# Mixed-Symmetry Interacting-Boson–Model States in the Nuclei $^{140}$Ba, $^{142}$Ce, and $^{144}$Nd with $N = 84$

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(Received 6 August 1984)

The $\gamma$ decay from the $2_3^+$ state at about 2-MeV excitation in the nuclei $^{140}$Ba, $^{142}$Ce, and $^{144}$Nd, with 84 neutrons, is shown to be consistent with its identification as the lowest state of mixed symmetry in the U(5) limit of the neutron-proton version of the interacting-boson model.

PACS numbers: 21.60.Gx, 21.60.Fw

The neutron-proton version of the interacting-boson model (IBM 2),[1] which distinguishes between neutron ($\nu$) and proton ($\pi$) bosons, can reproduce all the results of the original interacting-boson model (IBM 1) but in addition contains states of so-called mixed symmetry. Such states are not totally symmetric in the *sd* space and are allowed in IBM 2 because of the extra $\nu\pi$ degree of freedom. Because of the success of IBM 1 these states of mixed symmetry must in general lie above the totally symmetric states in energy and it is important to identify them experimentally.

In well-deformed nuclei, which are described in the IBM by the SU(3) limit, the lowest mixed-symmetry states belong to a $K = 1$ band and this is distinctive since IBM 1 does not contain any low-lying $K = 1$ bands. Some recent inelastic electron scattering experiments[2] claim to have excited the $J = 1$ member of this band in $^{156}$Gd with a large $B(M1)$ value of $(1.3 \pm 0.2)\mu_N^2$ and at an excitation energy of 3.075 MeV. This is in rough agreement with the calculated[3] IBM 2 value of $2.5\mu_N^2$ in the SU(3) limit allowing for spreading of the calculated $M1$ strength between different levels. The large calculated value results from the collectivity in the boson SU(3) scheme in which all bosons take part in the transition.

In this paper we explore the properties of the lowest mixed-symmetry state in the vibrational U(5) limit, which is a $2^+$ state, and show that the third excited $2_3^+$ state in $^{140}$Ba, $^{142}$Ce, and $^{144}$Nd at about 2 MeV has $\gamma$-decay properties consistent with such a description. Although the lower states of these nuclei are consistent with an IBM 1 analysis, close to the vibrational limit, the $2_3^+$ state cannot be fitted[4,5] within IBM 1. The experimental data are given in columns 2–5 of Table I.

These results were obtained in a group of $\gamma\gamma$ directional correlation experiments carried out at the Institut Laue-Langevin, Grenoble. The $^{140}$Ba[5] and $^{142}$Ce[6] isotones were studied following the $\beta$ decay of fission products in these mass chains which were selected by the OSTIS mass separator. Levels and transitions in $^{144}$Nd[7] were investigated by measuring secondary $\gamma$ rays following thermal-neutron capture by enriched $^{143}$Nd.

The small values for the mixing ratio $\delta$ suggest that there may be a strong $M1$ transition between $2_3^+$ and $2_1^+$. This would be consistent with a transition from mixed symmetry to full symmetry although, as we shall see, the boson-number enhancement factor is not present in the vibrational limit.

In the vibrational limit the ground state contains no $d$ bosons and there are two $2^+$ states with one $d$ boson, corresponding to full symmetry and mixed symmetry. We shall associate these two states with the $2_1^+$ and $2_3^+$ states, respectively, in Table I. They are given in terms of the ground state $|0^+\rangle$ by

$$|2_1^+\rangle = N^{-1/2}(d_\nu^\dagger s_\nu + d_\pi^\dagger s_\pi)|0^+\rangle$$

$$|2_3^+\rangle = \{(N_\pi/NN_\nu)^{1/2} d_\nu^\dagger s_\nu - (N_\nu/NN_\pi)^{1/2} d_\pi^\dagger s_\pi\}|0^+\rangle,$$

TABLE I. The energies (in megaelectronvolts), mixing ratios $\delta$, and branching ratios together with the derived values for certain combinations of the IBM transition parameters in units of $e\,\mathrm{b}/\mu_N$.

| | $E(2_3^+)$ | $E(2_1^+)$ | $\delta(2_3^+ \rightarrow 2_1^+)$ | $T(2_3^+ \rightarrow 0^+)/T(2_3^+ \rightarrow 2_1^+)$ | $(e_\pi \chi_\pi - e_\nu \chi_\nu)/(g_p - g_n)$ | $N^{1/2}(e_\pi - e_\nu)/(g_p - g_n)$ |
|---|---|---|---|---|---|---|
| $^{140}_{56}$Ba | 1.994 | 0.602 | 0.18(8) | 0.28(2) | 0.19(8) | 0.23(1) |
| $^{142}_{58}$Ce | 2.004 | 0.641 | 0.41(7) | 0.42(2) | 0.43(7) | 0.28(1) |
| $^{144}_{60}$Nd | 2.073 | 0.696 | 0.31(11) | 0.43(1) | 0.32(11) | 0.26(1) |

 

where $N_\nu$ and $N_\pi$ are the numbers of neutron and proton bosons, with $N=N_\nu+N_\pi$. With the usual definition of transition operators

$$T^{E2}=e_\pi Q_\pi+e_\nu Q_\nu,\quad Q=(s^\dagger d+d^\dagger s)+\chi(d^\dagger d)^{(2)},\quad T^{M1}=(3/4\pi)^{1/2}(g_p L_\pi+g_n L_\nu),$$

the relevant reduced matrix elements are

$$\langle 0^+ || T^{E2} || 2_1^+\rangle=(5/N)^{1/2}(e_\pi N_\pi+e_\nu N_\nu),\quad \langle 0^+ || T^{E2} || 2_3^+\rangle=(5N_\nu N_\pi/N)^{1/2}(e_\nu-e_\pi),$$
$$\langle 2_1^+ || T^{E2} || 2_3^+\rangle=(5N_\nu N_\pi/N^2)^{1/2}(e_\nu\chi_\nu-e_\pi\chi_\pi),\quad \langle 2_1^+ || T^{M1} || 2_3^+\rangle=3(5N_\nu N_\pi/2\pi N^2)^{1/2}(g_n-g_p),\tag{1}$$
$$\langle 2_1^+ || T^{E2} || 2_1^+\rangle=(\sqrt{5}/N)(e_\nu\chi_\nu N_\nu+e_\pi\chi_\pi N_\pi),$$

with Racah's definition of reduced matrix element.

Standard fomulas then give for the mixing ratio, δ, the equation

$$\frac{1.44\delta}{E(2_3^+)-E(2_1^+)}=\frac{e_\pi\chi_\pi-e_\nu\chi_\nu}{g_p-g_n}.\tag{2}$$

The analysis is simplified if we combine the branching ratio and the mixing ratio to eliminate the $E2$ component of the $2_3^+\rightarrow 2_1^+$ transition given

$$\frac{2.07(1+\delta^2)T(2_3^+\rightarrow 0^+)[E(2_3^+)-E(2_1^+)]^3}{T(2_3^+\rightarrow 2_1^+)[E(2_3^+)-E(0^+)]^5}=\frac{N(e_\pi-e_\nu)^2}{(g_p-g_n)^2}.\tag{3}$$

In both expressions (2) and (3) the energies are in megaelectronvolts, the effective charges in units of electron barns, and the $g$ factors in nuclear magnetons. The left-hand sides of these two equations are known from the data and hence we have values for the expressions on the right as given in the last two columns of Table I. We must now ask whether these values are consistent with those used for other data in this mass region.

Consider first the expression (3) since it is independent of the $\chi_\nu$ and $\chi_\pi$. For all three nuclei there is just one neutron boson and, measured from the $Z=50$ shell, $N_\pi=3,4,5$, respectively, for Ba, Ce, and Nd giving $N=4,5$, and 6. The increase shown in the last column of Table I from Ba to Ce is consistent with the $N^{1/2}$ factor and the absence of any further increase from Ce to Nd is consistent with the suggestion[3,8,9] that an effective boson number $N_\pi=4$ should be used for $Z=60$ rather than $N_\pi=5$ because of a tendency for shell closure at $Z=64$. Throughout this paper we use the value $N_\pi=4$ for Nd. An analysis[3] of $g$ factors of $2_1^+$ states in this region concludes that $g_p\sim 1$ and $g_n\sim 0$ so that from the last column of Table I we require

$$|e_\pi-e_\nu|=0.12\ e\text{b}.\tag{4}$$

There seems to be little evidence elsewhere in the literature for this difference and, for example, Puddu, Scholten, and Otsuka[10] chose $e_\pi=e_\nu=0.12\ e$ b. This would clearly be inconsistent with the conclusion (4) so we have analyzed a number of $2_1^+\rightarrow 0^+$ transitions since from the first of Eqs. (1) the dependence on $N_\pi$ and $N_\nu$ should tell us both $e_\pi$ and $e_\nu$. From (1) we have

$$B(E2;2_1^+\rightarrow 0^+)=(N_\pi e_\pi+N_\nu e_\nu)^2/N$$

so that the plot of the quantity

$$[NB(E2;2_1^+\rightarrow 0^+)/N_\pi^2]^{1/2}=e_\pi+e_\nu N_\nu/N_\pi$$

against $N_\nu/N_\pi$ should be linear, giving $e_\pi$ and $e_\nu$. Figure 1 shows this plot for some nuclei in the region of interest. The linearity is indeed present giving $e_\pi=0.12$, $e_\nu=0.24$. These values are in complete agreement with the result (4) from the decay of the $2_3^+$ state. The agreement is closer than the approximations deserve.

We now turn to the mixing ratio (2) in which the sign is also relevant, with the measured δ positive. In contrast to (3) the expression (2) is independent of $N$ so that the variation shown in column 6 of Table I cannot be reproduced by the model (unless the transition parameters change with $N$). However, the errors in this case are much greater and if we adopt a mean value of 0.31 and use the values for $g_n$, $g_p$, $e_\nu$, and $e_\pi$ as above we require

$$\chi_\pi-2\chi_\nu=2.6.\tag{5}$$

From both microscopic models and empirical analysis it is argued[10] that χ should be negative and large, $\chi\sim -1$, at the beginning of a shell, changing sign at about midshell. Since $N_\nu=1$ for all nuclei being considered we then expect $\chi_\nu\sim -1$. The un-

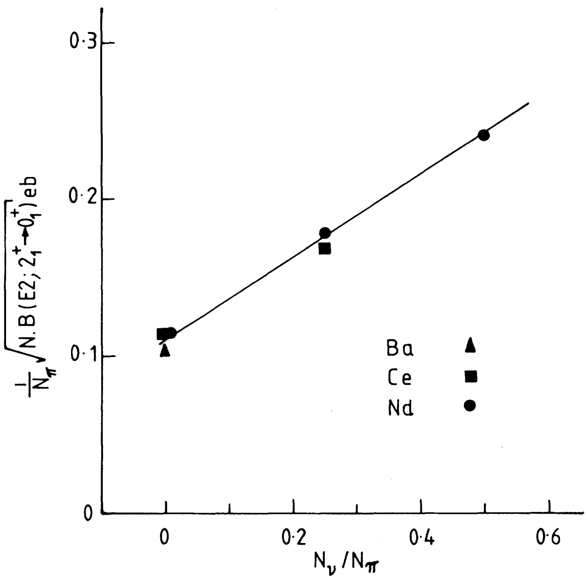


FIG. 1. The quantity $[B(E2;2_1^+\to 0_1^+)N]^{1/2}/N_\pi$ plotted vs $N_\nu/N_\pi$ for several nuclei where $N_\nu$ and $N_\pi = N - N_\nu$ are the neutron and proton boson numbers. The straight line is least-squares fitted to the data which are taken from Refs. 11–13. Experimental errors are insignificant compared to the size of the data symbols. The data relate to $^{138}$Ba; $^{140}$Ce, $^{142}$Ce; and $^{142}$Nd, $^{144}$Nd, $^{146}$Nd.

certainty about the closed shell at $Z = 64$ makes an estimate of $\chi_\pi$ more difficult but the effect of this closed shell would be to make $\chi_\pi$ vanish at about $Z = 57$ so that $\chi_\pi$ would be small for Ba, Ce, and Nd. The relation (5) is consistent with such values, $\chi_\pi \sim 0$, $\chi_\nu \sim -1.3$. Note that if the measured mixing ratio $\delta$ had been of opposite sign then the right-hand side of (5) would have been negative and the consistency with the IBM 2 would have been lost.

The value of $\chi$ is directly related to the quadrupole moment of the $2_1^+$ state and from the last of Eqs. (1) we have the expression for the quadrupole moment

$$Q(2_1^+) = 1.7(N_\pi e_\pi \chi_\pi + N_\nu e_\nu \chi_\nu)/N. \quad (6)$$

Unfortunately, the measured quadrupole moments have large errors, for example[14] in $^{142}$Ce $Q(2_1^+) = -0.12(9)$ b. If we use the parameter values chosen earlier in this Letter then Eq. (6) gives $Q(2_1^+) - = 0.11$ b which is again consistent and confirms the need for a negative $\chi_\nu$.

We conclude that the $\gamma$-decay properties of the $2_3^+$ state in these nuclei are well described by the lowest mixed-symmetry state in the vibrational limit of IBM 2. The validity of this limit is related to the proximity to the closed neutron shell at $N = 82$ and a more detailed calculation would be necessary for larger $N$ allowing departures from the vibrational limit. One should also allow admixture between states of full symmetry and mixed symmetry, i.e., states of different $F$ spin, but both of these generalizations introduce many unknown interaction parameters. The conclusions of this Letter are based on the simplest approximations. Our conclusion that $e_\nu > e_\pi$ is, at first sight, rather surprising since neutrons carry no charge but there are two points to bear in mind. Firstly, these parameters incorporate a (length)$^2$ factor and the neutrons are filling higher shells than protons, and, secondly, one expects a large effective neutron charge in this particular region just before the onset of deformation at $N = 88$. We do not suggest that $e_\nu > e_\pi$ in other regions. It must also be remembered that the notion of an effective charge refers to a specific model space. It may well be that in a more detailed calculation with a larger model space of the sort referred to above, the effective charges would move towards the bare values. Our conclusions are not sensitive to reasonable changes in $g_n$ and $g_p$ and the use of $N_\pi = 5$ for $Z = 60$ rather than the effective $N_\pi = 4$ would also have little effect. Finally, we remark that an alternative description of these states has been proposed[15] which involves the breaking of a 7/2 neutron pair.

We acknowledge helpful conversations with P. Halse, J. A. Evans, B. R. Barrett, and S. J. Robinson.

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