Domain Growth in the Ising Model in a Random Magnetic Field

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Simulations of the two-dimensional ferromagnetic Ising model in a random magnetic field with spin-flip dynamics are reported. After the system is deeply quenched into the unstable region of the phase diagram, novel dynamical behavior for the average size of the growing domains is observed.

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Random impurities are ubiquitous to condensed matter, metallurgical, and surface science systems. Some important aspects of these effects can be simulated by a random magnetic field.[1-3] This has been the subject of considerable recent interest.[1-7] The nature of the phase transition in simple Ising-type systems changes dramatically as a result of the random field. In particular, theory, experiment, and computer simulations have attempted to determine the dimensionality at which long-range order is broken. According to theory, fluctuations due to the random field raise the lower critical dimension in the Ising model from $d_l=1$ to either $d_l=2$[4] or possibly $d_l=3$.[5] Experiments[3] have been performed on an easily controlled analog[6] of the random-field Ising model: diluted antiferromagnetics in a uniform field. Those studies correspond to either (i) turning on the uniform field in the ordered state (slow field quench), or (ii) lowering the temperature when the field is already present (slow temperature quench). However, hysteresis effects complicate the interpretation of experiments. Several groups have studied random fields by Monte Carlo computer simulation.[7] Recently Stauffer *et al.*[7] have conducted extensive Monte Carlo simulations of slow field quenches and slow temperature quenches, after annealing at low temperature in the field. Their results provide a partial explanation for the observed hysteresis.

The behavior of a system after a rapid temperature quench is an interesting and fundamental problem in its own right.[8] It is useful to separate the evolution into different time regimes, such as early and late, for which different theories can be developed. We focus here on the early-time behavior. In an order-disorder transition in a two-state degenerate system (such as a binary alloy), domains of ordered phase form and grow as time evolves. The average size of a domain, $\bar{R}$, in an early to intermediate time regime[9] is given by $\bar{R}^2 \propto t$, in which the interface curvature drives domain growth. This has been observed in metallurgical systems, chemisorbed systems, and computer simulations of lattice-gas models.[10] Late-time theories describe the approach to equilibrium. On a two-dimensional substrate, random impurities enter such systems through, for example, steps, terraces, or vacancies. Experiments observe a slowdown in evolution which is partly attributed to these effects.[11] Theories of various aspects of domain growth in the presence of a random field have been recently proposed by two of us,[12] Villain,[13] and Grinstein and Fernandez.[14] These will be discussed below.

In this Letter, we present the first Monte Carlo computer simulation of domain growth in the random-field Ising model. Two typical configurations in the evolution are shown in Fig. 1. It is impossible to determine precisely the functional form of $\bar{R}(h,t)$ at the moment. Nevertheless, we find that the random field has crucial effects on domain growth: The evolution is dramatically slowed down, and the $\bar{R}^2 \propto t$ law (which describes the zero-field behavior) breaks down. An analysis in terms of existing theory (which, however, does not constitute a definitive test) is presented below. Our results for $\bar{R}^2(t)$ are given in Fig. 2. A more detailed discussion of our results will be given in a subsequent paper.

The Hamiltonian for the two-dimensional Ising model is

$$\mathscr{H} = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - \sum_{i=1}^{N} h_i\sigma_i,$$

where the interaction sum runs over nearest neighbors, and the N spins can take the values $\sigma_i = \pm 1$. We have considered the case where the random magnetic field h_i is given by the Gaussian probability distribution $P_i = [1/(2\pi)^{1/2}h]\exp[-h_i^2/(2h^2)]$, so that $\langle h_i\rangle = 0$, and $\langle h_i h_j\rangle = h^2\delta_{ij}$. Theoretical studies of the random-field Ising model often make use of this distribution.

After an instantaneous critical quench from temperature $T/J=\infty$ to $T/J=1$ (Boltzmann's constant set equal to unity), the system is allowed to evolve via the standard Metropolis spin-flip Monte Carlo

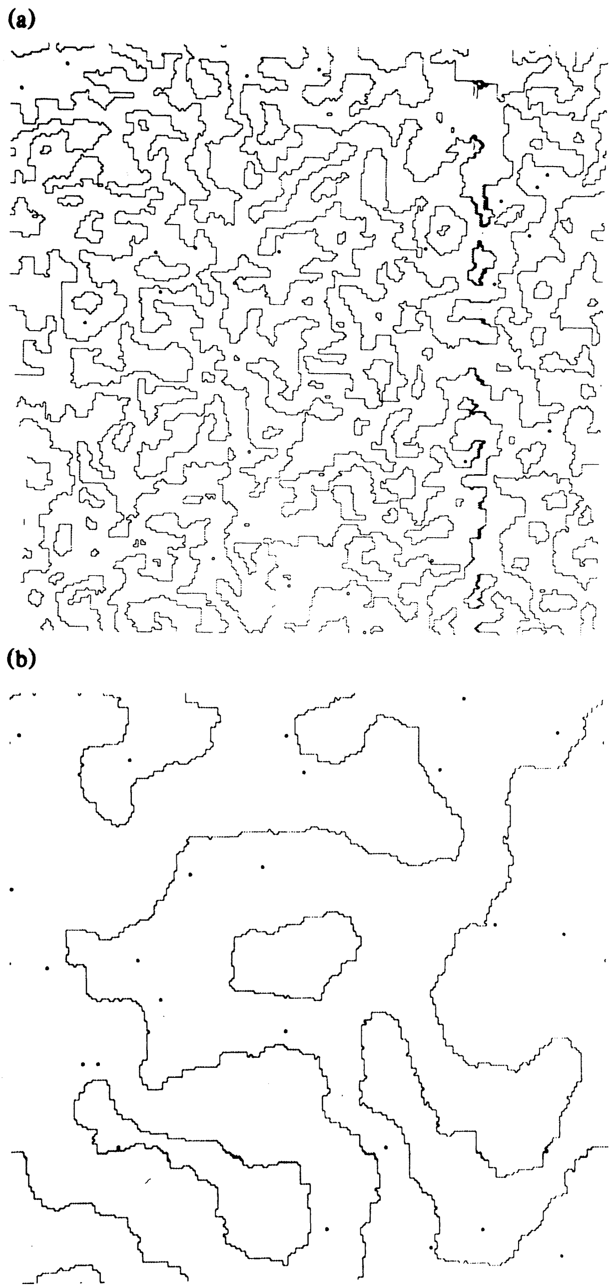

FIG. 1. Two typical configurations are shown at (a) $t = 12$ MCS and (b) $t = 500$ MCS. The system size is $N = 240^2$, the temperature is $T/J = 1$, and the field strength is $h = 0.311$.

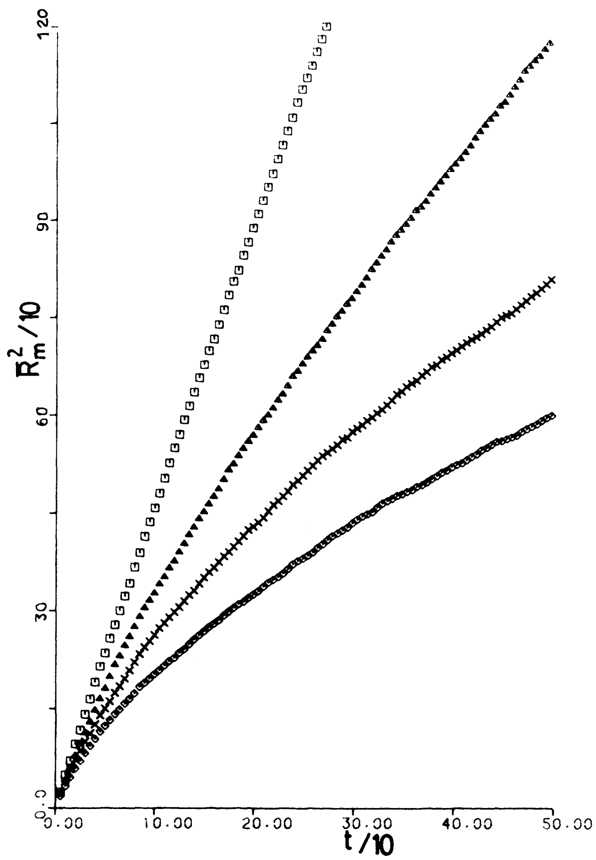

FIG. 2. $\bar{R}_M^2$ vs t. Squares, triangles, crosses, and lozenges, correspond to $h = 0$, 0.311, 0.415, and 0.518. Every fifth data point is plotted, from $t = 5$ MCS. Error bars are 7%; 352 quenches for $h = 0$; 450 quenches for each nonzero field; $N = 75^2$. $\bar{R}_M^2 \propto \bar{R}^2$, as discussed in text.

procedure. This corresponds to model A in field theory, where the scalar order parameter is nonconserved. The unit of time in the simulation is a Monte Carlo step (MCS) which consists of $N/15$ random updatings of groups of fifteen widely separated spins.[15] The average size of the domains, following the quench, is given by $\bar{R}^2$, the square of the inverse perimeter density. Although we have calculated $\bar{R}^2$ for several times during the simulation, it is difficult to evaluate it for all times of interest. Thus, as is common in the literature,[16,17] we present results for the following measures of evolving orientational order. Firstly we consider the nonequilibrium fluctuations in magnetization per spin, i.e.,

$$\bar{R}_M^2(t) \equiv N\langle [(1/N)\textstyle\sum_i \sigma_i]^2\rangle,$$

as proposed by Sadiq and Binder.[16] This is plotted in Fig. 2 for the different field strengths that we have considered, $h = 0$, $h = 0.311$, $h = 0.415$, and $h = 0.518$. Secondly we consider the average nonequilibrium energy per spin E, in the form[16,17] $\bar{R}_E(t) = 2/(2 + E/J)$, for the four field strengths. In the absence of a random field both $\bar{R}_M^2$ and $\bar{R}_E^2$ are proportional to $\bar{R}^2$. We have found that $\bar{R}_M^2 = C\bar{R}^2$, where C is a constant, while

$\bar{R}_E^2(h,t) = C'(h)\bar{R}^2(h,t)$, where $C'(h=0)=1$, for the times at which $\bar{R}$ was calculated. The factor $C'(h)$ arises from bulk energy due to the random field.[18]

We have chosen to simulate a 75^2 system over a large number of runs (352 runs for $h=0$, 450 runs for each nonzero h), rather than a larger system for a small number of runs, for the following reasons. The physics of the random field is such that it causes short-range roughening effects at interfaces in the growth process. These roughening effects result in noisy data. Thus we have done 450 runs for each nonzero field strength to get precise results. Since the random-field effects are predominately over short ranges, we expect $N=75^2$ to be large enough to encompass the necessary physics of the problem. We have checked this by conducting test runs over both larger and smaller lattices (from $N=60^2$ to $N=240^2$). The main effect we find due to finite sizes is that percolation effects (artifacts of the periodic boundary conditions we use) become important for $\bar{R} \sim 0.4\sqrt{N}$. This is a well-known result[16,17]; more subtle finite-size effects are also possible. Thus we have considered times $t < 500$ MCS, since we reach this value of $\bar{R}$ at $t \sim 400$ MCS for $h=0.311$. This effectively limits the time regime over which useful data can be taken.[19]

We now turn to the analysis of our data. It is possible to find a good effective power-law fit $\bar{R}^2 \propto t^n$ to our data, where, however, the *effective n* decreases as the random field increases. An effective power-law fit may have some implications for the evolution of, e.g., chemisorbed systems. Experimental studies have observed a slowdown of growth, which is partly attributed to surface heterogeneities. We emphasize, though, that the experimental slowdown usually involves the degeneracy of the ground state also: Sahni *et al.*[11] have shown that vertices in the Q-state Potts model slow domain growth through Q-dependent exponents in an effective power law. It may be that a combination of both impurity and degeneracy effects is responsible for the experimental observations.

The rather interconnected structure in Fig. 1(a) is typical of the initial stages of an unstable state's evolution. Previously, two of us have presented a theory for such domain growth in the random-field Ising model.[12] In two dimensions the growth law was found to be given by the functional form

$$\bar{R}^2 = \bar{R}^2(h=0,t)[1+h^2(a-b\ln t)],$$

where $\bar{R}^2 = (h=0,t) \propto t$, which is consistent with $d_l = 2$. Of course for $h^2 b << 1$ this will be numerically similar to a power law.[20] It is difficult to determine the range of validity of this theory since the approximations used to derive it are, to some extent, uncontrolled. However, for late times, as domains become compact, it is clear that the theory will break down because it is based on an assumption of isotropy. The logarithmic correction form is as good a fit to the time dependence as an effective power law.[21] However, the field dependence does not seem to be in agreement with theory[22] in that the coefficients a and b appear to have field dependence. There is a clear need, then, for further theoretical work.

Recently, Villain,[13] and Grinstein and Fernandez[14] have also presented dynamical theories for the random-field Ising model which address different issues than that of the theory discussed above. (For example, Villain studies the genesis of metastable states in a cooling process.) They predict logarithmic behavior for $\bar{R}(t)$. Since both theories assume compact domains with no curvature-driven growth, we expect that they describe the late stages of evolution. From Fig. 1(b), although domains are becoming compact, it is not clear that we have reached the time regime where these theories would apply. These late stages of growth would be inaccessible to our Monte Carlo simulations, except for much larger values of h than we have considered.[19] Therefore, as would be expected, we do not observe the logarithmic behavior for $\bar{R}$.

To conclude, we have investigated domain growth in the random-field Ising model and found a novel time dependence for the growth law. The dramatic slowing down that we observe may be related to growth in chemisorbed systems in the presence of heterogeneous impurities. The previous theory of two of us appears to provide a least a qualitative explanation of our data. A more quantitative comparison with theory will be given in a subsequent paper.

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[18]The energy per spin of a completely ordered domain is $E/J=-2$. Interfaces raise the energy of our system by an amount $2/\overline{R}$, while the random field raises it by an amount $\sim h/\overline{R}$. (This latter result follows straightforwardly from Imry and Ma's original argument, Ref. 1.) Thus we obtain $\overline{R}_E^2=C(h)\overline{R}^2$. Although $\overline{R}_E$ involves an uncertain field dependence, its time dependence is equivalent to that of $\overline{R}$. Hence, we have studied it, in addition to $\overline{R}_M$, because it is a somewhat less noisy measure than $\overline{R}_M$.

[19]An analysis, with precise data, of longer-time regimes would require much larger field strengths than we have considered. This could be of interest since the subtle breakdown of long-range order might be observed (see, e.g., Stauffer *et al.*, Ref. 7, where fields an order of magnitude larger than ours are studied). Nevertheless, looking at small fields minimizes the effects of fluctuations, and novel effects are still present. We note that the dynamics over the time regime that we have simulated are intrinsically nonlinear. Also, domain growth in real systems, in the presence of random impurities, would involve small fields. Thus the time-dependent deviations from $\overline{R}^2\propto t$ that we observe should be of interest. Finally, it is worth noting that, even at small fields, we can test the qualitative accuracy of the previous dynamical theory of two of us, which gives $d_l=2$.

[20]An estimate from theory gives $a\sim b\sim 1$. The parameters of the continuum theory, a and b, cannot, however, be precisely related to the appropriate parameters of the Monte Carlo simulation. Nevertheless the estimated values are of the correct order of magnitude.

[21]It may be worth noting that, according to theory, if the lower critical dimension were $d_l=3$ one would expect different dynamical behavior. One dimension below d_l one would naively expect $\overline{R}^2=At(1-B\sqrt{t})$. This is not consistent with our data.

[22]It is possible that the h^2 dependence of the logarithmic correction form is strictly valid only for the continuum model, since there is some evidence that the transition from continuum to lattice makes $h^2\rightarrow h^\alpha$, where we expect $\alpha<2$ (Refs. 4 and 12). We have attempted to fit our data by this form, but have been unable to draw any definitive conclusions.