Coordinate Time On and Near the Earth

Cohen, Moses, and Rosenblum[1] have recently called attention to certain relativistic effects which must be considered when synchronizing clocks near the Earth. These effects are actually well understood, have been verified experimentally,[2,3] and are used by most time standards laboratories when comparing remote clocks, as well as in Global Positioning System (GPS) operations. It is preferable to think of such effects not as "synchronization errors," but as well-determined corrections which, within the conceptual framework of general relativity, can be applied to clock readings to allow the construction of a globally distributed, mutually self-consistent network of synchronized coordinate clocks.

Such corrections may be applied to standard clocks near the rotating Earth to generate a coordinate clock time equalling at each instant the coordinate time of general relativity. In the local inertial frame, tidal potential effects due to other solar system bodies can be shown to have negligible effects on clocks[3]; only the Earth's gravitational potential V needs to be explicitly considered. When we keep first-order corrections, and transform to a frame rotating with angular velocity $\vec{\omega}$, the metric may be written[3]

$$ds^2 = [1 + 2(\phi - \phi_0)/c^2](c\,dt')^2 - 2\vec{\omega}\cdot\vec{r}'\times d\vec{r}'\,dt' - [1 - 2V/c^2]\delta_{ij}\,dx'^i\,dx'^j. \tag{1}$$

The potential ϕ includes centrifugal effects, ϕ_0 is the value of ϕ on the geoid, primes denote quantities measured in the rotating frame, and $t' = (1 + \phi_0/c^2)t$ is a new time scale which takes advantage of the combination of effects causing standard clocks on the geoid to beat at equal rates.

For portable clocks having velocity $\vec{v}'$, Eq. (1) may be solved for $c\,dt'$ and integrated along the path of the clock to yield

$$c\Delta t' = \int ds[1 - (\phi - \phi_0)/c^2 + v'^2/2c^2 + \vec{\omega}\cdot\vec{r}'\times\vec{v}'/c^2]. \tag{2}$$

For synchronization along a path by means of electromagnetic (em) signals, ds vanishes. Solving (1) for the elapsed coordinate time gives

$$c\Delta t' = \int d\sigma'[1 - (\phi - \phi_0)/c^2 + \vec{\omega}\cdot\vec{r}'\times\vec{c}'/c^2], \tag{3}$$

where $d\sigma'$ is the increment of proper distance along the signal path, and $\vec{c}'$ is the velocity of the em signal pulse observed in the rotating frame. The right-hand sides of Eqs. (2) and (3) are expressed in terms of measurable or calculable quantities. With use of the corrections indicated in the above equations, the coordinate clocks which read t' may be consistently synchronized by either portable clocks or em signals.

A portable clock has been carried between Washington, D.C. and Paris to measure the Paris Observatory clock [UTC(OP)] against the U.S. Naval Observatory (USNO) master clock [UTC(USNO MC)]. At 0802 UT 10 September 1983, it was found that UTC(USNO MC) − UTC(OP) = 1.01 μs ± 0.035 μs,[4] with a relativistic correction of 23 ns. Time comparisons were concurrently derived from common-view[5] em measurements of GPS satellites, conducted between National Bureau of Standards (NBS), Boulder, and USNO, and between NBS Boulder and Observatoire de Paris. The geometry-dependent Sagnac correction from Boulder to Paris varies from 71 to 112 ns and that from Boulder to Washington, D.C. varies from 11 to 13 ns. Using the appropriate relativistic corrections for each common-view time difference obtained via GPS NAVSTAR 4, 5, and 6 satellites yields UTC(USNO MC) − UTC(OP) = 1.100 μs ± 0.02 μs. The agreement between the two methods of comparison of coordinate clocks is well within the measurement uncertainties, and is evidence that a coordinate-time clock network can be established near the Earth for which synchronizations using portable clocks and em signals agree.

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[1] J. M. Cohen, H. E. Moses, and A. Rosenblum, Phys. Rev. Lett. **51**, 1501 (1983), and **39**, 1641 (1977).

[2] R. F. C. Vessot *et al.*, Phys. Rev. Lett. **45**, 2081 (1980); C. O. Alley, in Proceedings of the Thirty-third Annual Frequency Control Symposium, Fort Monmouth, New Jersey, May, 1979 (unpublished). (Available from Electronic Industries Association, Washington, D.C.); E. J. Post, Rev. Mod. Phys. **39**, 475 (1967); F. Aronowitz, *Laser Applications,* edited by M. Ross (Academic, New York, 1971), Vol. 1, p. 134.

[3] N. Ashby and D. W. Allan, Rad. Sci. **14**, 649 (1979).

[4] L. G. Charron, private communication.

[5] D. W. Allan and M. A. Weiss, in Proceedings of the Thirty-fourth Annual Frequency Control Symposium, Fort Monmouth, New Jersey, May, 1980 (unpublished).