Experimental Study of the Elastic Properties of a Percolating System

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Simultaneous measurement of the conductivity and the elasticity exponents in two dimensions for a percolating system made of metal and voids is reported. The elasticity exponent is $T = 3.5 \pm 0.4$, much larger than the conductivity exponent $t = 1.2 \pm 0.1$. This result is in excellent agreement with the theoretical prediction of Bergman.

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Recently, important progress has been made in the theoretical analysis of the elastic properties near a percolation threshold.[1-3] It seems established that the elastic properties belong to a different universality class than that of the electrical properties. In other words, the elastic constants near the percolation threshold approach zero with an exponent very different from that of the conductivity (t). Using a lattice model, Kantor and Webman[2] gave lower bounds of the elasticity exponent T, which are larger than t. Bergman[3] proposed a calculation of the exponent T in two dimensions, using a random lattice model in which bending is included (as in Ref. 2) and he obtained $T = 3.5 \pm 0.2$ very close to one of the lower bounds of Kantor and Webman.

However, from an experimental point of view the situation is less clear. It was proposed that a gel could be a good candidate for studying the elastic properties near the percolation threshold.[4] Several workers[5-7] reported an exponent for the shear modulus between 1.8 and 2.2, which is close to the value of t in three dimensions. However, in other cases[6] a higher exponent (3.8) was found. Although the identification of the gel process with percolation was made by several authors,[7] this point was questioned by others.[4,8] Therefore I thought it would be worthwhile to perform measurements on a system which seems well described by the lattice models of the Refs. 2 and 3: a sheet of metal in which holes are punched. The system also has another advantage, namely, there is the possibility of measuring simultaneously the conductivity exponent and the elasticity exponent. The important result reported in this Letter is that I found the two-dimensional conductivity exponent $t \simeq 1.1$–1.2 and a value of T equal to 3.5 ± 0.4, very close to that predicted by Bergman.

The experimental setup is shown schematically in Fig. 1. A foil of metal (copper or aluminium) of 0.2-mm thickness and of 20×20 cm^2 is clamped between two bars, and its elongation is measured under application of a force F. The strain Δl is measured as a function of F. The holes are punched (with the load off) at random (with use of a random-number generator from a computer) on the sites of a square lattice, with a diameter equal to the cell of the lattice (1 cm). I was careful with respect to the following points, when performing the experiment:

(a) In order to diminish the friction, ball bearings were used in the mobile parts, and the measurements were repeated several times with decreasing and increasing F.

(b) When increasing F, it is very easy to go beyond the elastic limit. In such a case the relation between Δl and F is not linear and different for increasing and decreasing F. In Fig. 2, I show one example of a curve below the elastic limit (Fig. 2 curve, *a*) and another example of a curve in which I went beyond the elastic limit (Fig. 2, curve *b*). Only in case (a) is it possible to measure the elastic constant. For this reason, I was very cautious to do measurements only in the linear and reversible regime, for Al as well as Cu. Since the elastic limit depends on the number of holes punched, the experiments were made twice: once to determine the elastic limit as a function of the number of holes and a second time on a new sample (it is a destructive experiment) to collect the data.

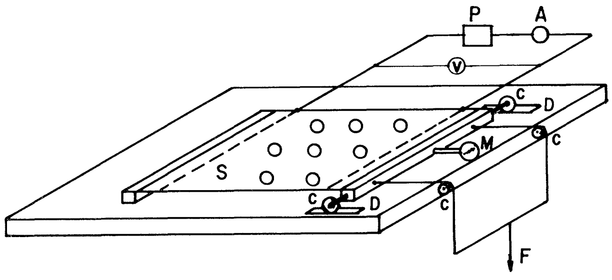

FIG. 1. Schematic view of the experimental setup: A, ammeter; V, voltmeter; P, power supply; C, ball bearing; D, groove; S, metallic sheet; M, micrometer.

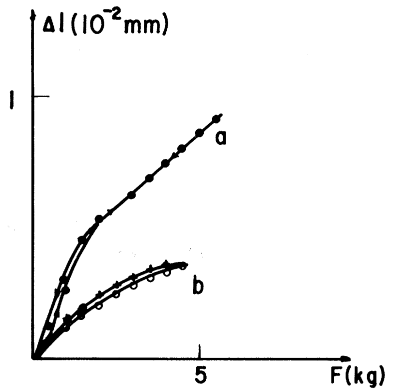

FIG. 2. Strain Δl vs the applied force. Curve *a*: After the rapid increase of Δl, there is the linear and reversible regime (Al with 30% voids). Curve *b*: In this case (Cu) the elastic limit is passed and there is hysteresis.

(c) When a hole is punched, it is inevitable that there is a small local deformation of the metal sheet. This deformation was eliminated to recover a plane sample. This point is essential in order to have reproducible results.

(d) The resistance of the sheet is measured by allowing constant dc current to flow in the sample and the voltage across it is measured. The measurements were made for the two polarities and the mean value was taken in order to eliminate spurious voltages.

When one begins to apply a force F, there is a rapid increase of Δl: It is merely the initial stretching of the foil in order to give it a plane shape. After that the linear region begins. The inverse of the slope in this regime is the elastic constant measured. In fact, a combination of C_{11} and C_{12}, $C = C_{11} - C_{12}^2/C_{11}$ was measured. Since it is believed[3] that C_{11} and C_{12} behave in the same manner, the measured quantity goes to zero like $(\phi_c - \phi)^T$. ϕ is the fraction of the surface which is removed. Since holes are punched at the sites of the square lattice, ϕ_c is very near the threshold for site percolation in a square lattice, $\phi_c = 0.6$.

Figure 3 is a linear plot of C and σ (electrical conductivity) for Cu. Figure 4 is a log-log plot of C and σ vs $(\phi - \phi_c)$. The slope yields $t = 1.1 \pm 0.1$ for Al and $t = 1.2 \pm 0.1$ for Cu, which are near the accepted values. The elasticity exponents are $T(\mathrm{Cu}) = 3.7 \pm 0.2$. and $T(\mathrm{Al}) = 3.3 \pm 0.2$, in excellent agreement with the lower bound of Kantor and Webman[2] and with the value of Bergman.[3] The results for Al are less dispersed and measurements closer to ϕ_c are possible.

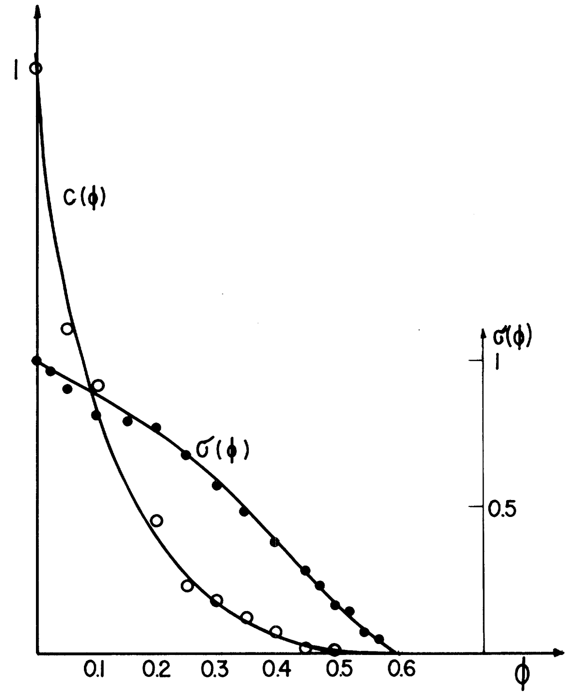

FIG. 3. Variations of the elastic modulus (C) and of the conductivity (σ) vs void content as a percentage of the total surface.

Since T was determined in a relatively narrow range of ϕ (and not too close to ϕ_c) maybe the values of the exponent T should not be taken too seriously. Even the fact that the known value of t was recovered should not automatically instill confidence in the value of T since the elastic constant and the conductivity belong to different universality classes. The important point is that these results are in good agreement with the theoretical values[2,3] and that T is far higher than the conductivity ex-

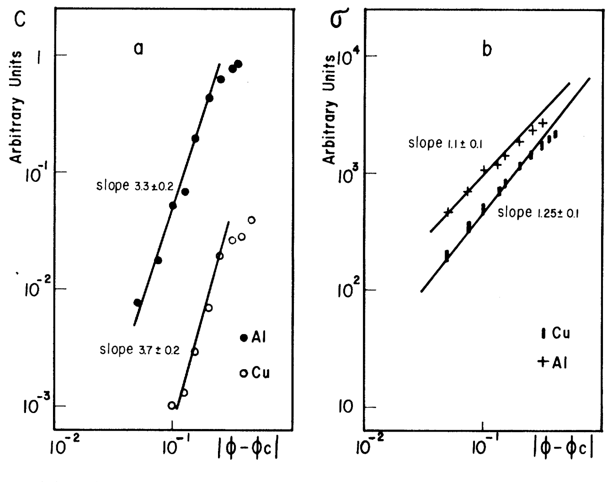

FIG. 4. Log-log plots of (a) C and of (b) σ vs $(\phi - \phi_c)$.

ponent t. Note also that this system is more complicated than the lattice systems[1-3] and thus the results confirm the universality of the elastic properties.

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