Comment on "Quantum Measurements and Stochastic Processes"

In a recent Letter,[1] Gisin has presented two interesting stochastic models to describe the dynamical reduction of the state vector. I wish to point out that, in these models, the state vector remains in a superposition for all finite times, i.e., it never completely reduces. This is not a desirable feature. More satisfactory stochastic models in this regard have previously been given.[2,3]

In the first model given by Gisin, the state vector describes a two-outcome experiment. The squared amplitudes multiplying the two states in the superposition are p and $1-p$, where p satisfies the stochastic differential equation

$$dp = 2p(1-p)\,d\alpha \tag{1}$$

[Eq. (9) in Ref. 1], where α is Brownian motion. Using the well-known method of Ito,[4] the ensemble of such solutions obeys the diffusion (Fokker-Planck) equation

$$\frac{\partial\rho}{\partial t} = 2\frac{\partial^2}{\partial p^2}p^2(1-p)^2\rho \tag{2}$$

[equivalent to Eq. (13) in Ref. 1] which I have discussed previously,[2] but rejected as a possible description of state-vector reduction for the reason given here.

The exact solution of Eq. (2) corresponding to the initial condition $\rho = \delta(p-p_0)$ at $t=0$ can be found [Eq. (2.3) in Ref. 2 or Eq. (14) in Ref. 1]. It consists of two peaks which travel toward $p=0$ and $p=1$. The areas under each peak are $1-p_0$ and p_0, respectively, as they should be for correct reduction. However, the peaks never reach $p=0$ or $p=1$. ρ vanishes at $p=0$ and $p=1$ for all $t \geqslant 0$: The peaks merely jam up closer and closer to these boundary points. In other words, each state vector in the ensemble is always in a superposition, and the reduction time for each state vector in the ensemble is infinite.

This "open boundary" behavior[4] occurs because of the strong singularity of the elliptic operator on the right-hand side of Eq. (2) at the boundary [$\sim p^2$ and $\sim (1-p)^2$]. The less singular "exit boundary" behavior[4] of the diffusion equation

$$\frac{\partial\rho}{\partial t} = \frac{\partial^2}{\partial p^2}p(1-p)\rho \tag{3}$$

satisfactorily describes the reduction process, with a finite mean reduction time, as I have shown.[2,3]

In the second model, in the simplest case of orthogonal projectors, the squared amplitudes $p_1, \ldots, p_n$ corresponding to an n-outcome experiment obey Eq. (16) of Ref. 1,

$$dp_k = 2p_k\left[\textstyle\sum_j p_j\,d\alpha_j - d\alpha_k\right] \tag{4}$$

where the α_j are independent Brownian motions. The diffusion equation in this case is readily found to be

$$\frac{\partial\rho}{\partial t} = 2\sum_{j,k}\frac{\partial^2}{\partial p_j\partial p_k}p_jp_k\left[\textstyle\sum_l p_l^2 - p_j - p_k + \delta_{jk}\right]\rho. \tag{5}$$

The simplest case of a two-outcome experiment will suffice. (The more general case behaves similarly.) If we change variables to $p = p_1$, $S = p_1 + p_2$, we obtain from (5)

$$\frac{\partial\rho}{\partial t} = 2\frac{\partial^2}{\partial p^2}p^2[(1-p)^2+(S-p)^2]\rho + 2\frac{\partial^2}{\partial S^2}(1-S)^2[S^2+2p(S+p)]\rho - 4\frac{\partial^2}{\partial p\partial S}(1-S)p[(S-p)^2-p(1-p)]\rho. \tag{6}$$

It is seen that $\rho = \delta(1-S)\bar{\rho}(p,t)$ is a solution of Eq. (6), so probability is conserved. The equation for $\bar{\rho}$

$$\frac{\partial\bar{\rho}}{\partial t} = 4\frac{\partial^2}{\partial p^2}p^2(1-p)^2\bar{\rho} \tag{7}$$

is identical in form to Eq. (2), so the reduction time is infinite for this model, too.

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