Pion Charge Exchange from Oriented, Deformed Nuclei

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(Received 23 July 1984)

We extend the theory of pion scattering to study charge-exchange reactions from oriented, deformed nuclei. We show that measurement of the orientation asymmetry for pion charge exchange can lead to a determination of the deformation, β_2^N, of the excess neutron distribution.

PACS numbers: 25.80.Fm, 21.10.Gv, 21.60.Ev

The measurement of the charge density in nuclei has been the object of intense experimental investigation with electromagnetic probes. The successes of this program are made possible by the coupling of the Coulomb interaction to the distribution of charge, and by the fact that this interaction is well understood. In particular, measurements made on nuclei having intrinsic ground-state deformation have revealed sufficient detail to allow the extraction of β_2^C, the quadrupole deformation of the charge distribution.[1] Comparison of these measurements with theoretical models has been in general quite favorable.[2,3]

Determination of the neutron distribution in nuclei is considerably more difficult. The strongly interacting probes n, p, d, α, and π have all been used for this purpose, but they suffer from the limitation that the forces are less well understood than in the electromagnetic case. Uncertainties associated with the strongly interacting probes have led to especially large uncertainties in the specification of the deformation of the neutron distribution.[4-6]

The selective sensitivity of the (π^+,π^0), (π^+,π^-) pion charge-exchange reactions to the neutrons, the development of reliable spectrometers for measuring these reactions, and the understanding[7] of recent charge-exchange measurements on spherical nuclei throughout the periodic table[8] have led us to reexamine the problem of determining the deformation of the neutron distribution. We have in mind an application of the theory of pion scattering that has been successful in these cases to a class of possible measurements that have not been made. To learn about β_2^N we must focus on an observable that has some chance of being sensitive to the deformation but insensitive to the less well-known aspects of the reaction dynamics. We thus define the orientation asymmetry

$$A_S(\theta)=\frac{d\sigma^{\perp}/d\Omega-d\sigma^{\parallel}/d\Omega}{d\sigma^{\perp}/d\Omega+d\sigma^{\parallel}d\Omega}, \qquad (1)$$

where $d\sigma^{\perp}/d\Omega$ and $d\sigma^{\parallel}/d\Omega$ are the cross sections for scattering from deformed nuclei oriented perpendicular and parallel, respectively, to the direction of the incident pions. Determination of A_S for single-charge exchange is difficult but feasible[9] with the techniques developed in connection with earlier measurements[10] of pion total cross sections from aligned ^{165}Ho.

The theory of charge-exchange reactions displays a strong sensitivity to neutron densities, and we next examine the theory to see whether it can help us learn about β_2^N. Because of the exploratory nature of our investigation, we have made some approximations, including the standard eikonal method. Such approaches are known to reproduce the striking regularities in single-charge-exchange measurements,[8] and the connection of the eikonal theory to the more detailed formulations has been explored.[11] These studies verify that the eikonal theory describes the sensitivity of the underlying theory to nuclear structure quite faithfully.

For highly deformed nuclei there exist a large number of low-lying levels easily excited by a medium-energy projectile. These states must be summed over in both the intermediate and final state. To accomplish this we use the closure approximation, which implies that the orientation of the nucleus is not changed during the scattering process. The scattering amplitude depends, therefore, on the orientation Ω of the body-fixed system in the laboratory. We will characterize this orientation by the Euler angles[12] $\Omega=(\alpha,\beta,\gamma)$. We project the amplitude $F(q,\Omega)$ onto the nuclear initial and final states,

$$F(q)=\langle I'M'K'|F(q,\Omega)|IMK\rangle, \quad (2)$$

where $F(q,\Omega)$ is the scattering from the intrinsic state whose orientation is Ω relative to the laboratory coordinate system.

For the wave function of the nuclear ground state, we take a product of the intrinsic wave function in the body-fixed system and the eigenstate $\psi_{IMK}(\Omega)$ of the collective variable describing the orientation of the nucleus. The latter is an eigenstate of the rigid-rotator Hamiltonian. These wave functions are the rotation matrices[10] $\mathscr{D}^{I}_{MK}(\alpha,\beta,\gamma)$. Two specific cases are needed for the calculation of $A_S(\theta)$: (1) If the nucleus is polarized along the Z axis, the projection M of the total angular momentum on the laboratory Z axis is $M=I$ and

$$\psi^{(\parallel)}_{IIK}(\Omega)=[(2I+1)/8\pi^2]^{1/2}\mathscr{D}^{I}_{IK}(\phi,\theta,0); \quad (3)$$

(2) in the case that the nucleus is polarized along the X axis, we rotate Eq. (3) by $\pi/2$ about the Y axis to obtain

$$\psi^{(\perp)}_{IIK}(\Omega)=[(2I+1)/8\pi^2]^{1/2}\sum_M \mathscr{D}^{I}_{IM}(0,\pi/2,0)\mathscr{D}^{I}_{MK}(\phi,\theta,0). \quad (4)$$

Equations (3) and (4) lead to the following expressions for the amplitudes:

$$F^{\parallel}(q)=\tfrac{1}{2}(2I+1)\int_0^{\pi}\sin\theta d\theta|d^{I}_{IK}(\theta)|^2F(q,\theta), \quad (5)$$

$$F^{\perp}(q)=\tfrac{1}{2}(2I+1)\sum_M|d^{I}_{IM}(\pi/2)|\int_0^{\pi}\sin\theta d\theta|d^{I}_{MK}(\theta)|^2F(q,\theta). \quad (6)$$

The charge-exchange amplitude to the analogs of the final state is generated from this result by applying isotopic spin invariance. The physical amplitude $F^{(ij)}$ for elastic $[(i,j)=(+,+)$ and $(-,-)$ and for π^+ and π^-, respectively], single charge exchange $[(i,j)=(0,+)]$, and double charge exchange $[(i,j)=(-,+)]$ may be expressed formally as

$$F^{(ij)}=\sum_\tau A^{(ij)}(\tau)F_\tau, \quad (7)$$

where $\tau=T$, $T+1$, and $T-1$ and where the set $A^{(ij)}(\tau)$ are Clebsch-Gordan coefficients and are given explicitly in Ref. 11.

We calculate F in the eikonal theory. The result is

$$F_\tau(q,\theta)=iK\int_0^{\infty}bdb\, J_0(qb)[1-G_\tau(b,\theta)],$$
$$G_\tau(b,\theta)=\frac{1}{2\pi}\int_0^{2\pi}d\phi'\, e^{-\chi_\tau(b,\theta,\phi')}, \quad (8)$$

where

$$\chi_\tau(b,\Omega)=\frac{1}{2K}\int_{-\infty}^{\infty}dZ\, U_\tau(b,Z,\Omega), \quad (8a)$$

U_τ is the projection of U onto the state of total isospin,

$$U_\tau(b,Z,\Omega)=\langle\tau|u_0+u_1\phi\cdot T_N+u_2(\phi\cdot T_N)^2|\tau\rangle, \quad (9)$$

and where u_0, u_1, and u_2 describe[11] the isoscalar, isovector, and isotensor components of U. We use a local form for the optical potential, so that

$$U_\tau=k^2\xi_\tau+\tfrac{1}{2}\nabla^2\xi_\tau, \quad (10)$$

where ξ_τ depends on the nuclear densities $\rho(\vec{r})$ and $\Delta\rho(\vec{r})$, which are the total and excess neutron densities. If we ignore the Coulomb interaction on nuclear structure, then

$$\rho(\vec{r})=N\rho_N(\vec{r})+Z\rho_p(\vec{r}),$$
$$\Delta\rho(\vec{r})=N\rho_N(\vec{r})-Z\rho_p(\vec{r}). \quad (11)$$

The quantities ξ_τ also depend on the free pion-nucleon scattering amplitude through an isoscalar $\lambda_0^{(1)}$ and an isovector $\lambda_1^{(1)}$ coefficient. We may write

$$\xi_\tau(\vec{r},\vec{\Omega})=\lambda_0^{(1)}\rho(\vec{r},\vec{\Omega})+\lambda_1^{(1)}\gamma^{(1)}(\tau)\Delta\rho(\vec{r},\vec{\Omega}), \quad (12)$$

where $\gamma^{(1)}(\tau)$ are Clebsch-Gordan coefficients (see Ref. 11) and where we have now displayed the explicit dependence of ρ and $\Delta\rho$ on the orientation Ω. We omit the higher-order medium modifications in this paper, although their influence could be included at 165 MeV using the results of Ref. 7. It is known that these scale the single-charge-exchange (SCX) cross sections in a simple fashion so that A_S is very insensitive to them. One must be more careful in the case of double-charge-exchange (DCX), where these corrections have a more subtle effect and can modify the shape of $d\sigma/d\Omega$.

We shall take the specific case of ^{165}Ho($N=98, Z=67$) to investigate the main ideas. This nucleus is known to be highly deformed, and studies with μ-atom techniques have determined the parameters of the charge density in the ground state.[13] A Woods-Saxon shape for the charge distribution was used in the analyses,

$$\rho_C(r,R)=\frac{\rho_0}{1+e^{(r-R)/a}}, \quad (13)$$

with the half-density radius R mapped onto the ellipsoidal surface

$$R=R_0[1+\beta_2^C Y_{20}(\theta)] \quad (14)$$

whose major axis lies along the Z' axis of the body-fixed coordinate system. The results of the analyses give

$$\beta_2^C=0.32, \quad R_0^C=6.15, \quad a=0.49. \quad (15)$$

The spin quantum number of the ground state of ^{165}Ho is $I=K=\frac{7}{2}$, where I is the total angular momentum and K is its projection on the body-fixed axis.

For our studies we imagine parametrizing the relevant densities $\rho_i(r,R_0,\beta_2)$ as follows:

$$\rho_i(r,R,\beta_2) = \rho_i^{(0)}(r,R_0)+R_0\beta_2 Y_{20}\rho_i^{(1)}(r,R_0). \quad (16)$$

This form is a good approximation for ^{165}Ho. For our actual calculations we have written $\rho^{(0)}$ to be the Woods-Saxon density in Eq. (13) and $\rho^{(1)}(r,R_0)=d\rho^{(0)}(r,R_0)/dr$. Making the dependence on ϕ' explicit in Eq. (8a),

$$\chi_\tau(b,\theta,\phi')=\chi_\tau^{(1)}(b,\theta)+\chi_\tau^{(2)}(b,\theta)\cos^2\phi' \quad (17)$$

and then

$$G_\tau(b,\theta)=\tfrac{1}{2}\exp[-\chi_\tau^{(1)}(b,\theta)-\tfrac{1}{2}\chi_\tau^{(2)}(b,\theta)]I_0(\tfrac{1}{2}\chi_\tau^{(2)}(b,\theta)), \quad (18)$$

where $I_0(x)$ is a Bessel function of imaginary argument of order zero.

We have calculated the differential cross sections and the asymmetries for pion incident energies ranging from 120 to 250 MeV and for different values of R_0^N and β_2^N to demonstrate the sensitivity of the cross sections $d\sigma^\perp/d\Omega$ and $d\sigma^\parallel/d\Omega$ and the asymmetry A_S to these quantities.

Figure 1 shows the quantity $A_S(0)$ plotted as a function of β_2^N/β_2^C at $T_\pi=180$ MeV for elastic, SCX, and DCX scatterings. Charge exchange displays a striking sensitivity to the neutron deformation. One notices that SCX and DCX are roughly equivalent in this regard. Elastic scattering shows very little sensitivity. One additional advantage of charge-exchange over elastic scattering is that elastic scattering is strongly affected by the Coulomb interaction, which was not included in our results. The Coulomb interaction dominates at small angles and would tend to hide the signal. No such problem exists for charge exchange. In order to utilize the sensitivity of DCX, it is necessary to include medium modifications, and extensions of the theory along this direction would be desirable.

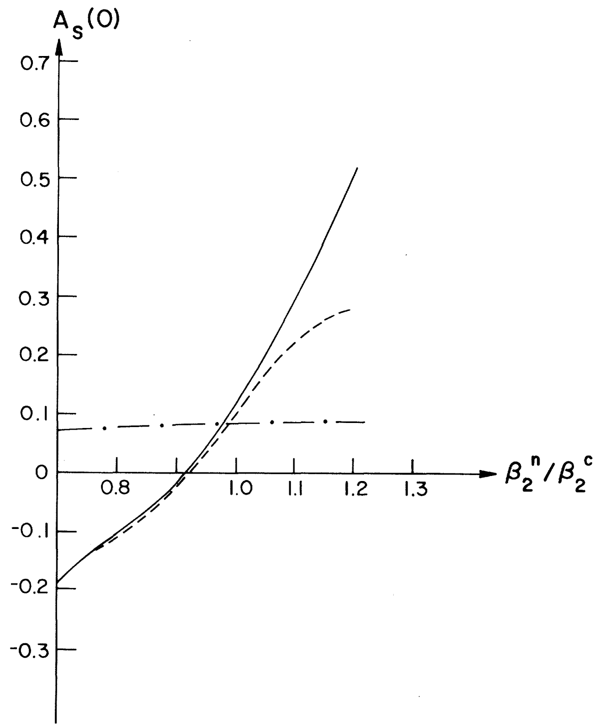

FIG. 1. The asymmetries at 0° from ^{165}Ho plotted as a function of β_2^N/β_2^C at $T_\pi=180$ MeV. Solid curve, single charge exchange (π^+,π^0); dashed curve, double charge exchange (π^+,π^-); dash-dotted curve, result for π^+ elastic scattering. We have taken β_2^C to be fixed at the value given in Eq. (15).

We conclude from our results that charge-exchange scattering is a sensitive measure of the excess neutron deformation in rare-earth nuclei. Single-charge-exchange experiments show promise for resolving discrepancies among the measurements that have been made in other less-sensitive reactions with strongly interacting probes. Plans for measuring the forward single-charge-exchange scattering from aligned ^{165}Ho to an accuracy of 10% are being made[9] at the Clinton P. Anderson Meson Physics Facility (LAMPF). This should be sufficient for a determination of β_2^N/β_2^C to an accuracy of 5%. In anticipation of the forthcoming results, predictions of $A_S(0)$ for realistic nuclear models (see, for example, Refs. 2 and 3) would be useful. A successful outcome of the initial experiment should motivate a much wider class of interesting measurements of aligned targets.

One of the authors (H.C.C.) is grateful to LAMPF, Los Alamos National Laboratory, for the kind hospitality extended to him.

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