Solving the Strong *CP* Problem without the Peccei-Quinn Symmetry

S. M. Barr
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 18 April 1984)

A simple theorem is proved about the $\bar{\theta}$ parameter in grand unified models with spontaneous *CP* symmetry breaking. This theorem states two conditions on such models which, if fulfilled, imply that $\bar{\theta}$ is zero at tree level. Models which fulfill these conditions are generalizations of a recent model of Nelson and are easy to construct.

PACS numbers: 11.30.Er, 11.30.Qc, 12.10.En

There are two currently viable approaches to solving the strong *CP* problem. The more familiar is the Peccei-Quinn mechanism.[1] In recent years Peccei-Quinn models have been studied considerably. This has led to the formulation of "invisible axion" models[2] with the concomitant astrophysical problems.[3] Though these problems are surmountable[4] it is worth studying the alternative to Peccei-Quinn models. In this other approach to the strong *CP* problem, *CP* invariance is imposed as a symmetry of the Lagrangian but is broken spontaneously. Thus, before symmetry breaking $\theta_{\rm QCD}=0$. It is necessary to impose additional symmetries on the theory to insure that the $\det M_Q$ is real at tree level. M_Q is the quark mass matrix. $\operatorname{Arg}[\det M_Q]$ is sometimes called $\theta_{\rm QFD}$. Several models of this type exist in the literature.[5] All of them appear contrived in comparison with, say, the Peccei-Quinn models. Also it has been found[6] to be difficult to construct grand unified models based on this approach. (It is desirable to have such models unified because the domain walls that result from spontaneous *CP* symmetry breaking could be removed by inflation were the *CP* symmetry breaking to occur at grand-unified-theory scales.) Recently, however, a quite elegant and comparitively simple model has been proposed by Nelson.[7] In this note I wish to state a theorem which gives a set of sufficient conditions for constructing such models. The conditions are few and simple and are satisfied by Nelson's model. This theorem thus reveals the essential features of her model, which allow it to be easily generalized.

Theorem.—Let us consider a model in which *CP* invariance is a symmetry of the Lagrangian but is spontaneously broken. Suppose that in this model the fermions may be classified in two sets: F consisting of fermions with the same $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers as n_f ordinary light families [for example in an SO(10) model they would be n_f $\underline{16}$'s], and R consisting of a *real* set of representations of $SU(3)\otimes SU(2)\otimes U(1)$. ($R$ may contain complex representations as long as it contains an equal number of conjugate representations.) Then $\bar{\theta}\equiv\theta_{\rm QCD}+\theta_{\rm QFD}$ will be zero at tree level if two conditions are satisfied: (1) $SU(2)\otimes U(1)$-breaking vacuum expectation values (VEV's) appear only in F-F Yukawa terms, not in F-R or R-R terms. (2) *CP*-nonconserving VEV's appear only in F-R Yukawa (or mass) terms, not in F-F or R-R terms.

Proof.—We will first examine the mass matrix M_- of the charge $-\frac{1}{3}$ quarks, q_-, and the charge $+\frac{1}{3}$ antiquarks, q_-^c. The q_- that belong to F will have $SU(2)\otimes U(1)$ quantum numbers $(-\frac{1}{2},+\frac{1}{6})$, where the first number is the third component of weak isospin, I_3, and the second number is the weak hypercharge, $\frac{1}{2}Y$. $Q=I_3+\frac{1}{2}Y$. The q_-^c belonging to F have quantum numbers $(0,\frac{1}{3})$. By condition (1) the q_- in R that mix in the mass matrix with the q_- in F have the quantum numbers $(-\frac{1}{2},+\frac{1}{6})$ or $(0,-\frac{1}{3})$. And the q_-^c in R that mix with the q_-^c in F have the quantum numbers $(\frac{1}{2},-\frac{1}{6})$ or $(0,\frac{1}{3})$. Thus we may represent M_- at tree level schematically by

$$q_-M_-^{(0)}q_-^c$$
$$=\left((-\tfrac{1}{2},\tfrac{1}{6})_i^{(F)}\quad(-\tfrac{1}{2},\tfrac{1}{6})_j^{(R)}\quad(0,-\tfrac{1}{3})_k^{(R)}\right)\begin{pmatrix}\langle(\frac{1}{2},-\frac{1}{2})\rangle_{ii'} & \langle(0,0)\rangle_{ij'} & 0\\ 0 & \langle(0,0)\rangle_{jj'} & 0\\ \langle(0,0)\rangle_{ki'} & 0 & \langle(0,0)\rangle_{kk'}\end{pmatrix}\begin{pmatrix}(0,\frac{1}{3})_{i'}^{(F)}\\ (\frac{1}{2},-\frac{1}{6})_{j'}^{(R)}\\ (0,\frac{1}{3})_{k'}^{(R)}\end{pmatrix}.$$

The indices i and i' run from 1 to n_f. The index j' runs over the same number of values as j because R is a *real* set of representations. Thus $\langle(0,0)\rangle_{jj'}$ is a square matrix. For the same reason $\langle(0,0)\rangle_{kk'}$ is also a square matrix. We have denoted VEV's or bare mass terms by their SU(2) ⊗ U(1) quantum numbers in an obvious notation. The zeroes in $M_-^{(0)}$ are explained by condition (1). They appear in F-R or R-R terms that would violate SU(2) ⊗ U(1). By condition (2) *CP* nonconservation only appears in the $\langle(0,0)\rangle_{ij'}$ and $\langle(0,0)\rangle_{ki'}$ terms that couple F and R. But these *CP* nonconserving terms do not contribute to $\det M_-^{(0)}$ as we now easily show. Consider the jth rows of $M_-^{(0)}$. The only nonvanishing entries are in $\langle(0,0)\rangle_{jj'}$. To get a nonvanishing contribution to $\det M_-^{(0)}$ we need one entry from each row of $\langle(0,0)\rangle_{jj'}$. But since this is a *square* matrix we will then have an entry from every column of $\langle(0,0)\rangle_{jj'}$ as well. Thus all the entries from the j'th columns of $M_-^{(0)}$ come from $\langle(0,0)\rangle_{jj'}$ and therefore *not* from $\langle(0,0)\rangle_{ij'}$. Similarly we need an entry from each column (and thus each row) of $\langle(0,0)\rangle_{kk'}$. Hence no entries of $\langle(0,0)\rangle_{ki'}$ contribute to $\det M_-^{(0)}$. Therefore $\det M_-^{(0)}$ does not contain any *CP*-nonconserving phase at tree level.

A parallel argument applies to the mass matrix, M_+, of the charge $\frac{2}{3}$ quarks, q_+, and the charge $-\frac{2}{3}$ antiquarks, q_+^c. In the same notation as before we may write, schematically, M_+ at tree level by

$$q_+ M_+^{(0)} q_+^c$$
$$= \left((\tfrac{1}{2},\tfrac{1}{6})_i^{(F)} \quad (\tfrac{1}{2},\tfrac{1}{6})_j^{(R)} \quad (0,\tfrac{2}{3})_k^{(R)} \right) \begin{pmatrix} \langle(-\frac{1}{2},\frac{1}{2}\rangle_{ii'} & \langle(0,0)\rangle_{ij'} & 0 \\ 0 & \langle(0,0)\rangle_{jj'} & 0 \\ \langle(0,0)\rangle_{ki'} & 0 & \langle(0,0)\rangle_{kk'} \end{pmatrix} \begin{pmatrix} (0,-\frac{2}{3})_{i'}^{(F)} \\ (-\frac{1}{2},-\frac{1}{6})_{j'}^{(R)} \\ 0,-\frac{2}{3})_{k'}^{(R)} \end{pmatrix}.$$

Repeating the above arguments one can show that $\det M_+^{(0)}$ does not contain any *CP*-nonconserving phase at tree level.

Finally one must consider color nonsinglet fermions which do not mix with the "family quarks" in F. These may have other charges than $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$, other colors than $\underline{3}$ and $\underline{3}^*$, or be in other than singlets or doublets of weak isospin. Since [by condition (1)] these are purely in R, we see immediately [by condition (2)] that the mass matrix of these quarks at tree level has no *CP* nonconservation.

We have shown that $\theta_{\rm QFD} \equiv \arg[\det M_Q]$ is zero at tree level. $\theta_{\rm QCD}$ is zero at tree level trivially since $\mathscr{L}$ is *CP* conserving. So the theorem is proved.

Though this theorem does not depend on whether the model is or is not grand unified, grand unification is the natural framework in which to apply it. One would expect in such a unified model, by Georgi's "survival hypothesis,"[8] that the light fermions will consist of n_f ordinary families, the other fermions acquiring superheavy masses of order the unification scale. One might wonder how the ordinary weak-interaction *CP*-nonconservation effects observed in the kaon system arise. The *CP*-nonconserving VEV's that connect F fermions to R fermions insure that the light families of fermions are linear combinations of F and R with coefficients that contain *CP*-nonconserving phases. Thus at low energies, where the heavy fermions decouple, it *appears* that the SU(2) ⊗ U(1)-breaking, Weinberg-Salam Higgs scalar(s) couple to the light fermions with complex Yukawa couplings. This will lead to complex light-quark mass matrices and to *CP* nonconservation in the weak interactions via the usual Cabibbo-Kobayashi-Maskawa mechanism. For an example the reader is referred to Ref. 7. We now will outline two examples of models fulfilling the conditions of our theorem. One is the SU(5) example of Ref. 7. The other is an SO(10) example.

Examples.—An example of this theorem is the simple model of Nelson.[7] That model has an $SU(5) \otimes SO(3)_{\rm Global} \times CP$ symmetry as well as other global symmetries. The F fermions are $(\underline{10},\underline{3})_L + (\underline{5}^*,\underline{3})_L$, while the R fermions are $(\underline{10},\underline{1})_L + (\underline{10}^*,\underline{1})_L + (\underline{5}^*,\underline{1})_L + (\underline{5},\underline{1})_L$. The scalars are in $(\underline{24},\underline{3})_{\rm H}$, $(\underline{1},\underline{3})_{\rm H}$, and $(\underline{5},\underline{1})_{\rm H}$ representations. The SU(2) ⊗ U(1)-breaking masses arise from $(\underline{10},\underline{3})_L(\underline{5}^*,\underline{3})_L(\underline{5},\underline{1})_{\rm H}^*$ and $(\underline{10},\underline{3})_L(\underline{10},\underline{3})_L(\underline{5},\underline{1})_{\rm H}$ Yukawa terms. Condition (1) is satisfied because global symmetries prevent $(\underline{10},\underline{1})_L(\underline{5}^*,\underline{1})_L(\underline{5},\underline{1})^*$ and other similar SU(2) ⊗ U(1)-breaking R-R terms from appearing in this model. Condition (2) is satisfied because *CP* nonconservation is arranged only to come from the VEV's of the $(\underline{1},\underline{3})_{\rm H}$ scalars. These only couple F to R. Thus at tree level this model has no strong *CP* nonconservation.

It is simple to construct such models based on larger groups than SU(5). Suppose we have an

SO(10) model. For the fermions in F we could choose three (or in general n_f) $\underline{16}_L$'s. For R we could choose for example any number of $\underline{10}$'s, $\underline{45}$'s, $\underline{120}$'s, $(\underline{16}+\underline{16}^*)$'s, that is, *any* real representations. Let us choose $\underline{126}_L+\underline{126}_L^*$. For the scalars let us make the usual choice of $\underline{10}_{\rm H}$, $\underline{16}_{\rm H}$, and $\underline{45}_{\rm H}$. Let us suppose that nonvanishing VEV's develop for components in the $\underline{1}(\underline{16})$, $\underline{1}(\underline{45})$, $\underline{5}(\underline{10})$, and $\underline{5}^*(\underline{10})$ [where the first number is an SU(5) representation and the number in parenthesis is an SO(10) representation]. Condition (1) is satisfied for example if $\underline{5}^*(\underline{16})$ does not develop a nonvanishing VEV, as there are no $\underline{16}_L\underline{126}_L\underline{10}_{\rm H}$ or $\underline{126}_L\underline{126}_L\underline{10}_{\rm H}$ couplings. If we have violated CP conservation only by the VEV's of the $\underline{1}(\underline{16})$ then condition (2) would be satisfied as well. The matrix $\underline{M}^{(0)}$ would, schematically, look like

$$q_-\underline{M}^{(0)}q_-^c=(\underline{10}(\underline{16})_i\quad \underline{10}(\underline{126})\quad \underline{5}(\underline{126}^*))\begin{pmatrix}\langle\underline{5}^*(\underline{10})\rangle & \langle\underline{1}(\underline{16})\rangle_i & 0\\ 0 & M & 0\\ \langle\underline{1}(\underline{16})\rangle_{i'} & 0 & M\end{pmatrix}\begin{pmatrix}\underline{5}^*(\underline{16})_{i'}\\ \underline{10}^*(\underline{126}^*)\\ \underline{5}^*(\underline{126})\end{pmatrix}.$$

Note that the zeroes here result because we have no Higgs representations which are in the products $\underline{16}\otimes\underline{126}$ and $\underline{126}\otimes\underline{126}$. This is only an example. It should be obvious that a huge variety of groups, representations, and coupling and breaking schemes are possible within the framework of the conditions of the theorem.

We have only shown that the *tree-level* mass matrix of the colored fermions, M_Q, has real determinant. At one-loop level one would expect that a CP-nonconserving phase would appear in $\det M_Q$. The value of the phase is somewhat model dependent. Generally $\delta\theta$ from one-loop corrections to M_Q goes like $(f^2/16\pi^2)\times$(phase) where f is some typical Yukawa coupling, or equivalently a ratio of a fermion mass to a VEV. Since Yukawa couplings can naturally be (and generally seem to be) rather small, one can have $\bar\theta$ small enough to be consistent with present bounds without much difficulty.

The models in Ref. 5 tend to have difficulties with Higgs-mediated flavor-changing neutral currents unless the neutral Higgs masses are greater than about 200 GeV. In these models spontaneous CP nonconservation arises from nonzero relative phases of the VEV's of several light SU(2) $\otimes$ U(1)-breaking Higgs scalars which contribute to M_Q. Because more than one light Higgs contributes to M_Q it happens that diagonalizing M_Q does not diagonalize, in general, the neutral Higgs couplings. However in models constructed along the lines suggested by our theorem this problem is less pressing. CP nonconservation occurs in the SU(2) $\otimes$ U(1)-singlet VEV's. It is thus not necessary to have a nonminimal light Higgs structure. It is well known that a minimal light Higgs structure will not in general give rise to unacceptably large Higgs-mediated flavor-changing neutral currents.

In conclusion, I would like to stress that there is a fairly simple class of unified models which solve the strong CP problem. They seem, from the standpoint of simplicity, competitive with the Peccei-Quinn models. Nelson's model[7] is the first of this type to be discovered but I wish to emphasize that an entire class of such models—satisfying fairly unrestrictive conditions—exists. It is probably possible to even loosen these conditions somewhat. It also should be emphasized that it is not necessary to have global symmetries (other than CP invariance) in these models.

[1]R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[2]S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978); Jinn E. Kim, Phys. Rev. Lett. **43**, 103 (1979); A. Vainshtein and V. Zakharov, Nucl. Phys. **B166**, 493 (1980); M. Dine, W. Fischler and M. Srednicki, Phys. Lett. **104B**, 199 (1981); H. P. Nilles and S. Raby, Stanford Linear Accelerator Center Report No. SLAC-PhB-2743 (1981).

[3]P. Sikivie, Phys. Rev. Lett. **48**, 1156 (1982), discovered the domain wall problem. See L. Abbott and P. Sikivie, Phys. Lett. **120B**, 133 (1983); J. Preskill, M. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); and M. Dine and W. Fischler, Phys. Lett. **120B**, 137 (1983), for the axion energy problem.

[4]G. Laxarides and Q. Shafi, Phys. Lett. **115B**, 21 (1982); R. Holdom, Stanford University Report No. ITP-712, ITP-713 (1982); S. Barr, D. Reiss, and A. Zee, Phys. Lett. **116B**, 227 (1982); S. Barr, X. C. Gao, and D. Reiss, Phys. Rev. D **26**, 2176 (1982), and several others.

[5]M. A. B. Beg and H.-S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R. N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 283 (1978); H. Georgi, Hadronic J. **1**, 155 (1978);

G. Segre and H. A. Weldon, Phys. Rev. Lett. **42**, 1191 (1979); S. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1654 (1979), and others.

[6]S. Barr, Phys. Rev. D **23**, 2434 (1981). In this paper the advantages of having both light and superheavy fermions was overlooked. The model of A. Nelson, Phys. Lett. **136B**, 387 (1984), obviates many of the difficulties encountered in this paper. [Attempts to find a third way are discussed in S. Barr and D. Seckel, Nucl. Phys. **B233**, 116 (1984).]

[7]See Nelson, Ref. 6.

[8]H. Georgi, Nucl. Phys. **B156**, 126 (1979).