Vector Polarization in Reactions with Spin-1 Particles

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(Received 17 February 1984)

It is shown that in any reaction involving, partially or entirely, spin-1 particles, the reaction amplitudes can be completely determined from a set of experiments that does not include measurements of vector polarization for any of the spin-1 particles in the reaction. In this sense, measurements of the vector polarization of the spin-1 particles in the reaction are dispensable. The result is applicable to any nuclear or particle reaction, for example, those containing deuterons, rho mesons, etc.

PACS numbers: 24.70.+s, 25.45−z

Both nuclear and particle physics are rich in reactions of considerable interest which contain particles with spin 1. Perhaps the most prevalent of such ingredients are the deuteron and the rho meson. For the latter the density matrix is explored in terms of correlation measurements of the two decay pions. For the deuteron, in recent years the technology of preparing polarized beams, or polarized targets, and of measuring the polarization of deuterons in the final state has made considerable progress, and therefore such polarization experiments constitute a crucial ingredient in our exploration of the dynamics of nuclear- and high-energy reactions involving deuterons. Yet, we are still far from being able to easily furnish deuteron polarization states of arbitrary prescription. The techniques for vector polarization are different from those for tensor polarization, and in some respects the former lags behind the latter.[1]

In order to explore the capabilities within such a technologically incomplete situation, it is of interest to define the extent to which information can be obtained about reactions including spin-1 particles without resorting to measuring the vector polarization of such spin-1 particles, that is, on the strength of only tensor polarization measurements. This note presents a very simple, most general, and quite far reaching result in that respect. We show that complete information can be obtained about the amplitudes of any reaction containing one or several spin-1 particles without ever resorting to use of vector-polarized spin-1 particles. Of course, the tensor polarization of the spin-1 particles must be measured in correlation with the polarization of other particles, as well, for completeness.

The theorem can also be used in a partial way. For example, if methods of measuring *final*-state vector polarization happen to be deficient, the theorem assures us that it is sufficient to be able to measure tensor polarization for the *final*-state deuterons, which we can, if we want, combine with specifications of vector, or tensor, or vector plus tensor polarizations in the *initial* state.

In practice it is almost always advisable to overdetermine the reaction amplitudes by measuring more observables then is theoretically necessary, in order to resolve remaining discrete ambiguities and in order to counteract the uncertainties caused by large experimental errors on the measurements. Fortunately there are a large number of tensor polarization observables, so that even if we want to add such overdetermining experiments, we may still not be forced to turn to vector-polarization measurements if we do not wish to.

Besides exploring what is feasible with currently available techniques, the theorem may also give stimulus to the future evolution of tensor polarization techniques in view of their potential to suffice by themselves. Furthermore, it is most likely that similar theorems can also be found for particles with spins higher than 1, an effort that may be stimulated by the present theorem.

In order to be specific, in the subsequent discussion we will use the deuteron as the spin-1 particle, with the understanding that everything in the discussion holds just as well for other spin-1 particles.

The proof is straightforward. We first consider the actual polarization analysis to establish the context of the theorem and some notation. The density matrix of the deuteron can be expanded into zeroth, first, and second rank tensors which can be expressed conveniently in Cartesian or spherical bases.[2] In the Cartesian basis these tensors are I, p_x, p_y, p_z, $p_{xx}-p_{yy}$, p_{zz}, p_{xy}, p_{yz}, and p_{xz}, where z is the deuteron's momentum direction, for specificity. The angular distribution of a rescattering or analyzing reaction such as $^3\mathrm{He}(d,p)^4\mathrm{He}$ depends on all but p_z of the deuteron's various vector and tensor polarizations. Since p_z is the only tensor of the "wrong" parity which is independent of the angle between the reaction plane and the analyzing plane, it is excluded by parity conservation. However, the contribution of a particular vector or tensor polarization to the angular distribution depends on the corresponding analyzing powers of the reaction, $A_y(\theta)$, $A_{xz}(\theta)$, $A_{xx}(\theta)-A_{yy}(\theta)$, and $A_{zz}(\theta)$, where θ is the center-of-mass (polar) scattering angle for the analyzing reaction.[2] For the ^{3}He reaction, and similar analyzing reactions, the vector analyzing power $A_y(\theta)$ can be small in the energy region below a few hundred megaelectronvolts, although below 10 MeV it is sizable for ^{3}He. Nevertheless, the determination of the vector polarizations p_x and p_y (as well as p_z, trivially) may not be possible for all relevant energies and angles. Measuring various azimuthal asymmetries at several polar angles does determine the tensor polarizations, nevertheless.[3]

The tensor polarizations p_{xz}, p_{yz}, $p_{xx}-p_{yy}$, p_{xy}, and p_{zz} correspond to the observables defined in the optimal formalism[4] as R^+-R^-, I^+-I^-, R^0, I^0, and $\Lambda\equiv(++)+(--)-2(00)$ (where the same axes x, y, and z are understood). This latter notation allows us to quickly relate the tensor observables to the amplitudes of the master reaction, i.e., the reaction that produced the polarized deuteron. To establish the method of proof for our theorem, consider the simplest master reaction first, the reaction with the spin structure $0+0\to 0+1$. For such a reaction there are three amplitudes, for helicities $+1$, 0, and -1. We call these complex amplitudes A, B, and C, respectively. Then the optimal observables for tensor polarization become

$$\Lambda=|A|^2+|C|^2-2|B|^2, \tag{1}$$

$$R^+-R^-=\mathrm{Re}(BC^*-AB^*), \tag{2}$$

$$I^+-I^-=\mathrm{Im}(BC^*-AB^*), \tag{3}$$

$$R^0=\mathrm{Re}AC^*, \tag{4}$$

$$I^0=\mathrm{Im}AC^*, \tag{5}$$

and we also have $|A|^2+|B|^2+|C|^2$, because that is just the completely unpolarized differential cross section. But we can see immediately that the above expressions for the observables determine A, B, and C completely apart from a single overall phase factor which of course is always arbitrary. From Eq. (1) and the unpolarized differential cross section we get $|A|^2+|C|^2$ and $|B|^2$. Then from this and Eqs. (4) and (5) we completely determine A and C. Thus we also know $A-C$, which then allows us, through Eqs. (2) and (3), to also determine B completely. Discrete ambiguities are not necessarily eliminated, however. (Certain of the above observables may be required to vanish by parity conservation and $A\to C^*$, $B\to -B^*$ leaves the equations unchanged.)

When dealing with the general master reaction involving a deuteron and any number of other particles with or without spin (some of which may also be deuterons), the amplitudes and observables will carry arguments for each of the participating particles. Let these amplitudes be $D(c,a;d,b)$ for which we will write A_{abc}, B_{abc}, and C_{abc}, where a, b, and c label the spin projection eigenvalues of the incoming particles a and b and of the other outgoing particle c, while A, B, and C still refer to the deuteron helicities $d=+1$, 0, and -1, respectively. A general observable in which both initial particles may be polarized, as described by density matrix $\rho^{(a)}_{aa'}$, and $\rho^{(b)}_{bb'}$, is given by an expression

$$\sum_{a,a',b,b'} D(c,a;d,b)\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}D^*(c',a';d',b'). \tag{6}$$

The deuteron's tensor polarization parameters for all other particles polarized (or not) in some particular way thereby are given by

$$\Lambda_{cc'}=\sum_{a,a',b,b'}[A_{abc}A^*_{a'b'c'}+C_{abc}C^*_{a'b'c'}-2B_{abc}B^*_{a'b'c'}]\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}, \tag{7}$$

$$R^+_{cc'}-R^-_{cc'}=\sum_{aa'bb'}\mathrm{Re}\{[B_{abc}C^*_{a'b'c'}-A_{abc}B^*_{a'b'c'}]\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}\}, \tag{8}$$

$$I^+_{cc'}-I^-_{cc'}=\sum_{aa'bb'}\mathrm{Im}\{[B_{abc}C^*_{a'b'c'}-A_{abc}B^*_{a'b'c'}]\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}\}, \tag{9}$$

$$R^0_{cc'}=\sum_{aa'bb'}\mathrm{Re}\{[A_{abc}C^*_{a'b'c'}]\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}\}, \tag{10}$$

$$I^0_{cc'}=\sum_{aa'bb'}\mathrm{Im}\{[A_{abc}C^*_{a'b'c'}]\rho^{(a)}_{aa'}\rho^{(b)}_{bb'}\}. \tag{11}$$

Each of the above quantities on the left-hand sides represents a double density matrix element of the two final particles in which the deuteron has a certain tensor polarization and the other final particle has polarization indices c and c', with the two initial particles characterized by the initial density matrices $\rho^{(a)}_{aa'}$ and $\rho^{(b)}_{bb'}$. Note that the vector polarizations p_x and p_y are equal to $(R^+ + R^-)_{cc'}$ and $(I^+ + I^-)_{cc'}$, for which the amplitude expressions are of the same form as Eqs. (8) and (9) with relative plus signs. To prove the theorem, we need only show that the $(R^+ + R^-)$ and/or the $(I^+ + I^-)$ measurements are superfluous. This must be the case, as we now show.

The initial density matrices are under our experimental control, so choose $\rho^{(a)}$ and $\rho^{(b)}$ to be diagonal elements. Then for each $c = c'$ (and the prepared $a = a'$, $b = b'$) the A_{abc}, B_{abc}, and C_{abc} can be determined by the same procedure used for Eqs. (1)–(5). For each set of indices, $\{a,b,c\}$, the relative phases among A,B,C are fixed thereby. To fix the five relative phases of A,B,C from one set $\{a,b,c,\}$ to another $\{a,b,c'\}$, an off-diagonal $(c \neq c')$ observable from among Eqs. (7)–(11) must be measured as well [e.g., Eq. (11) determines the phase between A_{abc} and $C_{abc'}$]. Repeating this process for each c and then $a \neq a'$ and $b \neq b'$ completes the phase specification and, therefore, by construction, the proof.

Alternatively note that A_{abc}, B_{abc}, and C_{abc} correspond to three "vectors" in the $a \times b \times c$ "complex vector space" and the five observables corresponding to tensor polarization are therefore scalar products of a particular kind [with metric given by $\rho^{(a)} \times \rho^{(b)} \times (c,c')$]. From this point of view there are only six independent scalar products of the A, B, and C multidimensional complex vectors, one of which will be the sum over the deuteron polarization. Of the remaining five scalar products, all will be combinations of the five generally independent linear combinations contained in the observables of Eqs. (7)–(11). That those observables are in general independent can be verified straightfowardly by evaluating the Jacobian for those quantities.

As a corollary, note that if one of the initial-state particles is also spin 1, then for each observable of Eqs. (7)–(11) it suffices to measure only those density matrix elements corresponding to tensor polarization to get all necessary information for that observable. Thus vector polarization for the deuteron is not needed for obtaining a complete description of an arbitrary reaction involving deuterons. In practice, a determination might still be made that measuring vector polarization may be easier and preferable to measuring some of the tensor polarization quantities, but if this is not the case, we now know that one can get by for all conceivable purposes without measuring vector polarization.

We gratefully acknowledge the comment of M. Simonius and A. Rinat. This work was supported in part by the U.S. Department of Energy.

[1]For an elementary and intuitive exposition of deuteron polarization quantities see S. E. Darden, Am. J. Phys. **35**, 727 (1967).

[2]See, for example, Gerald G. Ohlsen, Rep. Prog. Phys. **35**, 740 (1972).

[3]For an excellent and quite up-to-date account of the latest progress in deuteron technology, see *High Energy Spin Physics—1982,* edited by G. M. Bunce, AIP Conference Proceedings No. 95 (American Institute of Physics, New York, 1983), particularly pp. 367–637.

[4]Gary R. Goldstein and Michael J. Moravcsik, Ann. Phys. (N.Y.) **98**, 128 (1976), and various subsequent papers on this formalism.