Intrinsic Dynamical Instability in Optical Bistability with Two-Level Atoms

L. A. Orozco, A. T. Rosenberger, and H. J. Kimble
Department of Physics, University of Texas at Austin, Austin, Texas 78712
(Received 9 August 1984)

Self-pulsing instability is reported in an experiment in which two-level atoms couple to a single mode of a traveling-wave resonator to produce optical bistability. The threshold for the onset of oscillation together with the characteristics of the dynamical state are described. The self-pulsing instability observed is the analog in an absorbing system of the single-mode laser instability.

PACS numbers: 32.80.Kf, 05.70.Ln, 42.50.+q, 42.65.-k

Of the many examples of optical phenomena that display nonequilibrium phase transitions, one that has received considerable attention in recent years is optical bistability.[1-3] In addition to the bistability associated with time-independent solutions, bifurcations leading to a variety of dynamical instabilities have been predicted.[3-5] These "self-pulsing" instabilities have their roots in corresponding instabilities first analyzed for the homogeneously broadened laser[6,7] and share a common physical origin in that a strong intracavity field alters the absorption profile (gain profile in the case of the laser) of the intracavity medium to produce amplification at frequencies offset from that of the strong field. This basic mechanism is intrinsic to virtually all problems in optical physics involving the interaction with an intense field. For problems involving an optical resonator nonlinear dispersion must also be considered in order to complete the specification of the cavity boundary conditions.

The possibility of obtaining optical gain or amplification from a noninverted medium was long ago recognized in the context of parametric amplification[8] and in the absorption spectrum of two-level atoms driven by an intense field.[9] With regard to optical bistability, dynamical instability due to this intrinsic mechanism was first discussed by Bonifacio and Lugiato[10] and by Ikeda,[11] and involved longitudinal modes of the resonator. Single-mode instability in passive bistable systems was treated by Lugiato *et al.*[12] and by Ikeda and Akimoto[13] and included predictions of period doubling to chaos.[11] An instability involving only a single resonator mode is unique in optical bistability and in optical systems in general in that the atomic and field variables change collectively on a time scale much greater than that of the cavity round-trip time. As shown by Lugiato *et al.*[14], the occurrence of the single-mode instability involves a delicate interplay of nonlinear gain and dispersion in order that multiple frequencies can coexist in a resonator with effectively only a single mode. A model based upon "side-mode" gain alone does not suffice to explain the instability.

Of the numerous types of instabilities predicted to occur in optical bistability only a few have actually been observed.[15-19] Of these there have as yet been no observations of intrinsic instabilities for continuous-wave excitation, where intrinsic has the sense discussed above.[18,19] We have observed dynamical instability in a bistable system composed of homogeneously broadened, two-level atoms within a traveling-wave interferometer. Our experiment corresponds closely to one of the canonical theoretical models employed in quantum optics. The system is operated with atomic and cavity decay rates of comparable magnitude and in a domain such that only a single resonator mode need be considered. By exploring regions of atomic cooperativity C greater than 20 times that for the critical onset of (absorptive) bistability, we observe a complex phenomenology associated with the instability, including hysteresis in the appearance of self-pulsing, precipitation (or instability of the self-pulsing solution), and self pulsing in the absence of hysteresis in the steady-state characteristics.

In our experiment well-collimated beams of atomic sodium intersect at 90° the plane of a high-finesse interferometer, as shown in Fig. 1. The essential aspects of the apparatus are as described previously.[20] Optical prepumping of the atomic beams produces a two-state transition ($3^2S_{1/2}$, $F=2, m_F=2 \rightarrow 3^2P_{3/2}$, $F=3, m_F=3$) in the D_2 line of atomic sodium with an absorption width of 13 MHz. The dominant broadening mechanism above the 10-MHz natural linewidth is transit broadening. The interferometer is composed of two mirrors each of radius of curvature 5 cm and with transmission coefficients $T_1 = T_2 = (3.0 \pm 0.2) \times 10^{-3}$. The injected laser is matched in transverse profile to the fundamental TEM_{00} mode of the cavity with an efficiency of approximately 95%. In the absence of

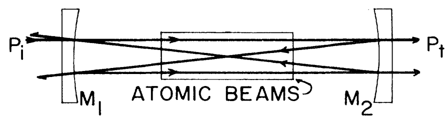

FIG. 1. Illustration of the confocal optical cavity containing well-collimated atomic beams directed out of the plane of the page.

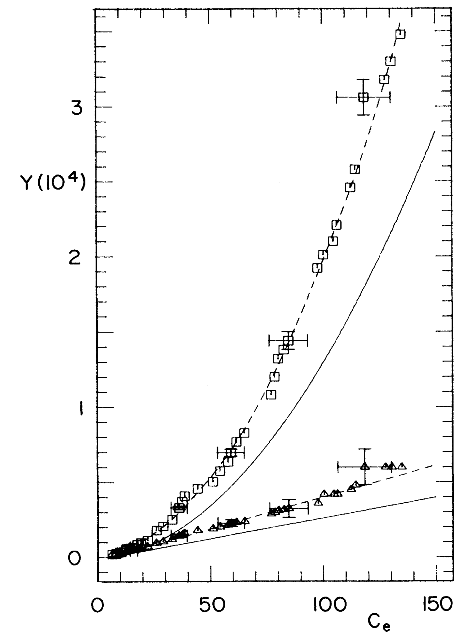

FIG. 2. Turning points Y for the steady-state hysteresis curve in absorptive bistability vs effective cooperativity C_e. $\Delta \approx 0 \approx \theta$. The full curve is the theoretical result of Ref. 21, as discussed in the text. This result is multiplied by 1.5 to give the dashed line.

the atomic beams (empty cavity) the peak transmission coefficient T_0 of the cavity is $(9.0 \pm 0.5) \times 10^{-2}$, and the cavity finesse F is 300 ± 20.

Observation of hysteresis in the cavity transmission characteristics is made following the procedure previously described.[20] We have been able to explore the range of effective cooperativity parameter C_e of $0 \leqslant C_e \leqslant 175$, where for the cavity geometry of Fig. 1, $C_e \equiv (\alpha_m LF)/4\pi$ with $\alpha_m L$ as the resonant atomic absorption for intensity loss in a round trip. Over the range of C_e we have recorded the incident and transmitted switching powers P_i and P_t of the hysteresis cycle in absorptive bistability with zero cavity detuning. These powers are in turn converted to scaled intensities through the relations[21] $Y \equiv P_i \epsilon / \pi \omega_0^2 I_s$ and $X \equiv P_t / \pi \omega_0^2 I_s T_2$, with ϵ giving the intracavity enhancement and with $I_s = 7.3$ mW/cm^2.[20] Figure 2 shows the result of our experiment for the onset and evolution of hysteresis compared to the prediction of theory in the single-transverse-mode approximation, including the small residual Doppler broadening.[21] The ratios of the input switching points in absorptive bistability have been used to calibrate our relative measurements of $\alpha_m L$ to absolute values of C_e through a single parameter over the entire range of C_e. A scaling of theory by a factor of 1.5, which is within our experimental uncertainty for Y, produces satisfactory agreement with experiment, as shown by the dashed line in Fig. 2.

Armed with this information about the reliability of theory as applied to the steady-state regime in absorptive bistability for our experiment, we next turn to an investigation of the dispersive regime. For C_e greater than about 50, and with small atomic detuning Δ and large cavity detuning θ of opposite sign each in units of its respective half-linewidth, self-pulsing is observed in the upper branch of the hysteresis cycle. We have recorded the time dependence of the output intensity X for a constant incident intensity Y and for fixed (C_e, θ, Δ), with the cavity length held constant by locking the cavity transmission to a Zeeman stablilized He-Ne laser. The oscillation is sinusoidal with a frequency ranging from 20 to 70 MHz and with a depth of modulation as great as 7 to 1. The general behavior of the self-pulsing instability for a variety of parameters (C_e, θ, Δ) is illustrated in Fig. 3. In this figure the detection bandwidth is small (~ 7 MHz) thus reducing the amplitude of the observed self-pulsing and allowing one to identify more clearly the characteristics of the instability with respect to the steady-state hysteresis cycle. Figures 3(a) and 3(b) show an instability occurring on the upper branch, where the arched segment of the trace represents the region of self-pulsation. Figure 3(c) illustrates the relationship of this oscillatory region to the bistable region for a scan in θ at approximately fixed (C_e, Δ). Note that the instability occurs even in the absence of hysteresis in the steady-state characteristics. When the instability extends into the region of steady-state hysteresis, precipitation to the lower branch has been observed. Figure 3(d) reveals a complex set of possible states on the upper branch; there is definitely a novel type of hysteresis associated with the appearance and range of stability of these states that is not yet well understood. Note that because of the small detection bandwidth, self-pulsing may be occurring in regions of the traces in Fig. 3 extending beyond the wide bands in X.

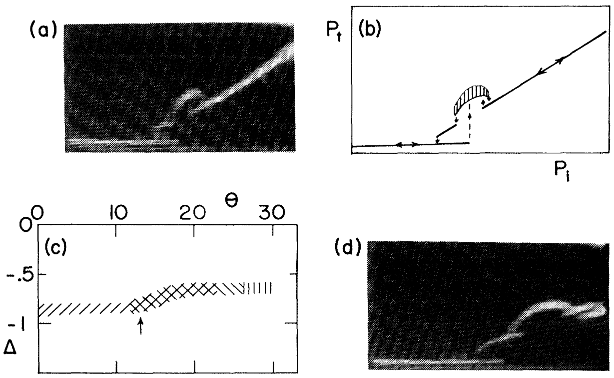

FIG. 3. Behavior of the self-pulsing instability. (a), (b) Photograph and explanatory drawing of the hysteresis cycle showing the instability as an arch separated from the otherwise straight-line segment of the upper branch ($C_e \approx 155$, $\Delta \approx -0.7$, $\theta \approx 13$, and $0 \leqslant X \leqslant 530$). (c) Evolution of the instability as a function of θ for approximately constant Δ, $C_e \approx 155$, and $Y < 1.3 \times 10^5$ (//, bistability; \\, instability; ||, no bistability, no instability). The arrow marks the position of the photograph in (a). (d) Photograph of the hysteresis cycle for $C_e \approx 157$, $\Delta \approx -0.5$, $\theta \approx 15$, and $0 \leqslant X \leqslant 320$. Several states are possible on the upper branch and are connected by a complex hysteresis pattern as the incident field is swept up and down.

While we are in the process of determining the precise boundaries of the self-pulsation in the (C_e, θ, Δ, X) space, several quantitative statements can nonetheless be made. The minimum value of C_e for self-pulsation appears to be in the range of 50–60. As in Fig. 3(c), for $C_e = 71$ and $\Delta = -1.5 \pm 0.2$ the instability in θ starts at $\theta = +5 \pm 1$ and extends to beyond $\theta = +20$. A change in the sign of Δ produces the same qualitative behavior if the sign of θ is also changed, indicating that self-focusing in the $F=2 \rightarrow F=3$ transition does not play an important role.[16] The frequency of the oscillation is apparently determined principally by the detuning θ; at a given θ, as X is varied by more than 2 to 1 along the upper branch over the range of instability, the frequency changes by less than $\pm 10\%$. Note that the mean value of the pulsation is displaced *above* the corresponding steady solution, contrary to the situation found for the multimode instability.[3,22]

Our observations are in reasonable accord with predictions for stability boundaries and frequencies of self-oscillation obtained from an analysis of the Maxwell-Bloch equations for the single-mode instability in optical bistability.[14,23] That our system is operated in the single-mode regime is evidenced by noting that (1) the free-spectral range of our cavity is large (1.5 GHz), (2) the detunings θ are small ($\theta = 10$ corresponds to a frequency detuning $\Delta\nu = 25$ MHz), (3) the Rabi frequencies Ω are small over the regions of the upper branch in Fig. 3 ($X = 100$ corresponds to a Rabi frequency $\Omega/2\pi = 80$ MHz), and (4) κ, $\gamma_\perp$, and $\gamma_\parallel$ are of the same order ($\kappa/\gamma_\perp = 0.4$, $\gamma_\parallel/\gamma_\perp = 1.6$, and $\gamma_\parallel = 2\pi \times 10^7$/sec). While period doubling is predicted by the plane-wave theory,[12,23] in a Fourier spectrum of the pulsation we have seen no evidence for subharmonics. We are attempting to extend our measurements to yet higher values of C_e in search of other dynamical states.

In summary, we have reported observations of intrinsic dynamical instability in a passive bistable system with cw excitation. The operating regime of our experiment corresponds to that appropriate for the single-mode instability in a homogeneously broadened system, which is the counterpart in an absorbing system of the single-mode laser instability.[6] Alternatively, the oscillation can be viewed as a result of amplification in four-wave mixing but with the important restrictions imposed by the cavity boundary conditions and by the strong coupling of cavity and atomic dynamics. The observed self-pulsing is robust, occurring over wide ranges in the parameter space (C_e, θ, Δ, X) with a complex behavior that includes hysteresis and precipitation. A preliminary comparison with theory supports our interpretation of the phenomenon.

This work was supported by the National Science Foundation under Grant No. PHY8211194, and by the Venture Research Unit of British Petroleum North America. We gratefully acknowledge correspondence with Professor L. A. Lugiato and Professor L. M. Narducci, who have provided a wealth of information through unpublished results.

[1] *Optical Bistability,* edited by C. M. Bowden, M. Ciftan, and H. R. Robl (Plenum, New York, 1981); *Optical Bistability 2,* edited by C. M. Bowden, H. M. Gibbs, and S. L. McCall (Plenum, New York, 1984).

[2] E. Abraham and S. D. Smith, Rep. Prog. Phys. **45**, 815 (1982).

[3] L. A. Lugiato, in *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1984), Vol. 21, p. 71.

[4] J. C. Englund, R. R. Snapp, and W. C. Schieve, Ref. 3, p. 357.

[5] H. J. Carmichael, in *Laser Physics,* edited by J. D. Harvey and D. F. Walls, Lecture Notes in Physics, Vol. 182 (Springer, New York, 1983), p. 64.

[6]For a review see L. W. Casperson, Ref. 5, p. 88.
[7]H. Haken, Z. Phys. **190**, 327 (1966); H. Risken, C. Schmid, and W. Weidlich, Z. Phys. **194**, 337 (1966); H. Risken and K. Nummedal, J. Appl. Phys. **39**, 4662 (1968).
[8]N. Bloembergen and Y.-R. Shen, Phys. Rev. **133**, A37 (1964).
[9]B. R. Mollow, Phys. Rev. A **5**, 2217 (1972); S. L. McCall, Phys. Rev. A **9**, 1515 (1974).
[10]R. Bonifacio and L. A. Lugiato, Lett. Nuovo Cimento Soc. Ital. Fis. **21**, 510 (1978).
[11]K. Ikeda, Opt. Commun. **30**, 257 (1979).
[12]L. A. Lugiato, L. M. Narducci, D. K. Bandy, and C. A. Pennise, Opt. Commun. **43**, 281 (1982), and **44**, 207 (1983).
[13]K. Ikeda and O. Akimoto, Phys. Rev. Lett. **48**, 617 (1982).
[14]L. A. Lugiato, R. J. Horowicz, G. Strini, and L. M. Narducci, Phys. Rev. A **30**, 1366 (1984).
[15]H. M. Gibbs, F. A. Hopf, D. L. Kaplan, and R. L. Shoemaker, Phys. Rev. Lett. **46**, 474 (1981).
[16]M. LeBerre, E. Ressayre, A. Tallet, K. Tai, F. A. Hopf, H. M. Gibbs, and J. V. Moloney, in *Thirteenth International Quantum Electronics Conference, Anaheim, 1984* (IEEE, New York, 1984), Paper A5.
[17]F. Mitschke, J. Mlynek, and W. Lange, Phys. Rev. Lett. **50**, 1660 (1983).
[18]H. Nakatsuka, S. Asaka, H. Itoh, K. Ikeda, and M. Matsuoka, Phys. Rev. Lett. **50**, 109 (1983).
[19]R. G. Harrison, W. J. Firth, and I. A. Al-Saidi, Phys. Rev. Lett. **53**, 258 (1984).
[20]A. T. Rosenberger, L. A. Orozco, and H. J. Kimble, Phys. Rev. A **28**, 2569 (1983), and to be published.
[21]P. D. Drummond, IEEE J. Quantum Electron **17**, 301 (1981); R. J. Ballagh, J. Cooper, M. Hamilton, W. J. Sandle, and D. M. Warrington, Opt. Commun. **37**, 143 (1981); L. A. Lugiato and M. Milani, Z. Phys. B **50**, 171 (1983).
[22]V. Benza and L. A. Lugiato, Z. Phys. B **47**, 79 (1982); M. Gronchi, V. Benza, L. A. Lugiato, P. Meystre, and M. Sargent, III, Phys. Rev. A **24**, 1419 (1981).
[23]L. A. Lugiato and L. M. Narducci, private communication.

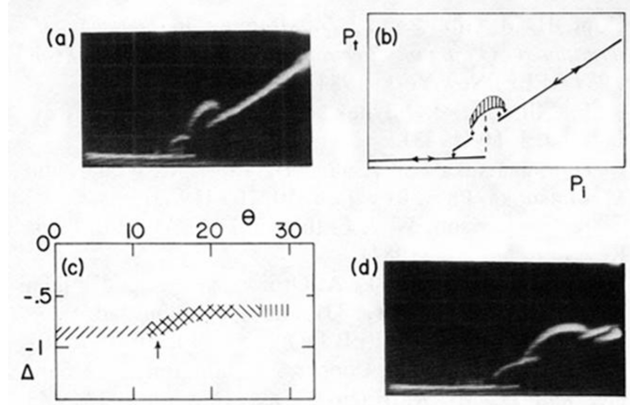

FIG. 3. Behavior of the self-pulsing instability. (a), (b) Photograph and explanatory drawing of the hysteresis cycle showing the instability as an arch separated from the otherwise straight-line segment of the upper branch ($C_e \approx 155$, $\Delta \approx -0.7$, $\theta \approx 13$, and $0 \leqslant X \leqslant 530$). (c) Evolution of the instability as a function of θ for approximately constant Δ, $C_e \approx 155$, and $Y < 1.3 \times 10^5$ (//, bistability; \\, instability; ||, no bistability, no instability). The arrow marks the position of the photograph in (a). (d) Photograph of the hysteresis cycle for $C_e \approx 157$, $\Delta \approx -0.5$, $\theta \approx 15$, and $0 \leqslant X \leqslant 320$. Several states are possible on the upper branch and are connected by a complex hysteresis pattern as the incident field is swept up and down.