Contribution of Three-Nucleon Potential to Triton Binding Energy

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Faddeev calculations for the triton are performed with three two-body potentials (RSC, URG, and Paris) taking eighteen channels, $J \leq 2$, for an interacting pair. Using these wave functions, we calculate the first-order perturbation energy for the Tucson-Melbourne three-body interaction. For the usual dipole pionic form factor with the cutoff parameter of $\Lambda = 800$ MeV, we get 0.89 MeV (RSC), 0.50 MeV (URG), and 0.67 MeV (Paris). Thus, the calculated triton binding energies amount to 8.13 MeV (RSC), 8.00 MeV(URG), and 8.23 MeV (Paris). These values are very close to the experimental value of 8.482 MeV.

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As is well known, realistic two-nucleon potentials that fit two-nucleon data underbind the triton by 1–1.5 MeV.[1] One possible explanation of this underbinding is the neglect of three-nucleon interactions. Among various three-nucleon potentials which have been discussed,[2,3] the two-pion–exchange three-nucleon potential (2PE-3NP) has received theoretical attention because of its long range nature.[4–6]

To construct the 2PE-3NP, we need the π-N scattering amplitude extrapolated to the pion off the mass shell. There are two basic approaches to the extrapolation: the model-independent and the model-dependent approach.[3] The model-independent approach was done by Coon *et al.*[4] and by Ueda, Sawada, and Takagi,[5] who used current algebra and partial conservation of axial-vector currents. On the other hand, the latest model-dependent approach is due to Coelho, Das, and Robilotta,[6] who used the effective Lagrangian which is approximately chiral invariant.

Recently, Muslim, Kim, and Ueda,[7] Bömelburg and Glöckle,[8] and Wiringa *et al.*[9] have investigated effects of the 2PE-3NP in the triton using solutions of the Faddeev equation with the Reid soft-core (RSC) potential.[10] They calculated the first-order perturbation correction (E_3) of the 2PE-3NP ($W_{2\mathrm{PE}}^{(3)}$) to the triton binding energy, $E_3 = \langle \Psi | W_{2\mathrm{PE}}^{(3)} | \Psi \rangle$, where Ψ is a normalized triton wave function. For Ψ, they used the solution of so-called five-channel Faddeev calculation in which the two-nucleon potential is restricted to the ${}^1S_0$ and ${}^3S_1 + {}^3D_1$ states. For the 2PE-3NP, they used the potential by Coon *et al.*[4] (Tucson-Melbourne potential) with a pionic form factor

$$H(\vec{Q}^2) = [(\Lambda^2 - \mu^2)/(\Lambda^2 + \vec{Q}^2)]^2 \qquad (1)$$

with the cutoff parameter $\Lambda \simeq 6\mu$, where μ is the pion mass. These authors obtained as E_3 the values 0.07 MeV,[7] −0.16 MeV,[8] and −0.41 MeV,[9] and concluded that the three-body effect is hopelessly small[7,8] or not enough.[9] Nevertheless, the result of Ref. 9 is encouraging because it suggests a possibility that a small change in the triton wave function may affect E_3 greatly.

In this Letter, we report on calculations of E_3 for the Tucson-Melbourne potential[4] using solutions of a full eighteen-channel Faddeev calculation for triton, in which all partial waves up to the total angular momentum $J = 2$ of the two-nucleon system are included. With inclusion of higher partial waves, the trition wave function slightly changes. But it turns out that the effect of this small change on E_3 is in fact very large, and first casts a hope of getting the triton binding energy.

We solve the Faddeev equation by a perturbational approach developed by Sasakawa and Sawada.[11,12] In this method, we divide the two-body t matrix of ${}^1S_0$ and ${}^3S_1 + {}^3D_1$ states into two parts, one separable and one nonseparable term. The separable term is treated as the zeroth-order term and all the remainders are treated as the perturbation. This method is very suitable for the treatment of small components. With this method, we have solved the three-channel Faddeev equation, ^{3}H in Ref. 11 and ^{3}He in Ref. 12, for the RSC potential. Since then, we have modified this method to treat also velocity-dependent potentials such as the Ueda-Riewe-Green (URG) model II[13] and the Paris potentials.[14] A detailed account is given by Sawada and Sasakawa.[15]

In Table I, we show results of the five- and eighteen-channel Faddeev calculations for the RSC, URG, and Paris potentials. From Table I, we see

that our five-channel result for the RSC potential (RSC-5) agrees with other recent Faddeev calculations[8,16,17] very well, although the result for the Paris potential is slightly different (0.18 MeV) from that of Hajduk and Sauer.[17] For RSC-5, the S-, S'-, and D-state probabilities in percent are given by 88.90, 1.67, and 9.34, respectively, in agreement with other authors.[17,18]

We express E_3 as

$$E_3 = 3\langle\Psi|W(123)|\Psi\rangle, \tag{2}$$

where $W(123)$ denotes the Tucson-Melbourne potential

$$\langle\vec{k}_1'\vec{k}_2'\vec{k}_3'|W(123)|\vec{k}_1\vec{k}_2\vec{k}_3\rangle = \delta\Big(\sum_{i=1}^{3}\vec{k}_i' - \sum_{i=1}^{3}\vec{k}_i\Big)\frac{1}{(2\pi)^6}\frac{g^2}{4M^2}\frac{H(\vec{Q}_1^2)}{\vec{Q}_1^2+\mu^2}\frac{H(\vec{Q}_2^2)}{\vec{Q}_2^2+\mu^2}(\vec{\sigma}_1\cdot\vec{Q}_1)(\vec{\sigma}_2\cdot\vec{Q}_2)$$
$$\times\{\vec{\tau}_1\cdot\vec{\tau}_2[-a+b\vec{Q}_1\cdot\vec{Q}_2-c(\vec{Q}_1^2+\vec{Q}_2^2)]-d(\vec{\tau}_3\cdot\vec{\tau}_2\times\vec{\tau}_1)(\vec{\sigma}_3\cdot\vec{Q}_2\times\vec{Q}_1)\}. \tag{3}$$

The coordinate representation of $W(123)$ is given by Ishikawa *et al.*[19]

The results of our calculations of E_3 for the form factor of Eq. (1) with the cutoff parameter $\Lambda = 800$ MeV are summarized in Table II. Comparing with previous calculations for RSC-5,[8,9] we see a satisfactory agreement between our value (−0.47 MeV) and the value by Wiringa *et al.* (−0.41 MeV).[9] The eighteen-channel calculation yields −0.89 (−0.50 and −0.67) MeV for RSC (URG and Paris). As a result, the calculated triton binding energy amounts to 8.13 (8.00 and 8.23) MeV for the RSC (URG and Paris) potential. These values are very close to the experimental value of 8.482 MeV, giving a bright prospect for further study of triton.

It is interesting to see what makes the difference between the five-channel and eighteen-channel calculations. In Table III, A_i ($i = 1, 2, 3$, and 4) denote the contributions to E_3 from the terms multiplied by $-a$, b, $-c$, and $-d$, respectively, of Eq. (3). Comparing the RSC-18 with RSC-5 (or RSC-18 → 5 with RSC-5 → 5) in this table, we see that the repulsive effect in the five-channel calculation is due to the term A_3. This effect is reduced in the eighteen-channel calculation. Comparing RSC-18 with RSC-18 → 5 (or RSC-5 with RSC-5 → 5), we see that small components give an attractive effect through the term A_2. Both of these effects make $|E_3|$ increase.

In Table IV, we list values of $E^{(3)}_{\alpha\alpha'} + E^{(3)}_{\alpha'\alpha}(1 - \delta_{\alpha\alpha'})$, where $E_3 = \sum_{\alpha\alpha'} E^{(3)}_{\alpha\alpha'}$, α denoting a spin-isospin-angular state given in Table I of Bömelburg.[8] In Table IV, we list the matrix elements which are larger than 0.1 MeV in absolute value for either one of RSC-5 or RSC-18. The main difference in the eighteen-channel and the five-channel cases is seen in the terms $(2,3)+(3,2)$ and $(2,4)+(4,2)$: In the eighteen-channel calculations, these two contributions cancel out, while in the five-channel calculations these terms yield a repulsive effect of 0.46 MeV. This difference is reflected in E_3 as seen in Table II. For other two-nucleon potentials, these contributions do not cancel completely in the eighteen-channel case, but the sum of these two states is smaller than that of the five-channel case.

TABLE I. Triton binding energies for three two-nucleon potentials in the five- and eighteen-channel calculations.

Two-nucleon potential	Binding energy (MeV) Five-channel	Eighteen-channel
RSC	7.03	7.24
URG	7.48	7.50
Paris	7.48	7.56

Another important remark from Table IV is that the sum Σ of the dominantly contributing terms for the eighteen-channel case is −0.24 MeV, which is only about 27% of the total value; $E_3 = -0.89$ MeV. The difference −0.65 MeV is due to the sum of terms, each of which is less than 0.1 MeV in absolute value. This is a result of slow convergence of the partial wave expansion.

Here we should make a remark about the agreement and the disagreement between Ref. 8 and the present work. The last two rows in Table III show that RSC-5 → 5 values by Bömelburg[8] and the present work almost agree with each other. On the other hand, the E_3 value of RSC-5 is −0.16 MeV by Bömelburg and Glöckle,[8] while −0.47 MeV by us (Table II). This shows that this difference was caused in the course of the coordinate transformation that results in small components. (For the coordinate transformation, see the caption for Table II.) In fact, for instance, Table IV gives a number −0.24 for $E_{2,13}+E_{13,2}$, while the corresponding value in Ref. 8 is 0.183. We see some such differences for a number of other small channels. In this

TABLE II. First-order perturbation calculations to the triton binding energy of the Tucson-Melbourne potential,[4] with $\Lambda = 800$ MeV, for various triton wave functions. The wave function Ψ in Eq. (2) is a sum of the Faddeev components $\Psi(ij,k)$: $\Psi = \Psi(12,3) + \Psi(23,1) + \Psi(31,2)$. In RSC-5 (-18), we take five (eighteen) channels for each component and solve the Faddeev equation. Then the components $\Psi(23,1)$ and $\Psi(31,2)$ are expressed in terms of the coordinates (12,3). This coordinate transformation results in an infinite number of channels for these components, but only eighteen (eighteen) channels are retained in the calculation of Eq. (2).

Two-nucleon Potential	E_3(MeV) Five-channel	Eighteen-channel
RSC	−0.47	−0.89
URG	−0.07	−0.50
Paris	−0.15	−0.67

regard, we think that our calculation is to be preferred, since our values of RSC-5 almost agree with Wiringa *et al.*[9] who did not project the total wave function onto a truncated basis set.

In Ref. 7, all terms with odd L values were not taken into account after the coordinate transformation. This resulted in a rather small value for RSC-5.

The strong sensitivity of E_3 to the two-body potential, the shape of the form factor, and the cutoff parameter is seen in Table V. With the values in this table, we will be able to obtain a "correct" binding energy of triton with any eighteen-channel calculation, if we choose the form factor and the cutoff parameter suitably. However, before we get

TABLE IV. Dominant contributions to E_3 for RSC-5 and RSC-18. The Tucson-Melbourne potential (Ref. 4) with $\Lambda = 800$ MeV is used.

α	α'	$E^{(3)}_{\alpha\alpha'} + E^{(3)}_{\alpha'\alpha}(1-\delta_{\alpha\alpha'})$ (MeV) RSC5	RSC18	Difference
1	1	−0.15	−0.15	0.00
1	2	0.14	0.14	0.00
1	3	−0.18	−0.22	−0.04
1	4	−0.23	−0.20	0.03
1	5	0.20	0.20	0.00
1	9	−0.34	−0.28	0.06
1	10	0.17	0.17	0.00
1	17	−0.11	−0.11	0.00
2	2	−0.14	−0.14	0.00
2	3	−0.96	−1.11	−0.15
2	4	1.42	1.11	−0.31
2	8	0.47	0.38	−0.09
2	9	0.28	0.21	−0.07
2	12	−0.21	−0.18	0.03
2	13	−0.24	−0.24	0.00
3	8	0.11	0.07	−0.04
8	13	0.14	0.11	−0.03
Sum Σ		0.37	−0.24	−0.61
E_3		−0.47	−0.89	−0.42

a reliable triton binding energy, we should find a reasonable (phenomenological) form factor by extensive analyses of other phenomena or by a more fundamental theory as a quark model.

In the present paper, we have reported the result with the Tucson-Melbourne potential. We got similar results for the Ueda potential,[5] which will be published elsewhere.[20]

Our conclusion in the present paper is that small

TABLE III. Contribution to E_3 from each term of Eq. (3). For A_i ($i = 1,2,3,4$), see the text. The row marked as RSC-18 (RSC-5) shows the values calculated as in the caption for Table II. The row marked as RSC-18 → 5 (RSC-5 → 5) shows the values obtained in the following manner: After eighteen- (five-) channel Faddeev calculations are performed, the coordinate transformations as described in the caption for Table II are done for the Faddeev components. Then, five (five) channels are retained in the calculation of Eq. (2). The values in the last row are due to Bömelburg (Ref. 8).

	A_1	A_2	A_3	A_4	Sum
RSC-18	0.05	−0.97	0.25	−0.22	−0.89
RSC-18 → 5	0.05	−0.54	0.26	−0.08	−0.31
RSC-5	0.07	−1.06	0.63	−0.11	−0.47
RSC-5 → 5	0.08	−0.54	0.69	−0.08	0.15
RSC-5 → 5	0.091	−0.579	0.813	−0.123	0.203

TABLE V. Dependence of E_3 on the form factor, the cutoff parameter Λ, and the two-body potential. The Tucson-Melbourne potential (Ref. 4) is used for the three-body interaction. The form factor of row I is given by Eq. (1), while that of row II is given by $H(\vec{Q}^2)=(\Lambda^2-\mu^2)/(\Lambda^2+\vec{Q}^2)$.

	Λ (MeV)	600	800	1000
RSC-18	I	−0.17	−0.89	−1.84
	II	−1.42	−2.60	−3.56
URG-18	I	−0.02	−0.50	−1.23
	II	−0.99	−2.01	−2.93
Paris-18	I	−0.07	−0.67	−1.53
	II	−1.21	−2.35	−3.33

components are very important to obtain the triton binding energy. Beyond this statement, we will not be able to draw any further conclusion from the present level of calculations, for instance, even if we perform fifty-channel calculations. At this moment, we can only say that getting the triton binding energy is not hopeless. Before we draw any further conclusion, we should take account of three-body effects due to meson exchanges other than two-pion exchange, and solve the three-nucleon problem with full three-body interactions, although comparisons between the binding energy obtained from the first-order solution with three-body interactions and the expectation value for the same interactions show that the difference between these is rather small.[21,22]

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Note added.—During the refereeing of this paper, the Los Alamos group performed RSC-18 calculations. Dr. G. L. Payne has shown one of us (T.S.) that their calculation yields the same or very close values to ours everywhere. For instance, their value for $E_{2,13}+E_{13,2}$ is −0.246, the same as ours. Thus, we conclude that the calculations of Ref. 8 should be checked.

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