Possible Form of Vacuum Deformation by Heavy Particles

R. MacKenzie and F. Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

and

A. Zee
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 20 July 1984)

The possibility is discussed that the lowest-energy state for certain quantum numbers involves a Higgs field polarized into a skyrmion-type configuration. In some models a new type of vacuum instability arises. Phenomenological consequences are indicated schematically.

PACS numbers: 11.10.Lm, 11.10.Ef, 11.30.Qc, 12.35.Ht

In theories where particles acquire mass through their interaction with a symmetry-breaking condensate, it has long been recognized that the lowest-energy state carrying the quantum numbers of a heavy particle—heavy, that is, if the condensate were to retain the value most favorable *in vacuo*—might be a state in which the condensate takes a form very different from its uniform vacuum value. For example, the heavy particle might carve out a spherical region[1] or a shell[2] wherein the condensate vanishes, thus reducing its effective mass at the cost of volume and gradient energy associated with the variation of the condensate from its vacuum value.

A more subtle possibility is implicit in recent work on fermion charges induced by vacuum polarization in the presence of a spatially variable condensate, which need not vanish anywhere.[3-5] Most of this work is concerned with computing the charges induced by a *given* topologically nontrivial configuration of the condensate.

In this work it is shown, among other things, that a spatially extended skyrmion[6] will in its ground state carry the quantum numbers of the heavy fermions (whose Compton wavelength is significantly smaller than the spatial scale), but not the quantum numbers of light fermions. However, since Derrick's theorem[7] shows, at least in the context of renormalizable field theories in 3+1 dimensions, that the possibilities for classically stable (e.g., for topological reasons) configurations are extremely limited, it becomes a nontrivial problem to find physically significant applications in this context. One possibility, which we explore here, is that a deformed condensate is stabilized, for energetic reasons, precisely because of the fermion charges it induces. In other words, it becomes stable if it is the lowest-energy state with the particular fermion quantum numbers it induces.

1+1 dimensional model.—To illustrate the idea in a simple context, consider fermions coupled to a boson field θ according to

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu\theta)^2-M^2(1-\cos\theta)+\bar{\psi}i\partial\!\!\!/\psi-g\bar{\psi}\exp(i\theta\gamma_5)\psi. \tag{1}$$

If θ is imagined to be steady at its vacuum value $\theta=0$, then ψ will create quanta of mass approximately g. To investigate what really happens in more detail, we can bosonize the theory,[8] replacing the fermion field ψ by a nonlocal expression in a boson field ϕ. In this mapping certain important bilinear expressions become local expressions in ϕ. In particular the Lagrangian becomes

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu\theta)^2-M^2(1-\cos\theta)+\tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{32}g^2\pi\cos(\theta-\sqrt{2\pi}\phi), \tag{2}$$

and the conserved fermion number current

$$\bar{\psi}\gamma_\mu\psi=(2\pi)^{-1/2}\epsilon_{\mu\nu}\partial_\nu\phi. \tag{3}$$

(There is a certain arbitrariness in the coefficient of the interaction term, corresponding to different methods of regularizing the theory. We have chosen it so that the soliton $\phi=4\tan^{-1}e^{mx}$, $m^2=g^2/32$ with $\theta=0$ fixed has mass g, as does the corresponding unit fermion number excitation in the original theory.)

While for $M >> m$ the θ field is rigid and the assumption that $\theta=0$ should be essentially correct for $M << m$ we can lower the energy considerably by allowing the θ field to follow the ϕ field, $\theta\approx\sqrt{2\pi}\phi$. By doing so, we find a state with fermion number unity and mass $\approx M$.

Four further remarks regarding this simple model are now appropriate, since they generalize:

(i) The effective position-dependent mass for the fermion generated by $\theta(x)$ in (1) does not go through zero or even vary in magnitude, though it does vary in phase.

(ii) The distinction between small and large deformations induced by fermion number f can be made sharp in the following sense. The θ field has its own conserved current $\tilde{j}_\mu = 1/2\pi\epsilon_{\mu\nu}\partial_\nu\theta$ and charge $\tilde{f}$. The sharp question is, For given f, what value of $\tilde{f}$ gives minimum energy?

(iii) One can change the model by expanding the internal space of the θ field into a larger manifold, say a two-dimensional plane. Let us replace the $\exp(i\theta\gamma_5)$ by $\phi_1 + i\gamma_5\phi_2$ in (1) and imagine a potential $V(\phi_1,\phi_2)$ which is minimized uniquely at $\phi_1 = 1$, $\phi_2 = 0$, and has a local maximum at $\phi_1 = \phi_2 = 0$. In this context the conserved current $\tilde{j}$ cannot be defined; there is strictly speaking no topological content in any spatial ϕ_1,ϕ_2 configuration. However, insofar as the point $\phi_1 = \phi_2 = 0$ represents a barrier the internal space may be approximated by a plane with the origin excised, for which the angle $\theta = \tan^{-1}\phi_2/\phi_1$ and thereby the current $\tilde{j}$ is defined (strictly speaking only $\partial\theta$ is defined, but this suffices to define $\tilde{j}$). The consequent "topological" solitons will be at best metastable in themselves, since the topology is undone by tunneling over the barrier at $\phi_1 = \phi_2 = 0$. (This costs finite energy—since $\phi_1 \to 1$ on both sides, only a finite volume energy is involved). The crucial point is that the configuration discussed above may still be the lowest-energy configuration for $f = 1$ (we expect this for sufficiently heavy fermions), which will make it absolutely stable.

(iv) Consider coupling N heavy fermions to θ as in (1). The deformed vacuum state with θ varying from 0 to 2π over length L and the bosonized fields following will have gradient energy of order N/L and potential energy M^2L, with the total minimized at

$$L = (N/M)^{1/2}, \tag{4}$$

$$E = N^{1/2}M. \tag{5}$$

This energy is of course $<< Ng$, the energy of the corresponding state with $\theta \equiv 0$. Moreover, the minimum energy for a state carrying, say, the first $N-1$ but not the final fermion number will be markedly higher.

3+1 dimensions.—We consider a linear σ model coupled to N fermion doublets. The coupling is taken to be

$$-\mathscr{L}_I = \sum_{j=1}^{N} g\bar{\psi}_j(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)\psi_j, \tag{6}$$

so that the fermions acquire mass $\mu = gv$ in the vacuum $\langle\sigma\rangle = v$. We have in mind primarily weak interaction Higgs fields coupling to heavy quarks or leptons.[4,5,9]

Spread-out skyrmion configurations of the type

$$(\sigma,\vec{\pi}) = v(\cos\theta(r),\hat{r}\sin\theta(r)), \tag{7}$$

$$\theta(0) = -\pi, \quad \theta(\infty) = 0,$$

$$\partial\theta \sim 1/R, \quad \mu R >> 1 \tag{8}$$

induce unit fermion numbers of each type, i.e.,

$$\int\langle\bar{\psi}_1\gamma_0\psi_1\rangle\, d^3x = \int\langle\bar{\psi}_2\gamma_0\psi_2\rangle\, d^3x = \ldots = 1, \tag{9}$$

as can be proved by the adiabatic method.

The condition $R\mu >> 1$ ensures that the skyrmion does indeed carry the corresponding fermion quantum number; in the opposite limit the background field is essentially invisible to the fermion, being averaged over by zero-point motion.

The gradient scalar energy associated with such a configuration is

$$E_\nabla \sim v^2R, \tag{10}$$

which is to be compared with the energy for N approximately free fermions, $N\mu$. We can reconcile the consistency requirement (8), $R\mu >> 1$, with the energetic requirement $v^2R < N\mu$ if $g^2N >> 1$.

Controlled calculations can be done in the limit $N\to\infty$, with g^2N fixed; the leading order expression for the effective Lagrangian of the scalar involves summing one-loop fermion graphs with no internal lines.[10] We assume that the minimum energy for a spatially uniform σ field occurs at $\langle\sigma\rangle = v$, and we expand the energy for spatially variable σ and $\vec{\pi}$ (assuming, for convenience, that $\sigma^2 + \vec{\pi}^2 = v^2$) in powers of gradients. One readily estimates that successively higher powers of gradients are accompanied by factors $1/\mu R$, so that they will be smaller under condition (8). The terms with four gradients can be determined according to the familiar interpretation of the effective Lagrangian as a generating function as follows. The calculations are done in the framework of the linear σ model; the restriction to the nonlinear model is made only when the coefficients are evaluated.

The effective action $\Gamma[\phi]$ can be expanded in two ways: in powers of ϕ,

$$\Gamma[\phi] = \sum\nolimits_n \frac{1}{n!}\int d^4x_1 \cdots d^4x_n\, \Gamma^{(n)}_{a_1\cdots a_n}(x_1\ldots x_n)\phi_{a_1}(x_1)\cdots\phi_{a_n}(x_n), \tag{11}$$

and in powers of gradients. There are ten terms with four derivatives:

$$\begin{aligned}\Gamma[\phi]=\int d^4x\,[&x_1(\phi^2)\partial^2\phi_a\partial^2\phi_a+X_2(\phi^2)\partial_\mu\phi_a\partial^\mu\phi_a\partial_\nu\phi_b\partial^\nu\phi_b\\&+X_3(\phi^2)\partial_\mu\phi_a\partial_\nu\phi_a\partial^\mu\phi_b\partial^\nu\phi_b+Y_1(\phi^2)\phi_a\partial_\mu\phi_a\phi_b\partial^\mu\partial^2\phi_b\\&+Y_2(\phi^2)\phi_a\partial_\mu\phi_a\partial^\mu\phi_b\partial^2\phi_b+Y_3(\phi^2)\phi_a\partial_\mu\phi_a\partial^\mu\phi_b\partial^\nu\partial_\nu\phi_b\\&+Y_4(\phi^2)\phi_a\partial_\mu\phi_a\phi_b\partial_\nu\phi_b\phi_c\partial^\nu\phi_c+Y_5(\phi^2)\phi_a\partial_\mu\phi_a\phi_b\partial^\mu\phi_b\partial_\nu\phi_c\partial^\nu\phi_c\\&+Y_6(\phi^2)\phi_a\partial_\mu\phi_a\phi_b\partial_\nu\phi_b\partial^\mu\phi_c\partial^\nu\phi_c+Y_7(\phi^2)\phi_a\partial_\mu\phi_a\phi_b\partial^\mu\phi_b\phi_c\partial_\nu\phi_c\phi_d\partial^\nu\phi_d].\end{aligned}\tag{12}$$

Only the first three terms survive in the nonlinear model since $\phi_a\partial_\mu\phi_a=\frac{1}{2}\partial_\mu\phi^2=0$; by computation of judiciously chosen vertex functions they can be isolated. For example, only the first and fourth terms contribute to the momentum-space vertex functions with only two nonvanishing external momenta. A further simplification can be made with the assumption that $\phi_0=v$, $\vec{\phi}=0$ at the point about which the gradient expansion is made. Then only the first term contributes to

$$\Gamma_{0,\ldots,0,i,j}(0,\ldots,0,p,-p)\quad(i,j\neq 0).$$

If we sum over all possible diagrams of this type, the zero-momentum vertices simply give the fermion a mass $\mu\equiv gv$, and

$$\Gamma_{ij}^{(2)}(p,-p)_m=2p^4\delta_{ij}X_1(v^2)+\text{other terms}.\tag{13}$$

The left-hand side is easily calculated; the result is

$$X_1=N/48\pi^2v^2.\tag{14}$$

In a similar manner the second and third terms can be isolated; one finds

$$\Gamma_{ijkl}^{(4)}(p,-p,q,-q)_m=8\delta_{ij}\delta_{kl}[X_2(v^2)p^2q^2+X_3(v^2)(pq)^2]+\text{other terms},\tag{15}$$

from which

$$X_2=-N/32\pi^2v^4,\quad X_3=N/48\pi^2v^4.\tag{16}$$

From Eqs. (14) and (16) the energy is[11]

$$\begin{aligned}E_s=\int d^3x\,\{\tfrac{1}{2}(\partial_i\phi_a)^2+(N/\pi^2)[-(48v^2)^{-1}\partial^2\phi_a\partial^2\phi_a+(32v^4)^{-1}\partial_i\phi_a\partial_i\phi_a\partial_j\phi_b\partial_j\phi_b\\-(48v^4)^{-1}\partial_i\phi_a\partial_j\phi_a\partial_i\phi_b\partial_j\phi_b]\}.\end{aligned}\tag{17}$$

It is easy to show that

$$\int\partial^2\phi_a\partial^2\phi_a\geqslant\int\partial_i\phi_a\partial_i\phi_a\partial_j\phi_b\partial_j\phi_b\geqslant\int\partial_i\phi_a\partial_j\phi_a\partial_i\phi_b\partial_j\phi_b.$$

The signs of the coefficients of the higher-order terms are rather disappointing in the following sense. If they were all positive (or, more generally, if they summed to a manifestly positive quantity) they would represent a contribution to the gradient energy which would scale with the characteristic size R as

$$E_\nabla\sim c_1v^2R+c_2g^2N/R\quad(c_1,c_2\sim 1),\tag{18}$$

which would be minimized at

$$R\sim N^{1/2}/v=(g^2N)^{1/2}/\mu,\tag{19}$$

and we would have had a fully self-consistent [cf. (8) and notice the role of large N here] stable semiclassical description of the soliton, circumventing Derrick's theorem. The result actually obtained requires quite a different interpretation. We find that for $Ng^2\gg 1$ we can make the total energy negative for R such that (8) is still obeyed, indicating an instability of the originally assumed ground state. Similar results have been found in models containing many heavy scalars.[12] Our search has not been exhaustive, and a model with positive gradient energy may yet be possible.

For $g^2N\sim 1$, $N\to\infty$ the gradient expansion is useless. To see if the skyrmion is energetically favored the energy associated with polarizing the Dirac sea must be calculated; fermion determinants similar to what we require are routinely calculated in lattice simulations and we plan to pursue this approach further.

Phenomenology.—There is an important qualitative distinction between the skyrmion-type bags discussed above and more traditional bags, which leads to striking phenomenological consequences. It is that the skyrmion-type bag is intrinsically associated with $f_1 = \ldots = f_N = 1$ in its ground state, so that, for instance, $f_1 = \ldots = f_{N-1} = 1$, $f_N = 0$ is *higher* in energy. In traditional bags, removing fermions of course tends to lower the energy. The reason for the difference is that while the scalar fields associated with traditional bags lower the energy of positive-energy fermions (by turning off their mass in a finite region) they do not send these energies through zero. In continously turning on a skyrmion-type bag from the vacuum, on the contrary, we find a positive-energy level going negative. In the former case, it always costs energy (though less than before) to add fermions, while in the latter case there is actually energy gained by filling the level.

In the real world, heavy quarks, if they exist at all, will probably not give us new absolutely conserved quantum numbers but rather analogs of charm and strangeness which are violated by W-boson interactions. (In our sense "heavy" means $g^2 N \sim 1$, where g is the fermion-Higgs coupling, or $m_F \sim 300\ \mathrm{GeV}/\sqrt{N}$.) If the skyrmion state with $f_1 = \ldots = f_N = 1$ is produced it may be very long lived and decay into many-jet final states: Since sequential decay is blocked, its decay requires that all N fermions decay simultaneously.

Another noteworthy effect is common to all types of bags stabilized by approximately conserved quantum numbers. That is, after the fermions decay, they leave behind an "empty bag" that has lost its *raison d'être;* it will decay by contracting and emitting a coherent shower of Higgs particles, if the semiclassical description is valid.

We may summarize the main points of this Letter as follows:

(i) Calculations of fermion quantum numbers induced by topologically nontrivial background scalar fields may in certain cases be turned around; the fermion quantum numbers will induce scalar condensates to deform.

(ii) By calculation of the energy associated with effects of this kind, the gradient expansion can be a powerful tool. When combined with a certain large-N limit, it gives tractable expressions for quasirealistic models. Here it was used to show that the Higgs field in a standard $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge model will deform if many different heavy quarks are present. In many cases, a new vacuum instability arises.

(iii) The phenomenology associated with this type of "weak bag" is distinguishable from that of other bags that have been discussed as a consequence of the fact that quite specific quantum numbers are energetically favored.

This material is based upon research supported in part by the National Science Foundation through Grant No. PHY77-27084, supplemented by funds from the National Aeronautics and Space Administration. We thank J. R. Schrieffer for helpful remarks.

[1]T. D. Lee and G. Wick, Phys. Rev. D **9**, 2291 (1974); R. Friedberg, T. D. Lee, and R. Sirlin, Phys. Rev. D **13**, 2739 (1976); R. Friedberg and T. D. Lee, Phys. Rev. D **15**, 1694, and **16**, 1096 (1977); A. Chodos, R. Jaffe, K. Johnson, C. Thorn, and V. Weisskopf, Phys. Rev. D **9**, 3471 (1974).

[2]W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein, and T. M. Yan, Phys. Rev. D **11**, 1094 (1974); M. Creutz, Phys. Rev. D **10**, 1749 (1974).

[3]J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 968 (1981).

[4]R. MacKenzie and F. Wilczek, Institute of Theoretical Physics, University of California, Santa Barbara, Reports No. NSF-ITP-84-32 and No. NSF-ITP-84-33, 1984 (to be published).

[5]D. D'Hoker and E. Farhi, Nucl. Phys. **B241**, 109 (1984).

[6]T. Skyrme, Proc. Roy. Soc. London, Ser. A **260**, 127 (1961); A. Balachadian, V. Nair, S. Rajeev, and A. Stern, Phys. Rev. Lett. **49**, 1124 (1982); E. Witten, Nucl. Phys. **B223**, 422 (1983).

[7]M. Derrick, J. Math. Phys. (N.Y.) **5**, 1252 (1964).

[8]S. Mandelstam, Phys. Rev. D **11**, 3026 (1975). For some remarks related to our discussion see E. Witten, Nucl. Phys. **B145**, 110 (1978).

[9]J. Gipson and H. Tze, Nucl. Phys. **B183**, 524 (1981).

[10]In reality the effective potential for many fermions becomes unbounded below for large scalar fields: S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973); we ignore this problem, which occurs outside the domain in which we are interested. This calculation is presented in greater detail by R. MacKenzie, Institute of Theoretical Physics, University of California, Santa Barbara, Report No. NSF-ITP-84-135, 1984 (to be published).

[11]In this regard, see S. Coleman, L. Jackiw, and H. D. Politzer, Phys. Rev. D **10**, 2491 (1974).

[12]As this manuscript was being typed we received a paper by I. Aitchison and C. Fraser, where the fermion gradient energy is calculated in a different way with equivalent results.