Evidence for Corrections to Soft-Pion S-Wave Nonleptonic Hyperon-Decay Amplitudes

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Evidence is found for corrections to the soft-pion S-wave nonleptonic hyperon-decay amplitudes from the observed deviation from the generalized Lee-Sugawara relation $2A(\Xi^-) + A(\Lambda^0_-) + (\frac{3}{2})^{1/2}A(\Sigma^-) = 0$. These corrections, assumed to come from the low-lying $\frac{1}{2}^-$ excited baryon intermediate states, are estimated experimentally with use of the expressions suggested by the quark model and are found to reduce significantly the soft-pion results.

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A long standing problem in nonleptonic hyperon decays is the failure of current algebra (CA) to predict the P-wave amplitudes in terms of the measured values for the S wave. Except for the decays $\Lambda \rightarrow p \pi^{-}, n \pi^{0}$, all other calculated *P*-wave amplitudes are too small compared with experiments.^{1, 2} The most serious case is the vanishingly small value predicted for $B(\Sigma_{\pm}^{+})$, in strong disagreement with data. This has led many people to correct for the soft-pion results by including terms of the order $O(\Delta m)$ relative to the main contributions. These corrections are usually¹ assumed to come from the K^* , $O(1^+)$ pole in the t channel and the low-lying baryon resonances B^* pole in the s and u channel. According to Gronau, the K^* -pole terms can reduce significantly the soft-pion S-wave amplitude.³ On the other hand, Le Yaouanc et al. have argued that the $\frac{1}{2}$ low-lying baryon resonances with their masses close to the ground state are more likely to produce an important correction to the soft-pion Swave amplitude because of the small mass difference in the energy denominator.⁴ Using a nonrelativistic quark model they found that these corrections are important in reducing the S-wave soft-pion amplitudes. It thus appears that the S-wave amplitudes are reduced by the K^* -pole and the $B^*(\frac{1}{2})$ pole terms and it is important to have a direct evidence for these corrections. For the S-wave amplitudes, since the K^* -pole terms satisfy the Lee-Sugawara (LS) relation⁵ to first order in SU(3)breaking, the large observed deviation from this relation as measured by the quantity

 $\Delta(LS) = 2A (\Xi_{-}) + A (\Lambda_{-}^{0}) + (\frac{3}{2})^{1/2} A (\Sigma_{-}^{-})$ (1)

is then evidence for the $B^*(\frac{1}{2})$ contributions, *in*dependent of the magnitude of the commutator terms. The main purpose of this note is to point out the evidence for corrections to S-wave soft-pion amplitudes given by the measured value of $\Delta(LS)$. Since this has never been done before I believe it is important, both from an experimental and a theoretical stand point, to present this evidence as a test of the validity of the soft-pion theorem for S-wave nonleptonic hyperon decays. In the past, because of lack of accurate data, it has not been possible to find evidence for a deviation of CA soft-pion amplitudes, but with the very precise data⁶ presently available on $A(\Sigma^{-})$ we have now unambiguous evidence for the extra terms, independent of the amount of the (27,1) piece. In fact, without these extra terms we would have, according to the standard soft-pion result,¹

$$A\left(\Lambda_{-}^{0}\right) = \frac{1}{\sqrt{3}} \frac{1}{f_{\pi}} \left(d' + 3f'\right) + \frac{2}{\sqrt{15}} a_{27},$$

$$A\left(\Xi_{-}^{-}\right) = \frac{1}{\sqrt{3}} \frac{1}{f_{\pi}} \left(d' - 3f'\right) + \frac{2}{\sqrt{15}} a_{27},$$

$$A\left(\Sigma_{-}^{-}\right) = -\frac{\sqrt{2}}{f_{\pi}} \left(d' - f'\right) - \frac{2\sqrt{2}}{\sqrt{5}} a_{27},$$

$$A\left(\Sigma_{0}^{+}\right) = \frac{\Lambda}{f_{\pi}} \left(d' - f'\right) - \frac{4}{3\sqrt{5}} a_{27},$$

$$A\left(\Sigma_{+}^{+}\right) = \frac{2}{3} \sqrt{10} a_{27}.$$
(2)

For simplicity I have written down the (27,1) terms without the factor $1/\sqrt{2}f_{\pi}$.

From Eqs. (2) we obtain the usual two sum $rules^{1}$:

$$A(\Sigma_0^+) = -\frac{1}{\sqrt{2}} [A(\Sigma_-^-) + \sqrt{2}A(\Sigma_+^+)]$$
(3)

and

$$\Delta(LS) = 2A (\Xi^{-}) + A (\Lambda^{0}_{-}) + (\frac{3}{2})^{1/2}A (\Sigma^{-}_{-}) = 0.$$
(4)

The first sum rule [Eq. (3)] reduces to the usual

 $\Delta I = \frac{1}{2}$ rule in the limit $a_{27} = 0$. The second sum rule is independent of the knowledge of a_{27} and can therefore be tested experimentally. It is a generalized (LS) relation in the presence of the (27,1)piece¹ and is reduced to the usual LS relation for the octet (8,1) piece by setting $a_{27} = 0$ and $A(\Sigma_{-}) = -\sqrt{2}A(\Sigma_{0}^{+})$. It is important to realize that to check the validity of the soft-pion theorem for S waves, it is this generalized LS relation which must be tested, rather than the usual LS relation which is not rigorously valid. This fact has been overlooked in recent analysis where only a test of the usual LS relation is given in terms of $A(\Sigma_0^+)$ and the measured value of $A(\Sigma^{\perp})$ has not been used.⁶ Had one used $A(\Sigma_{\pm}^{-})$ and $A(\Sigma_{\pm}^{+})$ one would discover immediately a large deviation from the LS relation. I take a different approach in this paper and I believe that a test of the generalized LS relation can be done without using data on $A(\Sigma_0^+)$ and $A(\Sigma_{+}^{+})$, which should be measured with greater precision. [The error in $A(\Sigma_0^+)$ is now much smaller than that given many years ago.]

The first sum rule gives

 $A(\Sigma_0^+) = -1.42 \pm 0.02$

to be compared with the measured value

 $A(\Sigma_0^+)_{exp} = -1.48 \pm 0.05.$

The agreement is rather good but one may need some small deviation from the soft-pion theorem for the (27,1) contribution to remove a possible small discrepancy. Also, the extracted data on $A(\Sigma_0^+)$ must be corrected for by the final-state interaction effects which, as pointed out by Scadron and Thebaud, are important in Σ_0^+ decay⁷ and can reduce $A(\Sigma_0^+)$ by 3%, thereby removing most of the discrepancy with the predicted value obtained with the first sum rule. [Note that the extracted values of $A(\Sigma_+^+)$ and $A(\Sigma_-^-)$ are unaffected by these final-state effects, which are negligible for Σ_+^+ and Σ_-^- decays as shown in Table II of their paper.]

For the second sum rule we find, however,

$$\Delta(\text{LS})_{\text{exp}} = -0.246 \pm 0.04, \tag{5}$$

showing a large deviation from the generalized LS relation. This deviation cannot come from the K^* -pole terms which satisfy the LS relation to first order in SU(3) breaking. [The $B_f B_i K^*$ couplings are universal and from a SU(3) stand point are the same as the vector-current couplings to baryons which are purely of the *F* type.] More precisely, we have

$$\Delta(\text{LS})_{K^*} = (\frac{3}{2})^{1/2} C' [3m_{\Lambda} + m_{\Sigma} - 2(m_N + m_{\Xi})] = O(\epsilon^2),$$

which vanishes to first order in SU(3) breaking.

Thus some extra terms other than the K^* -pole terms must be present. One such contribution is naturally the low-lying excited-baryon $B^*(\frac{1}{2})$ pole terms which in the quark model can be expressed as⁴

$$A'(\Sigma_{-}^{-}) = -(\sqrt{2}/f_{\pi})(18\sqrt{3}C\Delta M'),$$

$$A'(\Lambda_{-}^{0}) = -(\sqrt{2}/f_{\pi})(9\sqrt{2}C\Delta M),$$

$$A'(\Xi_{-}^{-}) = (\sqrt{2}/f_{\pi})(18\sqrt{2}C\Delta M'''),$$

(6)

which gives

$$\Delta (\text{LS})_{B^*} = -(18C/f_{\pi})(3\Delta M' + \Delta M - 4\Delta M'''), \quad (7)$$

where

$$\Delta M = m_{\Lambda} - m_{N}, \quad \Delta M' = m_{\Sigma} - m_{N},$$

$$\Delta M'' = m_{\Xi} - m_{\Sigma}, \quad \Delta M''' = m_{\Xi} - m_{\Lambda},$$

in the notations of Marshak, Riazuddin, and Ryan. Using the experimental value of Δ (LS), we find

$$18C \simeq 0.25,\tag{8}$$

from which we deduce

$$A'(\Sigma_{-}^{-}) = -1.24, \quad A'(\Lambda_{-}^{0}) = -0.35, A'(\Xi_{-}^{-}) = 0.80.$$
(9)

Thus the $\frac{1}{2}^{-}$ low-lying baryon resonances produce a large correction to the soft-pion S-wave amplitudes (given by the commutator terms $\langle B_f | [Q_i^5, H_w] | B_i \rangle$). It is significant that from a simple algebraic relation among the $B^*(\frac{1}{2}^-)$ terms as given by the quark model and from $\Delta(\text{LS})_{\text{exp}}$, we obtain a reduction of the soft-pion amplitude as indeed found in the nonrelativistic quark model.

If we now assume that these corrections are given empirically as in (9), then from the measured Swave amplitudes, we deduce the following soft-pion amplitudes

$$f' = 1.35 f_{\pi}, \quad d' = -0.89 f_{\pi}, \tag{10}$$

independent of the detailed dynamics of the quark model. [In obtaining (10) I have neglected a negligible amount of the (27,1) piece.]

From (10) we get

 $(d'/f')_{NL} = \frac{2}{3},$

which differs significantly from the usual value¹ $-\frac{1}{3}$ obtained without the correction terms. It should be stressed that the usual S-wave soft-pion

fit with

$$(d'/f')_{NL} = (d/f)_{mass term} = -\frac{1}{3}$$

is no guarantee for the validity of the CA soft-pion theorem. Under SU(3)_L \otimes SU(3)_R, the mass term u_8 is a (3, 3^{*}) + (3^{*}, 3) piece while H_w is an octet (8,1) piece; with respect to SU(3) they are two entirely different objects so that their d/f ratio is not related to each other and $(d'/f')_{NL}$ need not be $-\frac{1}{3}$.

I end this note with some comments on the Pwave amplitudes. In general we expect similar corrections for the *P*-wave amplitudes, as found recently in a quark-model calculation.⁸ The magnitude of these corrections are difficult to obtain in a reliable manner because of uncertainties in the evaluation of the Born terms in which the s- and uchannel ground-state baryon pole contributions tend largely to cancel out so that the values obtained for the P-wave amplitudes are sensitive to d', f' and to possible SU(3) violation effects in the meson-baryon coupling constants. If, from a chiral $SU(3) \otimes SU(3)$ effective Lagrangian standpoint^{8,9} one assumes that the axial vector-current baryon matrix elements are SU(3) symmetric (i.e., determined by D and F) rather than the meson-baryon pseudoscalar coupling constants, then, with the new d', f' in (10), apart from $B(\Sigma^{-})$ and $B(\Lambda^{0})$ which agree well with data, the calculated values for $B(\Sigma_0^+)$, $B(\Sigma_+^+)$, and $B(\Xi_-^-)$ are still too small

compared with experiments. This suggests that the *P*-wave amplitude cannot be consistently described by the Born terms alone.

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