Applicability of Perturbative Quantum Chromodynamics to High-p_t Exclusive Scattering

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Relations among high-p_t exclusive cross sections, particularly for $\gamma\gamma \rightarrow$ baryon-antibaryon and for meson-nucleon scattering, test the applicability of perturbative QCD in the kinematic regime of the experiments. This permits assessment of the importance of higher-order corrections and non-leading-twist contributions to the Born approximation, the extent of the "Sudakov suppression," and the validity of the asymptotic approximation to the wave functions.

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Ten years ago it was argued that exclusive hadron scattering at large momentum transfer is essentially a "short-distance" process, with the observed regularities in the energy dependence reflecting the quark composition of the hadrons, and with the absolute normalization, energy, angular, and helicity dependences computable (in principle) by use of perturbative QCD.[1] In the intervening time progress has been slow, largely because of the difficulty of evaluating Born amplitudes involving many quarks and gluons, and of analyzing the higher-order corrections to them. Several processes have by now been calculated to lowest order. The first and cleanest theoretically was the pion form factor[2]; then came nucleon form factors,[3] $\gamma\gamma \rightarrow \pi\pi$ and $\rho\rho$,[4] $\gamma\gamma \rightarrow p\bar{p}$,[5,6] and a numerical study of $\pi^+\pi^+ \rightarrow \pi^+\pi^+$.[7] Recently an algebraic computer program has been completed with which the hundreds of thousands of Born diagrams required for more complicated processes such as photoproduction, meson-nucleon scattering, and nucleon-nucleon scattering can be evaluated.[6,8] Meanwhile studies of the higher-order corrections in QCD have clarified, but not completely resolved, the question of which large-momentum-transfer processes are governed by perturbative QCD and which may inevitably involve infrared and confinement effects. Briefly, the form factors should be simply calculable within perturbative QCD while purely hadronic amplitudes seem to factorize into a perturbatively calculable piece and a "Sudakov factor" which damps the contribution of nonperturbative regions of internal momenta.[3,9] The Sudakov factors may be calculable, at least if one is ready to re-sum perturbation theory in a not entirely uncontroversial way. However, incorporating them correctly and integrating the quark Born amplitudes over the hadron wave functions is a major task. It is not yet known with certainty whether, with the Sudakov suppression correctly incorporated, the nonperturbative integration regions are completely or only partially eliminated.[7,9]

Nonetheless, the complete evaluation of the Born amplitudes sets the stage for a major theoretical advance in our ability to predict the cross sections including their dependence on helicity, energy, and angle. It is therefore particularly timely to point out experimental means of directly determining whether these perturbative calculations are actually relevant for the processes at hand in the regimes of energy and momentum transfer being measured.[10]

If perturbative QCD is correct, the calculation of physical hadron-scattering amplitudes can be expressed as a convolution of fundamental quark-scattering amplitudes with wave functions of the hadrons in terms of quarks. Apart from the overall normalization, these factors are computable in the asymptotic limit of infinite momentum transfer, putting aside for the moment the problem of nonperturbative contributions and the Sudakov suppression. The question is, how accurate are the asymptotic approximations at experimentally accessible momentum transfers? Specifically:

Are the wave functions well approximated by their minimal Fock-state components, $q\bar{q}$ and qqq for mesons and baryons, respectively?

Do they have the SU(6) flavor-spin structure predicted asymptotically?

Can the internal p_t's of the quarks inside the hadrons be neglected?

Are the quark scattering amplitudes well approximated by lowest-order (i.e., Born approximation) QCD?

Can higher-twist effects (e.g., from quark masses) be ignored in predictions of the quark-scattering amplitudes in experimentally interesting kinematic regimes?

Most importantly of all, are the exclusive hadron-scattering amplitudes completely governed by perturbatively calculable quark-scattering amplitudes?

Every one of these questions can be answered without recourse to absolute predictions, as we shall see below, because the comparative study of a suitable set of experimental cross sections permits separate evaluation of the legitimacy of each approximation being made.

I. *Higher-twist effects due to quark masses and intrinsic p_t's are measured by hadron-helicity nonconservation.*—Chirality is conserved at every vertex by vector interactions, so that chirality conservation is true to all orders in QCD. In the limit that quark masses can be neglected, the helicity and chirality are identical for quarks, and for antiquarks the helicity is the negative of the chirality. Since the helicity of a hadron is the sum of the helicities of its quarks, as long as the p_t's of the quarks inside the hadron are negligible compared to their longitudinal momenta, it follows that $\sum h_i = \sum h_f$, where h_i and h_f are the initial and final hadron helicities.[11] Nonvanishing values for cross sections which violate helicity conservation thus measure higher-twist effects, independently of all other approximations (including truncating the perturbative calculation, e.g., in Born approximation), and should decrease as a power of s.

A new prediction which is stronger than chirality conservation alone is that the amplitude for $\gamma_L + \gamma_L$ and $\gamma_R + \gamma_R \to B_{\pm 3/2} + \bar{B}_{\mp 3/2}$ vanishes identically, if only the qqq Fock state of the baryon contributes. That this is true diagram by diagram in Feynman-gauge Born approximation was discovered by computer.[6] I have proven that the result holds diagram by diagram to all orders;[12] this is most easily demonstrated by use of the two-component formalism of Ref. 8, which I extended to diagrams with loops. The size of $\gamma_{L(R)} + \gamma_{L(R)} \to B_{\pm 3/2} + \bar{B}_{\mp 3/2}$ therefore is sensitive to the contribution of nonvalence components of the baryon wave function, as well as the $p_t/p_{\parallel}$ and masses of the quarks, at the actual s and t of the experiment.

II. *The flavor-chirality wave functions and the importance of nonvalence components in the Fock-state wave functions are tested by relations among cross sections.*—At short distance, the hadron flavor-helicity wave functions are those of SU(6)[3]; e.g., a positive-helicity proton has the wave function $6^{-1/2}[2u_+u_+d_- - u_+d_+u_- - d_+u_+u_-]$. The scattering amplitudes for any hadronic process are obtained by projecting the fundamental quark amplitudes relevant to that process onto the SU(6) flavor wave functions of the physical hadrons. Therefore nontrivial relations exist between various hadron cross sections following from their dependence on the same set of quark amplitudes.[12]

For $\gamma\gamma \to B\bar{B}$ the fundamental quark amplitudes, A_i, are shown in Fig. 1. There are fifteen different reactions of the type $\gamma\gamma \to B\bar{B}$ with either the baryon or antibaryon in the octet, which depend (for each photon helicity combination) on only four amplitudes: A_{0-3}.[13] Eleven relations are thus predicted among the hadron amplitudes. Nine of these follow from U-spin invariance (the $\gamma\gamma$ initial state is a U-spin singlet) and are therefore sensitive to the importance of mass differences, but not to the importance of an SU(3) singlet "sea":

$$\Sigma^+\bar{\Sigma}^- = p\bar{p}; \quad \Xi^0\bar{\Xi}^0 = n\bar{n}; \quad \Xi^-\bar{\Xi}^+ = \Sigma^-\bar{\Sigma}^+;$$
$$\Lambda\bar{\Sigma}^0 = \sqrt{3}(n\bar{n} - \Lambda\bar{\Lambda}); \quad \Sigma^0\bar{\Sigma}^0 = -2n\bar{n} - 3\Lambda\bar{\Lambda};$$
$$\Sigma^+\bar{Y}^{*-} = p\bar{\Delta}^-; \quad n\bar{\Delta}^0 = \Xi^0\bar{\Xi}^{*0};$$
$$\Lambda^0\bar{Y}^{*0} = \sqrt{3}\Sigma^0\bar{Y}^{*0} = \tfrac{1}{2}\sqrt{3}n\bar{\Delta}^0 \ .$$

Note that U-spin cannot relate $B_8\bar{B}_8$ and $B_8\bar{B}_{10}$ amplitudes. The two additional relations among the amplitudes hold only if components of the baryon wave functions containing additional $q\bar{q}$ pairs or valence glue can be neglected, because if the baryon Fock state contains more than three quarks, the set of fundamental amplitudes must be enlarged beyond the four A_i of Fig. 1. These two new relations relate $B_8\bar{B}_8$ and $B_8\bar{B}_{10}$ amplitudes and can be written[12] as

$$p\bar{\Delta}^- = \sqrt{2}(p\bar{p} + n\bar{n} - 2\Lambda\bar{\Lambda} - \Sigma^-\bar{\Sigma}^+);$$
$$n\bar{\Delta}^0 = 2\sqrt{2}(n\bar{n} - \Lambda\bar{\Lambda}) \ .$$

Including final states with helicity-$\frac{3}{2}$ baryons adds ten reactions (twenty if the baryon helicities are measured) and involves two more amplitudes, $A_{4,5}$, six fewer than U-spin alone would imply. A similar analysis of meson-nucleon scattering is even more impressive. For example, there are 66 reactions of the type $\{\pi^{\pm}$ or $K^{\pm,0}\} + p \to \{\pi, K, \eta, \eta'$ or $\phi\} + B_8$

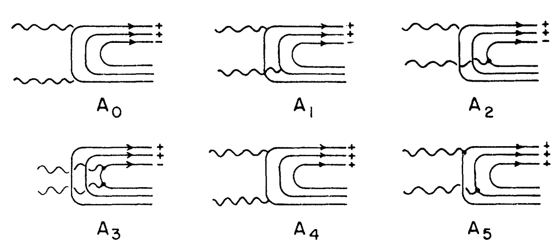

FIG. 1. Independent quark amplitudes necessary to completely determine $\gamma\gamma \to B_8\bar{B}_8$, $B_8\bar{B}_{10}$, and $B_{10}\bar{B}_{10}$, with photon symmetrization and gluon connections understood. The + (−) labels the chirality of the fermion line.

TABLE I. Amplitudes for $\{\pi^{\pm}$ or $K^{\pm,0}\} + p \to \{\pi, K, \eta, \eta'$ or $\phi\} + B_8$ or B_{10} in terms of the fundamental quark amplitudes defined in Ref. 12. Reactions followed by the same letter are proportional to one another, usually as a consequence of U-spin and isospin invariance at large p_t.

REACTION	$(\text{COEF})^{-1}$	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
$\pi^+p \to \pi^+p$	12	6	6	1	4	5	2	1	2	2				
$\to K^+\Sigma^+$	12			1	4			1	2	2				
$\to \pi^0\Delta^{++}$	$4\sqrt{3}$ a			-1	2	1	-2	-1	1	-2				
$\to \eta\Delta^{++}$	12 b			-1	2	-1	2	-1	1	-2				
$\to \eta'\Delta^{++}$	$6\sqrt{2}$ c			-1	2	-1	2	-1	1	-2	-6	6	-3	3
$\to \phi\Delta^{++}$	$2\sqrt{6}$ d										-2	-2	-1	-1
$\to \pi^+\Delta^+$	$6\sqrt{2}$ a			-1	2	1	-2	-1	1	-2				
$\to K^+Y^{*+}$	$6\sqrt{2}$			-1	2			-1	1	-2				
$\pi^-p \to \pi^-p$	12	6	6	5	2	1	4	1	2	2				
$\to \pi^0n$	$12\sqrt{2}$			4	-2	-4	2							
$\to \eta n$	$12\sqrt{6}$			-4	2	-4	2	-4	4	-2				
$\to \eta' n$	$12\sqrt{3}$			-4	2	-4	2	-4	4	-2	-6	6	-12	12
$\to \phi n$	12										-2	-2	-4	-4
$\to K^0\Lambda$	$4\sqrt{6}$			3				1		1				
$\to K^0\Sigma^0$	$12\sqrt{2}$			-1	-4			1	-4	-1				
$\to K^+\Sigma^-$	12 g							-2	2	-1				
$\to \pi^-\Delta^+$	$6\sqrt{2}$			1	-2	-2	2	-1	-2	1				
$\to \pi^0\Delta^0$	12			-1	2	1	-2	3	3					
$\to \eta\Delta^0$	$12\sqrt{3}$ b			1	-2	1	-2	1	-1	2				
$\to \eta'\Delta^0$	$6\sqrt{6}$ c			1	-2	1	-2	1	-1	2	6	-6	3	-3
$\to \phi\Delta^0$	$6\sqrt{2}$ d										2	2	1	1
$\to \pi^+\Delta^-$	$2\sqrt{6}$ e							2	1	1				
$\to K^0Y^{*0}$	12			1	-2			-1	-2	1				
$\to K^+Y^{*-}$	$6\sqrt{2}$ e							2	1	1				
$K^+p \to K^+p$	12	6	6			5	2							
$\to K^0\Delta^{++}$	$2\sqrt{6}$ f					-1	2							
$\to K^+\Delta^+$	$6\sqrt{2}$ f					1	-2							
$K^-p \to K^-p$	12	6	6	5	2									
$\to \bar{K}^0n$	12			-4	2									
$\to \pi^0\Lambda$	$8\sqrt{3}$ i					3		1	1					
$\to \eta\Lambda$	24			-6		3		3	-3					
$\to \eta'\Lambda$	$12\sqrt{2}$			3		3		3	-3				9	-9
$\to \pi^-\Sigma^+$	12					1	4	1	2	2				
$\to \pi^0\Sigma^0$	24					-1	-4	-3	-3					
$\to \eta\Sigma^0$	$24\sqrt{3}$ j			2	8	-1	-4	-1	1	4				
$\to \eta'\Sigma^0$	$12\sqrt{6}$ k			-1	-4	-1	-4	-1	1	4	12	-12	-3	3
$K^-p \to \pi^+\Sigma^-$	12 m							-2	-1	2				
$\to K^0\Xi^0$	12 g							-2	2	-1				
$\to K^+\Xi^-$	12							4	-1	-1				
$\to K^-\Delta^+$	$6\sqrt{2}$ h			1	-2									
$\to \bar{K}^0\Delta^0$	$6\sqrt{2}$ h			1	-2									
$\to \pi^-Y^{*+}$	$6\sqrt{2}$					-1	2	-1	-2	1				
$\to \pi^0Y^{*0}$	$12\sqrt{2}$					1	-2	3	3					
$\to \eta Y^{*0}$	$12\sqrt{6}$ n			-2	4	1	-2	1	-1	2				
$\to \eta' Y^{*0}$	$12\sqrt{3}$ c			1	-2	1	-2	1	-1	2	6	-6	3	-3
$\to \pi^+Y^{*-}$	$6\sqrt{2}$ e							2	1	1				
$\to K^0\Xi^{*0}$	$6\sqrt{2}$							-1	-2	1				
$\to K^+\Xi^{*-}$	$6\sqrt{2}$ e							2	1	1				
$K^0p \to K^0p$	12	6	6			1	4							
$\to K^+n$	6					-4	2							
$\to K^0\Delta^+$	$6\sqrt{2}$ f					-1	2							
$\to K^+\Delta^0$	$6\sqrt{2}$ f					1	-2							
$\bar{K}^0p \to \bar{K}^0p$	12	6	6	1	4									
$\to \pi^+\Lambda$	$4\sqrt{6}$ i					3		1	1					
$\to \pi^0\Sigma^+$	$12\sqrt{2}$ l					-1	-4	1	-1	-4				
$\to \eta\Sigma^+$	$12\sqrt{6}$ j			-2	-8	1	4	1	-1	-4				
$\to \eta'\Sigma^+$	$12\sqrt{3}$ k			1	4	1	4	1	-1	-4	-12	12	3	-3
$\to \pi^+\Sigma^0$	$12\sqrt{2}$ l					-1	-4	1	-1	-4				
$\to K^+\Xi^0$	12 m							-2	-1	2				
$\to K^-\Delta^{++}$	$2\sqrt{6}$ h			-1	2									
$\to \bar{K}^0\Delta^+$	$6\sqrt{2}$ h			-1	2									
$\to \pi^0Y^{*+}$	12 o					1	-2	-1	1	-2				
$\to \eta Y^{*+}$	$12\sqrt{3}$ n			2	-4	-1	2	-1	1	-2				
$\to \eta' Y^{*+}$	$6\sqrt{6}$ c			-1	2	-1	2	-1	1	-2	-6	6	-3	3
$\to \pi^+Y^{*0}$	12 o					1	-2	-1	1	-2				
$\to K^+\Xi^{*0}$	$6\sqrt{2}$							-1	1	-2				

or B_{10} which depend on only thirteen quark amplitudes in linear combinations completely determined by the SU(6) wave functions, as tabulated in Table I. Reference 12 gives complete results for all exclusive processes of experimental interest and a detailed discussion of what is tested by each relation.

In the event the predicted relations among the amplitudes are not exact, the rate at which they improve with s (as a power, or logarithmically, or not at all) determines (respectively) whether their failure is due to quark masses or nonvalence Fock-state contributions, nonasymptotic flavor wave functions, or unequal short-distance normalizations of the baryon wave functions.[13]

III. *The validity of perturbative* QCD *and of the Born approximation to the quark-scattering amplitudes can be determined by studying the phases of hadron amplitudes.*—In QCD, a large-p_t exclusive hadron-scattering amplitude is determined by a sum of quark-scattering diagrams, such as those shown in Fig. 1 for $\gamma\gamma \to$ baryon-antibaryon. Each amplitude A_i shown in the figure is actually a sum of many diagrams (one for each possible way of connecting the quarks by the minimal number of gluons), which must be evaluated for arbitrary momentum fractions $\{x_j\}$ for the external quarks, and then integrated over the amplitudes for the quarks of the hadrons to carry the fractions $\{x_j\}$. The quark amplitude for any process is real in Born approximation; however, for a general hadronic process there are kinematic configurations of the quarks (pinch singularities in the integration over the $\{x_j\}$ [14]) in which intermediate states are "on mass shell." Therefore by dispersion relations the hadron-scattering amplitude has an imaginary part. Confinement, not present at any finite order of perturbation theory, presumably prevents quarks from propagating freely; it is not known what that implies for the phases of the hadron-scattering amplitudes.[15]

For $\gamma\gamma \to B\bar{B}$, $\gamma\gamma \to$ two mesons, and form factors, explicit examination of all diagrams shows that the integrations over the wave functions do not take any virtual intermediate quarks on mass shell. Nonzero phases of $\gamma\gamma$ annihilation amplitudes therefore are only due to higher-order corrections to the Born approximation and "should" be small—$O(\alpha_s)$—and decreasing as $\ln s$. These phases can be experimentally determined for $\gamma\gamma \to B\bar{B}'$, because with polarized photons 25 cross sections (35, if helicities of final decuplet baryons can be measured) depend on six magnitudes and four relative phases of the quark amplitudes.[12]

It can be seen from Table I that in $\{\pi^{\pm}$ or $K^{\pm,0}\} + p \to \{\pi, K, \eta, \eta'$ or $\phi\} + B_8$ or B_{10} there are 66 independent measurable reactions of which 45 depend on distinct linear combinations of thirteen (in general complex) A_i. With the absolute values of 45 distinct linear combinations of the A_i available for measurement, the thirteen $|A_i|$ and their relative phases are highly overdetermined. Together with the 21 proportionalities among cross sections, these relations provide an extraordinary arena for testing the theory. Comparison of phases in the $\gamma\gamma$ annihilation reactions where nonzero values are due only to higher-order corrections, with those, say, in meson-nucleon scattering where they are potentially due both to higher-order perturbative corrections and to contributions from nonperturbative regions of the wave-function integration, allows assessment of the importance of nonperturbative effects.

The tests discussed here of the assumptions underlying the perturbative calculations do not rely on the results of the calculations themselves. If we are fortunate, these tests will demonstrate the validity of the perturbative QCD approximations for large-p_t exclusive hadron scattering in experimentally accessible kinematic regimes. Then, thanks to the great variety of hadrons and the diverse character of available hadronic processes, including real and virtual photons, the theory can be quantitatively tested in literally hundreds of different reactions.

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[11]S. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981), suggested that these relations be used as a test of the spin-1 nature of the gluon. Here, instead, I am advocating assumption of the correctness of QCD and use of such relations to test whether the asymptotic regime has been reached in specific experiments. The superficially straightforward method of studying higher-twist effects by seeing whether the energy dependence of the cross sections is that given by the leading power-law prediction (Ref. 1) is actually not satisfactory because a variety of complications, including running coupling constants, anomalous dimensions of the wave functions, and Sudakov factors, make the actual asymptotic energy dependence a nontrivial prediction requiring detailed calculation.

[12]G. R. Farrar, Rockefeller University Report No. RU-83-34 (unpublished), and to be published, and Rockefeller University Report No. RU-83-46, in Proceedings of the LAMPF II Workshop, Los Alamos, July 1983 (to be published).

[13]This assumes that the short-distance normalization of all the baryon wave functions is the same. Defining ϕ_B to be the normalization of the leading-twist component of the baryon wave function at short distance, we have

$$\Gamma(\psi \to B\bar{B})/\Gamma(\psi \to p\bar{p}) \sim (\phi_B/\phi_p)^4 (p_B/p_p).$$

For the Λ this ratio is $[(0.11 \pm 0.02)/(0.22 \pm 0.02)]$, giving $\phi_\Lambda/\phi_p = 0.9$, which is consistent, within the errors, with $\phi_\Lambda = \phi_p$, as are the ratios for n, Σ, and Ξ, although these have larger errors. There are no data yet to give ϕ_Δ, but it is reasonable to expect that the decuplet has the same short-distance normalization as the octet. If not, that will become apparent when cross sections involving 8's are compared with ones involving 10's, such as $\gamma\gamma \to \Lambda\bar{\Sigma}^0$ and $\gamma\gamma \to \Lambda\bar{Y}^{*0}$.

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