relations for some self-bound systems such as diatomic molecules and metals have a universal shape.[4] Consequently, the experimentally measured zero-temperature equation of state of metals can be predicted in the absence of phase transitions from the equilibrium values of the interactomic spacing, the binding energy per atom, and the compressibility. The analogous physical quantities are known reasonably well for nuclear matter. Assuming that the "shape" of the energy versus interparticle-separation relation is of the universal form for nuclear matter, we predict the equation of state using the equilibrium values described above.

The existence of a universal binding-energy–distance relation was illustrated in Ref. 4. There it was shown for (1) metallic cohesion, (2) metallic adhesion, (3) chemisorption of gas atoms on metal surfaces, and (4) the binding of some diatomic molecules that the total binding energy per particle can be written as

$$E(a) = \Delta E\, E^*(a^*). \tag{1}$$

Here ΔE is the equilibrium binding energy and a is the coordinate describing the separation of the constituents (e.g., the distance between the two nuclei of a diatomic molecule). The scaled coordinate a^* is given by $a^* = (a - a_0)/l$. Here a_0 is the equilibrium spacing while l is a length scale. $E^*(a^*)$ is a scaled function chosen to have $E^*(a^* = 0) = -1$ and $E^{*\prime\prime}(a^* = 0) = 1$. $E^*(a^*)$ describes the *shape* of the universal binding-energy relation.

Since $E^*(a^*)$ represents a quite general scaled form for the binding-energy relation in molecules and solids, it is of some interest to see if it can be related to the binding of nuclear matter. Recently Wiringa, Smith, and Ainsworth[5] have empirically fit nucleon-nucleon data and have obtained a simple potential which yields a high-quality description of deuteron properties and nucleon-nucleon phase shifts below 330 MeV. We average the spin-isospin–dependent s- and p-wave components to obtain a form we call $\bar{v}_{14}$ which should be representative for nuclear matter (a spin-isospin saturated system) at moderate density. After scaling the $\bar{v}_{14}$ potential[6] we plot the result as the curve on Fig. 1. For reference[4] we have included scaled binding-energy relations for the H_2^+ molecule, the Al-Zn interface, and the bulk solid Mo. These are typical examples of systems that have been shown to exhibit universal behavior. We see that in the region of strong binding the $\bar{v}_{14}$ potential is virtually indistinguishable from the universal binding-energy relation. Thus there is a *quantitatively* accurate relationship between the binding energy versus atomic spacing in some metals and molecules and this form

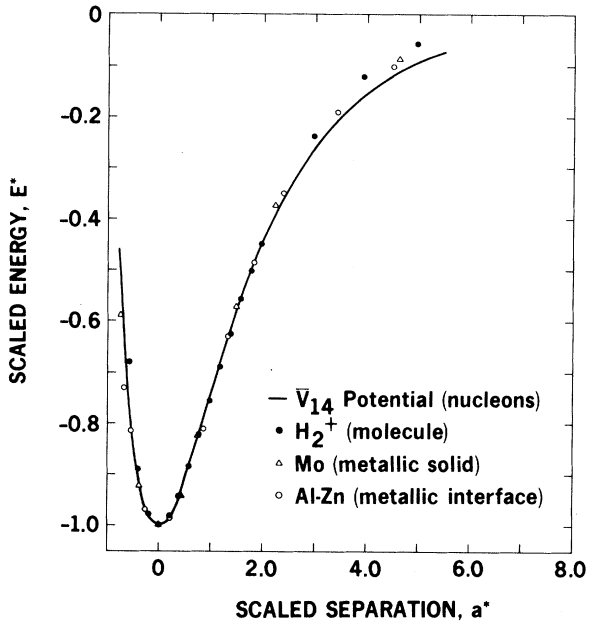

FIG. 1. Comparison of a scaled two-nucleon potential, referred to as $\bar{v}_{14}$ in the text (Ref. 5) with scaled binding-energy relations for the molecule H_2^+, the bulk metal Mo, and the bimetallic interface Al-Zn. The scaling is described in the text.

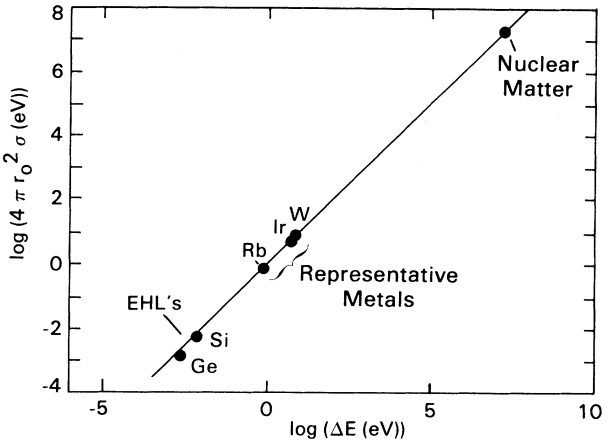

FIG. 2. The predicted surface-energy (σ) –cohesive-energy (ΔE) relation from Eq. (2) is plotted as the solid line. Experimental results for various materials are plotted. The electron-hole-liquid values are obtained from Refs. 9 and 10, the nuclear values from Ref. 7, and the bulk metal values from Ref. 4.

of the nucleon-nucleon potential.

For metals the scaled form of the binding-energy–separation relation can be connected[4] to both surface energy σ and the bulk cohesive energy ΔE. This leads to the relation

$$4\pi r_0^2 \sigma \approx 0.82 \Delta E. \tag{2}$$

The equilibrium (Wigner-Seitz) radius of a sphere containing one atom in the bulk solid is denoted by r_0. Given the result in Fig. 1 it is of interest to see if Eq. (2) also holds for nuclear matter. We take parameters for nuclear matter from the mass formula.[7] The relevant terms are

$$\begin{aligned} E/A &= \Delta E + BA^{-1/3} + \ldots \\ &= (16 + 4\pi R^2 \sigma / A + \ldots)\ \text{MeV} \\ &= (16 + 18A^{-1/3} + \ldots)\ \text{MeV}. \end{aligned}$$

Here R is the radius of the nucleus if we assume a constant value of the density corresponding to the saturation limit for nuclear matter and A is the mass number. The radius of a sphere containing one nucleon, r_0, is also taken from the saturation limit and is given explicitly below.

Figure 2 shows a plot of $4\pi r_0^2 \sigma$ vs ΔE for (1) nuclear matter, (2) several representative metals, and (3) the electron-hole-liquid in Si and Ge. The agreement with Eq. (2) over 9 orders of magnitude in energy (and ~ 21 orders of magnitude in density, i.e., 10^{17} vs 10^{38} particles/cm^3) is striking. Surface energies for the electron-hole liquid were taken from the theoretical calculations of Shore and coworkers.[8–10] Their values are in good agreement with experiment for Ge,[11] and consistent with scattered experimental results for Si.[10,11] A relation similar to Eq. (2) describes the curvature energy, E_c, of nuclear matter and the electron-hole liquid. The curvature energy is functionally of the form $E_c \sim 0.7 \Delta E A^{1/3}$. The constant was found by fitting to the electron-hole liquid results of Ref. 11. The scaled relation predicts that $E_c \sim 12$ MeV $A^{1/3}$ for nuclear matter, in good agreement with the many-body calculations of Negele and Vautherin.[12]

We therefore conjecture that the binding energy per nucleon in nuclear matter can be represented by Eq. (1). Then the entire equation of state at zero temperature in the absence of phase transitions can be predicted given the equilibrium values of the binding energy, ΔE, the compressibility, K, and the Wigner-Seitz radius (internucleon separation), r_0. We choose[7] $\Delta E = 16$ MeV/nucleon, $K = 220 \pm 10$ MeV, and $r_0 = 1.12$ fm. Here $K = r_0^2 d^2E/dr^2|_{r_0}$. The length scale l for nuclear matter is defined (see Ref. 4) by

$$l = \left(\frac{\Delta E}{d^2E/dr^2|_{r_0}} \right)^{1/2} = 0.30 \text{ fm}. \tag{3}$$

What can one predict from this conjecture for

$E(a)$? Basically, if given the harmonic properties, the entire relation is given, including the anharmonic properties.

The zero-temperature equation of state is given by $P(V) = -\Delta E[E^{*\prime}(a^*)/4\pi l r^2]$. Here the volume per nucleon is denoted by $V = 4\pi r^3/3$. Using the simple, approximate form for $E^*(a^*) = -(1+a^*)e^{-a^*}$,[13] we find the binding-energy–volume relation shown in Fig. 3. The left-hand side of the curve represents the region of large positive pressure. The right-hand side represents the effects of a *negative* hydrostatic pressure. There is a maximum negative hydrostatic pressure which nuclear matter can sustain at zero temperature (corresponding to the inflection point in the energy-volume relation). Our results yield a rupture pressure, $P_r \sim -1$ MeV/fm^3. Similarly, there is a lowest density for which nuclear matter is stable under negative hydrostatic pressure, the number density being $\sim 60\%$ of the saturation density.

Numerous parametrizations have been proposed for the nuclear-matter equation of state[14] which assume a power series in the density with a small number of terms. In Fig. 3 we present for comparison such an energy per nucleon (E) curve

$$E = \sum_{l=2}^{5} a_l \left(\frac{n}{n_0}\right)^{l/3}, \tag{4}$$

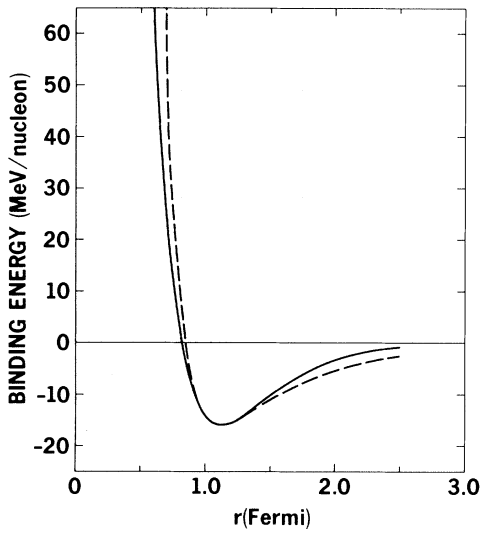

FIG. 3. Conjectured zero-temperature binding-energy relation for nuclear matter (solid line) and a conventional power-series form (Ref. 14) (dashed line). The universal binding-energy relation, $E^*(a^*) = -(1+a^*)e^{-a^*}$, of Ref. 4 is combined with Eq. (1) and the following nuclear matter data to make this prediction: $\Delta E = 16$ MeV, $r_0 = 1.12$ fm, and $l = 0.3$ fm.

in the form proposed in Ref. 14. Here n_0 is the saturation density (0.170 nucleon/fm^3) and the four coefficients are given by our choices of nuclear-matter saturation properties (see above) and the requirement of Fermi-gas behavior at low density. These conditions yield $a_2 = 23.04$ MeV, $a_3 = -119.12$ MeV, $a_4 = 89.12$ MeV, and $a_5 = -9.04$ MeV. In the range of densities with which we are concerned the unacceptable high-density behavior is not relevant. It should be noted that at low temperatures including absolute zero, a liquid-phase equation of state will not connect at low densities with ideal Fermi-gas behavior. While the proposed form of Ref. 14 expressed here in Eq. (4) is therefore flawed, we include it in Fig. 3 because it is representative of a number of power-series forms currently in use in nuclear physics.

Using our equation of state we may estimate the critical density n_c and temperature T_c of the liquid-to-gas phase transition in nuclear matter. To make these estimates we assume, as in Ref. 14, that the entropy of nuclear matter is that of a noninteracting degenerate Fermi gas. With this we can evalute the pressure as a function of density and temperature and look for the critical point. This leads to the equation of state for $T \geq T_c$,

$$P = -\frac{\Delta E}{3l}\left(\frac{3}{4\pi}\right)^{1/3} n^{2/3} a^* e^{-a^*} + \frac{n^{1/3}}{6}\frac{m}{\hbar^2}(bT)^2. \tag{5}$$

In this expression m is the nucleon mass and $b = 1.809$ for nuclear matter. We obtain a $T_c = 20.5$ MeV and $n_c = 0.33 n_0$. This critical temperature is somewhat higher and the critical density is somewhat lower than most predictions.[14] Experimental information on these quantities is not yet available.

The results of Figs. 1 and 2 lead one to examine the features common to nuclear matter, metals, and the electron-hole liquid. At least three features are common: (1) Nucleons, electrons, and holes are all fermions. The Pauli exclusion principle has a direct effect on energy levels and allows for exchange interactions. (2) The forces in these systems are effectively of an exponential nature. (3) Long-range unscreened interactions must be negligible or accounted for explicitly. Nuclear forces tend to saturate at short distances as do screened Coulombic interactions in metals and some molecules.

Nuclear matter will not exhibit the rich structurally rigid behavior shown by other self-bound atomic systems such as solids and molecules. Rather, the nucleus is more comparable to a liquid metal (especially an electron-hole liquid) since the nu-

cleons are free to move over distances comparable to the dimensions of the nucleus. The energy difference between liquid and solid metal is very small compared to the binding energy. Continuing our analogy, we do not expect the liquid nature of the nucleus to cause a large change in the expected equation of state. In the preceding development we have ignored Coulombic interactions within the nucleus. In the application of our results to finite nuclei they (as well as surface, asymmetry, and other contributions) must be included explicitly.

Palmer and Anderson[3] have made empirical correspondence between molecules and nuclear matter. While their two-body potential was determined in an *ad hoc* fashion, the spirit of our approach of relating binding energies of solids to nuclear energetics is consistent with their thesis. We see now that the universal binding-energy relation obviates the need for an *ad hoc* two-body potential.

In conclusion we have shown that the partial-wave–averaged realistic nucleon-nucleon potential of Wiringa *et al.*[5] over the region of strong binding can be scaled to the form of the universal binding-energy relation recently discovered for solids. Consequently it was supposed that the surface and cohesive energies of nuclei could be related as in metals and the electron-hole liquid. This supposition was demonstrated to be correct. We have proposed a form for the zero-temperature equation of state of nuclear matter in the absence of phase transitions. A prediction is made for both the density and pressure of nuclear matter when it ruptures under a negative hydrostatic load. Finally, as compared to other postulated equations of state for nuclear matter,[1] which have power-law shapes, ours has an exponential form.

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