Some Decay Modes of 1^{-+} Hybrid Mesons

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Using the QCD sum-rule approach, we study the two-body decays into ordinary mesons of $J^{PC}=1^{-+}$ quark-antiquark-gluon hybrid mesons containing u and d quarks. It is found that the $I=0$ state is stable for these decays, while the $I=1$ states decay predominantly into $\pi\rho$ with a characteristic width of 10 to 100 MeV.

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The use of the QCD sum-rule approach to the hadronic spectrum[1] has given remarkably successful predictions for masses, couplings, form factors, and partial decay widths of "usual" hadronic states.[2]

It is worthwhile, therefore, to apply the same techniques to predict the properties of "unusual" resonances, such as glueballs or hybrid states.

Recently, we did such a study for quark-antiquark-gluon ($q\bar{q}g$) hybrid mesons containing u and d quarks, with $J^{PC}=1^{-+}, 0^{++}, 1^{+-}, 0^{--}$ and $I=0, 1$.[3]

In particular, the mass of the 1^{-+} states was predicted at 1.3 GeV, in reasonable agreement with the lowest bag-model results[4,5] but in slight disagreement with another QCD sum-rule calculation.[6]

The absence of mixing of these 1^{-+} exotic mesons with ordinary $q\bar{q}$ mesons or gg glueballs, and the expected small mixing effects with other "unusual" mesons such as ggg glueballs or $q\bar{q}q\bar{q}$ mesons, makes the study of their properties quite interesting since such a theoretical situation could simplify their experimental identification.

In this Letter, we present our results for two-body decay widths of the 1^{-+} mesons into ordinary mesons, using the QCD sum-rule approach. Such estimates have also recently been considered in other models for QCD.[7]

With a mass of 1.3 GeV, the 1^{-+}, $I=0$ hybrid meson $h\,(I=0)$ is kinematically stable for two-body decays into ordinary $q\bar{q}$ mesons, while the $h\,(I=1)$ states could decay into $\pi\eta$, $\pi\eta'$, and $\pi\rho$. However, the QCD sum-rule prediction has to be understood with a precision of 10%, and with a mass of 1.43 GeV new decay modes become accessible, such as $h\,(I=0)\to A_1(1270)$ or $h\,(I=1)\to\pi B(1235)$. But, as a result of phase-space suppression, these modes have then presumably small branching ratios, and we do not consider them here.

Other decay modes may also be allowed, such as $h\to 3\pi$ or $h\to 4\pi$, or even $h\to\pi(q\bar{q}g0^{-+})$ if these hybrid mesons are light enough,[5,7] but again we expect them to have small branching ratios since these are three- or four-body decays or have phase-space suppression factors (and we do not have yet a QCD sum-rule prediction for the 0^{-+} hybrid state).

Therefore, we consider in the following only the two-body decays into ordinary mesons, i.e., $h\to\pi\eta$, $\pi\eta'$, $\pi\rho$, these being presumably the dominant decay modes of the $J^{PC}=1^{-+}$, $I=1$ states.

To make any predictions for the corresponding widths using the QCD sum-rule approach, one has to know the operator product expansion (OPE) of three-point functions of operators coupling to h, π, and η (or η', ρ) states. Such operators are obtained as linear combinations of

$$J_\mu^{(h)}(x)=\bar{\psi}_1(x)\gamma^\alpha G_{\alpha\mu}(x)\psi_2(x), \tag{1a}$$

$$J^{(\pi,\eta,\eta')}(x)=\bar{\psi}_1(x)i\gamma_5\psi_2(x), \tag{1b}$$

$$J_\nu^{(\rho)}(x)=\bar{\psi}_1(x)\gamma_\nu\psi_2(x), \tag{1c}$$

where ψ_1, ψ_2 are quark fields of definite flavor, $G_{\mu\nu}=gG^a_{\mu\nu}T^a$, and $G^a_{\mu\nu}$ is the gluon field strength.

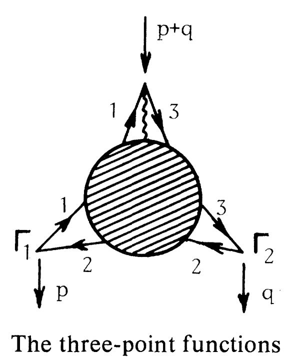

FIG. 1. The three-point functions Eq. (2).

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It is therefore useful to first compute the OPE for the following three-point functions (see Fig. 1):

$$\int d^4x\, d^4y e^{ipx} e^{iqy} \langle 0|T\{\bar{\psi}_1(x)\Gamma_1\psi_2(x)\bar{\psi}_2(y)\Gamma_2\psi_3(y)\,\bar{\psi}_3(0)\gamma^\alpha G_{\alpha\mu}(0)\psi_1(0)\}|0\rangle, \tag{2}$$

where $(\Gamma_1,\Gamma_2)=(i\gamma_5,i\gamma_5)$, $(i\gamma_5,\gamma_\nu)$, or $(\gamma_\nu,i\gamma_5)$.

We used the background-field method[3] to compute these OPE's but, because of the double Fourier transform, one cannot work in configuration space and has to use the quark and gluon propagators (in the background fields) in momentum space.[3]

On the other hand, since we consider decays into mesons containing only the light u and d quarks and cannot hope for precise predictions, we neglect the quark masses in our calculations.

And finally, in order to simplify the computation of some of the Wilson coefficients, we consider the three-point functions (2) at the symmetrical point $p^2=q^2=(p+q)^2=-Q^2$.

Our results for the OPE's are then as follows:

For $(\Gamma_1,\Gamma_2)=(i\gamma_5,i\gamma_5)$,

$$(p+q)_\mu\left\{\frac{i\alpha_s}{12\pi^3}Q^2\ln^2\frac{Q^2}{\mu^2}+\left[\frac{-i}{12}\right](I-2)\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G_a^{\mu\nu}\right\rangle\frac{1}{Q^2}+\cdots\right\} \tag{3}$$

with

$$I=-2\int_0^1 dt\frac{\ln t}{t^2-t+1}=2(1.171\,953).$$

For $(\Gamma_1,\Gamma_2)=(i\gamma_5,\gamma_\nu)$,

$$\begin{aligned} i\epsilon_{\mu\nu\rho\sigma}p^\rho q^\sigma\Bigg\{&\frac{1}{Q^2}\ln\frac{Q^2}{\mu^2}\frac{\alpha_s}{9\pi}[2\langle\bar{\psi}_1\psi_1\rangle+3\langle\bar{\psi}_2\psi_2\rangle+\langle\bar{\psi}_3\psi_3\rangle] \\ &+\frac{1}{(Q^2)^2}\frac{-1}{12}[3\langle\bar{\psi}_1\sigma^{\mu\nu}G_{\mu\nu}\psi_1\rangle+\langle\bar{\psi}_2\sigma^{\mu\nu}G_{\mu\nu}\psi_2\rangle-3\langle\bar{\psi}_3\sigma^{\mu\nu}G_{\mu\nu}\psi_3\rangle] \\ &+\frac{1}{(Q^2)^3}\frac{-\pi^2}{9}\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G_a^{\mu\nu}\right\rangle\langle\bar{\psi}_3\psi_3\rangle+\cdots\Bigg\}. \end{aligned} \tag{4}$$

For $(\Gamma_1,\Gamma_2)=(\gamma_\nu,i\gamma_5)$,

Eq. (4) with $(\psi_1\leftrightarrow\psi_3)$, (5)

where $\alpha_s=g^2/4\pi$ and μ is some typical hadronic mass ($\epsilon^{0123}=+1$).

The Feynman diagrams contributing to (3)–(5) are those of Figs. 2 and 3, and we used the vacuum dominance approximation in the evaluation of the $\langle\bar{\psi}\psi G^2\rangle$ matrix elements in (4) and (5).

The contribution of the 1^{-+} hybrid mesons to these expressions is obtained by projecting them on $[g_{\mu\lambda}-(p+q)_\mu(p+q)_\lambda/(p+q)^2]$. In the cases $(\Gamma_1,\Gamma_2)=(i\gamma_5,\gamma_\nu)$, $(\gamma_\nu,i\gamma_5)$ this shows that Eqs. (4) and (5) include only the spin-1 contributions, but in the case $(\Gamma_1,\Gamma_2)=(i\gamma_5,i\gamma_5)$ it vanishes identically.

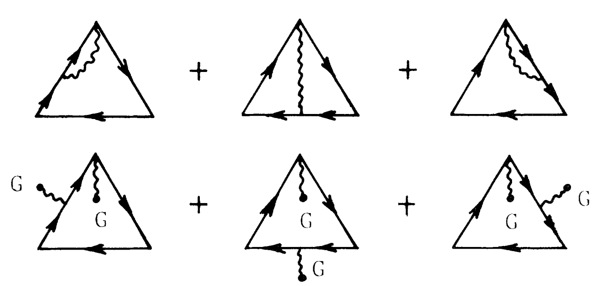

FIG. 2. The OPE for $(\Gamma_1,\Gamma_2)=(i\gamma_5,i\gamma_5)$.

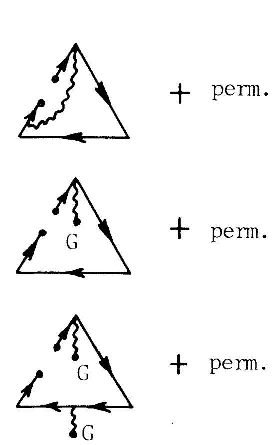

FIG. 3. The OPE for $(\Gamma_1,\Gamma_2)=(i\gamma_5,\gamma_\nu)$, $(\gamma_\nu,i\gamma_5)$.

Actually, it is possible to show in the last case that for massless quarks, in the given kinematical situation, the Wilson coefficients of the operators 1 and G^2 at lowest order in α_s are proportional to $(p+q)_\mu$ not only for the short-distance singularities but for all values of p and q.

However, for small but nonzero quark masses, terms such as $(m^2/Q^2)Q^2\ln^2Q^2$ and $m/Q^2\langle\bar\psi\psi\rangle$, proportional to $(p-q)_\mu$ which include spin-1 contributions, would presumably be obtained but they are small for light quarks.

In higher order in α_s, one expects contributions proportional to $(p-q)_\mu$, even in the case of massless quarks, but again they are small when compared to other decay modes.

We therefore conclude that in the real world the decay modes $h\to\pi\eta$, $\pi\eta'$ must be suppressed compared to $h\to\pi\rho$, which should be the dominant decay mode of the $1^{-+}(I=1)$ hybrid meson.

This result is in disagreement with Tanimoto's predictions.[7]

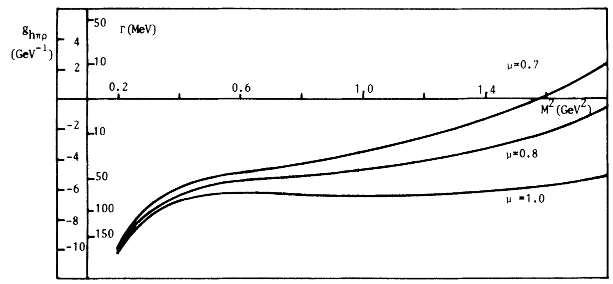

FIG. 4. The prediction for $g_{h\pi\rho}$ and $\Gamma(h\to\pi\rho)$.

For definiteness, let us consider the decay $h^-(I=1)\to\pi^-\rho^0$. The corresponding operators are

$$J_\mu^{(h)}(x)=\bar d(x)\gamma^\alpha G_{\alpha\mu}(x)u(x), \tag{6a}$$

$$J^{(\pi)}(x)=\bar u(x)i\gamma_5 d(x), \tag{6b}$$

$$J_\nu^{(\rho)}(x)=\tfrac{1}{2}[\bar u(x)\gamma_\nu u(x)-\bar d(x)\gamma_\nu d(x)], \tag{6c}$$

while the three-point function reads

$$\int d^4x\, d^4y\, e^{ipx}e^{iqy}\langle 0|T\{J^{(\pi)}(x)J_\nu^{(\rho)}(y)J_\mu^{(h)}(0)\}|0\rangle = i\epsilon_{\mu\nu\rho\sigma}p^\rho q^\sigma A(Q^2). \tag{7}$$

From Eqs. (4) and (5), we have

$$A(Q^2)=\frac{-\alpha_s}{3\pi}[\langle\bar uu\rangle+\langle\bar dd\rangle]\frac{1}{Q^2}\ln\frac{Q^2}{\mu^2}+\frac{1}{24}[\langle\bar\mu\sigma^{\mu\nu}G_{\mu\nu u}\rangle+\langle\bar d\sigma^{\mu\nu}G_{\mu\nu d}\rangle]\frac{1}{(Q^2)^2}$$
$$+\frac{\pi^2}{18}\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G_a^{\mu\nu}\right\rangle[\langle\bar uu\rangle+\langle\bar dd\rangle]\frac{1}{(Q^2)^3}+\dots . \tag{8}$$

On the other hand, by definition of the $h\pi\rho$ coupling through the introduction of a phenomenological interaction Lagrangian

$$\mathscr{L}_{\text{eff}}^{\text{int}}=g_{h\pi\rho}\epsilon_{ijk}\epsilon_{\mu\nu\rho\sigma}\partial^\rho h_i^\mu\partial^\sigma\rho_j^\nu\pi_k, \tag{9}$$

where $i,j,k=1,2,3$ are isospin indices, one obtains for (7)

$$A(Q^2)=g_{h\pi\rho}\sqrt{2}\frac{f_\pi m_\pi^2}{m_u+m_d}\frac{m_\rho^2}{g_\rho}\sqrt{2}\frac{m_h^4}{g_h}\frac{1}{(Q^2+m_\pi^2)(Q^2+m_\rho^2)(Q^2+m_h^2)}. \tag{10}$$

We did not include in the phenomenological model for $A(Q^2)$ other contributions, such as higher-lying resonances or continuum states in each of the h, π, or ρ channels. After Borel transformation, these contributions are not suppressed as much as they are in the case of two-point functions. Had we worked at a nonsymmetric kinematical configuration, these contributions would have been suppressed by the use of double Borel transforms of double dispersion relations.[8]

However, since we only try to make an estimate for the widths, we will assume in the following that these corrections are small and neglect them.[2, 9]

Then, identifying the Borel transforms[1] of Eqs. (8) and (10), we obtain for the $h\pi\rho$ coupling

$$g_{h\pi\rho}=\frac{(m_h^2-m_\rho^2)(m_h^2-m_\pi^2)(m_\rho^2-m_\pi^2)}{[f_\pi m_\pi^2/(m_u+m_d)]\sqrt{2}(m_\rho^2/g_\rho)\sqrt{2}(m_h^4/g_h)}\frac{-1}{3\pi}[\langle\bar uu\rangle+\langle\bar dd\rangle]$$
$$\times\frac{\alpha_s(\mu^2)[\ln(M^2/\mu^2)-\gamma]+(-\pi/8)(m_0^2/M^2)+(-\pi^3/12)\langle(\alpha_s/\pi)G^a_{\mu\nu}G_a^{\mu\nu}\rangle[1/(M^2)^2]}{(m_h^2-m_\rho^2)e^{-m_\pi^2/M^2}-(m_h^2-m_\pi^2)e^{-m_\rho^2/M^2}+(m_\rho^2-m_\pi^2)e^{-m_h^2/M^2}}, \tag{11}$$

where γ is Euler's constant, and we used

$$\langle \bar{u}\sigma^{\mu\nu}G_{\mu\nu}u\rangle + \langle \bar{d}\sigma^{\mu\nu}G_{\mu\nu}d\rangle = m_0^2[\langle \bar{u}u\rangle + \langle \bar{d}d\rangle] \tag{12}$$

with $m_0^2 = 0.5$ to $1.0\ \text{GeV}^2$.

The decay width is

$$\Gamma(h^- \to \pi^-\rho^0) = \frac{1}{12\pi} g_{h\pi\rho}^2 \left[\left(\frac{m_h^2 + m_\rho^2 - m_\pi^2}{2m_h}\right)^2 - m_\rho^2\right]^{3/2}. \tag{13}$$

Some of our results are shown in Fig. 4, with $m_0^2 = 0.8\ \text{GeV}^2$ and $\Lambda = 100$ MeV. We used

$$\langle \bar{u}u\rangle = \langle \bar{d}d\rangle = (-0.25\ \text{GeV})^3, \quad m_u + m_d = 11\ \text{MeV}, \quad m_\pi = 137\ \text{MeV},$$
$$f_\pi = 95\ \text{MeV}, \quad m_\rho = 770\ \text{MeV}, \quad \frac{g_\rho^2}{4\pi} = 2.4\ (\text{Ref. 1}), \quad m_h = 1.3\ \text{GeV}, \tag{14}$$
$$\frac{1}{g_h} = 1.1\times 10^{-2}\ (\text{Ref. 3}), \quad \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu}\right\rangle = (0.33\ \text{GeV})^4, \quad \alpha_s(\mu) = \frac{4\pi}{9\ln(\mu^2/\Lambda^2)}.$$

The values for M^2 have to be chosen in some interval, so that the contributions of the higher power corrections and of the continuum states are expected to be small. We consider the interval [0.6; 1.4] to be reasonable, since at $M^2 = 0.6\ \text{GeV}^2$ the contribution of the last term in the numerator of Eq. (11) compared to that of the second term is less than 20%, and 1.4 GeV^2 is somewhat less than the threshold of the ρ continuum[1] (the h continuum is higher[3]). With these restrictions in mind, the QCD sum-rule predictions are

$$g_{h\pi\rho} \simeq (-2)\ \text{to}\ (-7)\ \text{GeV}^{-1}, \qquad \Gamma(h^- \to \pi^-\rho^0) \simeq 10\ \text{to}\ 100\ \text{MeV} \tag{15}$$

as can be seen from Fig. 4.

These numbers depend somewhat on the values of Λ and m_0^2, because the first two terms in the numerator of Eq. (11) are of the same order of magnitude. Taking $\Lambda = 200$ MeV or $m_0^2 = 0.5$, 1.0 GeV^2 affects $g_{h\pi\rho}$ on the order of 20%, which is the characteristic precision for QCD sum-rule predictions.

We therefore conclude that, within all our approximations, the QCD sum-rule approach predicts the $h(I=1)\pi\rho$ coupling and the corresponding decay widths as being those given in Eq. (15). This agrees with the *lowest* order of magnitude estimate made in an earlier work,[10] but disagrees with Tanimoto's predictions.[7]

These hybrid states could be produced in radiative decays of heavy quarkonia,[5,6] and we find it important to look for them in e^+e^- experiments.

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