Phase Transition in a Dzyaloshinsky-Moriya Spin-Glass

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A mean-field model of a Heisenberg spin-glass with weak random anisotropy is solved. It is shown that an Ising-like phase transition in a finite magnetic field H occurs at $T_c(H) \sim T_c(0) - AH^{2/3}$ in the limit of small H. The crossover to an Ising behavior occurs when $D/J > (\mu H/J)^{2/3}$, where D and J are the anisotropy and exchange coupling constants, respectively.

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Spin-glass (SG) phase transitions occur, according to mean-field theory, also in the presence of a magnetic field H. In an Ising system, this transition occurs at the de Almeida–Thouless[1] (AT) line, which has the form

$$\tau_c \simeq (3h^2/4)^{1/3}, \tag{1}$$

where $\tau_c = 1 - T_c(H)/T_c(0)$, and $h \equiv \mu H/k_B T_c(0)$ is assumed to be small. The paramagnetic state $T > T_c(H)$ is a reversible phase whereas below $T_c(H)$ irreversibility and remanence set in.[2] In an isotropic m-component SG, the transition occurs along the Gabay-Toulouse[3] (GT) line which is

$$\tau_c \simeq h^2(m^2+4m+2)/4(m+2)^2 \tag{2}$$

for small h. Here, the line marks the freezing of degrees of freedom transverse to the field direction as well as the onset of irreversibility. Near the GT line the irreversibility is predominantly in the *local transverse* response. However, in an isotropic system it does not affect the *uniform* transverse response since there are no barriers to uniform spin rotations. On the other hand, the longitudinal irreversibility is weak near $T_c(H)$ and becomes "strong" only below a "crossover" temperature whose field dependence is similar to Eq. (1). Indeed, it has been proposed[4,5] that the AT-like lines which have been observed in numerous experiments[5,6] in vector SG's are not critical lines but strong-irreversibility crossover lines. Of course, it is also possible that the observed lines are finite-time effects which are not necessarily related to the mean-field transitions, as was recently demonstrated in simulations of two-dimensional SG models.[7]

So far, most of the discussions of the finite-field transitions have treated the vector SG's as isotropic systems. Spin-glasses, however, are known to have small amounts of anisotropy. In particular, Ruderman-Kittel-Kasuya-Yosida (RKKY) SG's contain weak randomly anisotropic Dzyaloshinsky-Moriya (DM) interactions.[8] Random anisotropy is expected to play an important role in the SG transitions, in particular, in the presence of a field. First, because of the random mixing of spin components, all of them are frozen as soon as a field is turned on. Thus, for sufficiently strong anisotropy a crossover to an Ising behavior is expected. Second, the random anisotropy couples the uniform transverse response to the local transverse irreversibility, thus making it possible, at least in principle, to measure the strong transverse irreversibility near $T_c(H)$.

In this Letter we study the effects of random anisotropy on the SG transition using an infinite-ranged Hamiltonian of an m-component SG with weak DM interactions,

$$H = -\tfrac{1}{2}\sum_{ij} J_{ij}\vec{S}_i\vec{S}_j - \tfrac{1}{2}\sum_{ij,\mu\nu} S_i^\mu D_{ij}^{\mu\nu} S_j^\nu - \mu H \sum_i S_i^z. \tag{3}$$

The interaction constants J_{ij} and D_{ij} are random variables with zero means and variances $\langle J_{ij}^2\rangle_r = J^2/N$ and $(D_{ij}^{\mu\nu})^2 = D^2/N$, where N is the total number of spins and $\langle\,\rangle_r$ denotes an average over the disorder. The spin variables are normalized as $|\vec{S}_i|^2 = m$. The matrix D_{ij} obeys $D_{ij}^{\mu\nu} = -D_{ij}^{\nu\mu}$ which is a generalization of the DM interactions in the Heisenberg $(m=3)$ case. The SG order is characterized by the Cartesian components of the Edwards-Anderson[9] (EA) order parameter Q^μ and the irreversible part of the local susceptibility[2] Δ^μ. If $H=0$, the transition from the paramagnetic phase ($Q=\Delta=0$) to the SG phase occurs at $T_g \equiv T_c(H=0)=[J^2+D^2(m-1)]^{1/2}$. If H and $D \neq 0$, *all* components of Q are nonzero *at all T*, and $T_c(H)$ is defined as the temperature below which ergodicity is broken giving rise to anomalously long relaxation times and nonzero Δ. We have calculated $T_c(H)$ and the properties near it. We limit ourselves to the case of $D/J \ll 1$, which is a reasonable assumption for most SG's, and also assume that $h \equiv \mu H/k_B T$, $d \equiv D/k_B T$, and $\tau \equiv 1 - T/T_g$ are small. Our main results are the following:

(1) The random anisotropy modifies significantly both the form and the nature of the finite-field transition, even when D is much smaller than J and therefore does not have much effect on the zero-field transition. In fact, *in the limit of* $h \to 0$, the transition has an Ising character[10] with

$$\tau_c \simeq [h^2(m+2)/4m]^{1/3} \tag{4}$$

for any fixed d. This result holds in *the range* $h^{2/3} \ll d$, which we term *the strong-anisotropy regime*. In this regime both Q^μ and Δ^μ are, to leading order, isotropic already near T_c, because of the *strong* mixing of the spin components by D, and their T and H dependence is similar to the Ising case.

(2) In the *weak-anisotropy limit*, $d \ll h^{5/2}$, the transition is essentially identical to the GT one, Eq. (2), with $q^z \gg q^y$, $\Delta^z \ll \Delta^y$ (z and y denote longitudinal and transverse directions, respectively) near the transition, as in the $D=0$ case. In the intermediate regime $h^{5/2} \ll d \ll h^{2/3}$ the degrees of freedom which trigger the transition are still mainly the transverse spin components implying that $\Delta^z \ll \Delta^y$ near T_c. However, the shape of the critical line is modified by D as will be described below.

(3) For all values of D, a macroscopic anisotropy constant K appears below $T_c(H)$ and gives rise to uniform transverse ireversibility which can be observed in transverse susceptibility or torque measurements. In the weak-anisotropy regime, K behaves *near the GT line as* $K \propto T_g d^2 h(\tau-\tau_c)^2$ and in the strong anisotropy limit as $K \propto T_g d^2(\tau^3-\tau_c^3)$. *In all cases K is coupled mainly to* Δ^y *and is not expected to exhibit further crossover at a lower temperature.*

Besides the obvious limitations of the mean-field approximation, the lack of a reliable estimate of the microscopic coupling constant D makes it difficult to compare the above results with experiments. In principle, the observed AT shape of the critical lines is consistent with the theory if the applied fields are sufficiently small so that the strong-anisotropy limit has been reached. However, on the basis of the available data on the low-T values of K it seems likely that the anisotropy in binary alloys, such as pure CuMn, is not large compared to fields of the order of a few kilogauss.[11] If this is indeed the case and the observed lines are only strong-irreversibility crossover lines then *measurements of the anisotropy constant K should yield a transition temperature which is higher in the presence of a field than that observed in "longitudinal" measurements.* Furthermore, this field-dependent temperature should be quite sensitive to the addition of a few hundred parts per millon of nonmagnetic impurities which increase[8] the DM anisotropy by orders of magnitude, although they do not affect T_g. It would be very interesting to test these predictions by accurate measurements of $K(T,H)$ in e.g., transverse susceptibility or torque experiments. We proceed to give more details of the theory.

We study model (3) by adding relaxational dynamics to the system and using the SG dynamic theory developed earlier.[2] We define the average local susceptibility and correlation functions,

$$\chi^{\mu\nu}(t-t') = \langle\langle(\partial S_i^\mu(t)/\partial H_i^\nu(t'))\rangle\rangle_r,$$

$$C^{\mu\nu}(t'-t) = \langle\langle(S_i(t)S_i^\nu(t'))\rangle\rangle_r,$$

where the inner brackets $\langle\,\rangle$ indicate an average over the thermal noise. As in the Ising case,[2] the low-T phase is characterized by a hierarchy of infinitely long relaxation times t_s, $s \in [0,1]$, with $t_s \gg t_p$ if $s < p$. This gives rise to time-dependent order parameters $Q^{\mu\nu}(s) \equiv C^{\mu\nu}(t_s)$ and $\chi^{\mu\nu}(s) = \chi^{\mu\nu}(\omega_s) = \chi^{\mu\nu}(1) + \Delta^{\mu\nu}(\omega_s)$ where 1 refers to the nonequilibrium (i.e., ac) limit, $\omega = \omega_1 = 1/t_1$. We use a coordinate system in which both Q and χ are diagonal, $Q^{\mu\nu} = q^\mu \delta^{\mu\nu}$, and similarly for χ^μ and Δ^μ. The order parameters are determined via the partition function $Z = \int D\vec{S} \exp(-\beta H_{\rm MF} + \beta\mu H \times S^z)$ where

$$-H_{\rm MF} = \sum_{\mu,\nu} A^{\mu\nu}\left[\frac{1}{2}(S^\mu)^2\chi^\nu(1) - S^\mu\int_0^1 ds\,[z^\nu(s) - m_s^\mu d\Delta^\nu/ds]\right], \tag{5}$$

and $A^{\mu\nu} = (J^2-D^2)\delta^{\mu\nu} + D^2$ and $\vec{z}(s)$ are random Gaussian vectors with variance

$$[\vec{z}(s)\vec{z}(p)] = \delta(s-p)\frac{d}{ds}[(J^2-D^2)(A^{-1}QA^{-1}) + D^2A^{-2}(\mathrm{Tr}Q)].$$

$\vec{m}_s$ is the partially averaged magnetization, $\vec{m}_s=[(\partial \ln Z/\partial\beta\vec{H})]_s$, and the order parameters are $Q^\mu(s)=[(m_s^\mu)^2]$ and $\chi^\mu(s)=\sum_\nu A_{\mu\nu}^{-1}[\delta m^\mu/\delta z_s^\nu]$. Here, the square brackets with and without subscript s denote the averages $\int_{p>s}\pi\, dz_p$ and $\int_p \pi\, dz_p$, respectively. In order to determine $T_c(H)$ and to obtain information about the low-T state, we expand the self-consistent equation for $\Delta(s)$ in powers of Δ, yielding to leading order the following equation:

$$\sum_\nu [\delta^{\mu\nu}-\chi_2^{\mu\nu}(s)-2\Delta^\mu(s)\delta^{\mu\nu}+O(\Delta^2)]A^{\nu\rho}d\Delta^\nu/ds=0, \qquad (6)$$

where $\chi_2^{\mu\nu}(s)$ is the average of the square of the *partially* averaged nonequilibrium susceptibility matrix,

$$T^2\chi_2^{\mu\nu}(s)\equiv\{([\langle S^\mu S^\nu\rangle]_s-[m^\mu m^\nu]_s)^2\}.$$

Substitution of $s=1$ in Eq. (6) yields the marginal stability condition

$$\det[(A^{-1})^{\mu\nu}-\chi_2^{\mu\nu}(1)]=0, \qquad (7)$$

which holds everywhere in the SG phase. The critical temperature $T_c(H)$ is the temperature at which the high-T phase $[Q(s)=Q,\ \Delta=0]$ satisfies Eq. (7). It was noted earlier[13] that, in the SG phase, one must distinguish between the *local* transverse response $\chi^y(s)$ and the *uniform* one, which we denote by $\chi^T(s)$. The former corresponds to a small rotation of the field on a *particular* spin while keeping intact the uniform field which acts on the rest of the system, and may develop long time tails even when $D=0$. On the other hand, χ^T is the response to a uniform rotation of the external fields which does not develop irreversibility *if* $D=0$. Thus, for all values of D, the equilibrium susceptibility $\chi^T(0)$ is equal to M/H, where $M=[\langle S^z\rangle]$, as a result of the *average* isotropy of the system. On the other hand,

$$\chi^T(1)\simeq M/H-\Delta^y(0)(1+HM_r/K)^{-1},$$

where M_r is the remanent magnetization $\sim H\Delta^z$ and K is the macroscopic anisotropy constant. Extending previous calculations[13] of K to finite fields yields

$$K=-D^2\int_0^1 ds\{[2q^y(s)+q^z(s)]d\Delta^y/ds+q^y(s)\,d\Delta^z/ds\}. \qquad (8)$$

Since K is coupled to the (strong) transverse irreversibility Δ^y, it is not expected to show a sharp crossover at lower temperatures.

Expanding Eq. (7) and the equations of state we have derived Eq. (4) for $d>>h^{2/3}$. In this regime,

$$\Delta^\mu(s)\simeq 3[q^2(1)-q^2(0)]/(m+2),$$

$q^\mu(0)\simeq[h^2(m+2)/4m]^{1/3}$, and $q^\mu(1)\simeq\tau$ independent of μ. These results as well as Eq. (4) agree, when $m=1$, with the Ising ones. In the weak-anisotropy regime $d<<h^{5/2}$ the results of the isotropic case $(D=0)$ are unchanged to leading order, giving $q^z\simeq h/\sqrt{2}$, $q^y(1)-q^y(0)\sim 1-T/T_c$, $\Delta^y(s)\propto[q^y(1)]^2-[q^y(0)]^2$, and $\Delta^z(s)\propto[q^y(1)^3-[q^y(s)]^3$. The anisotropy induces a nonzero value of $q^y(0)\propto(hd^2)^{1/3}$ which is very small compared to $q^z(0)$. Also, $\Delta^z<<\Delta^y$ near $T_c(H)$ and the two become comparable only below a crossover temperature $\tau^*\propto h^{2/3}$. In the intermediate regime, the zero eigenvector of Eqs. (6) and (7) is mostly in the transverse directions, implying that $\Delta^y>>\Delta^z$ near the transition. However, the anisotropy is sufficiently strong to modify the shape of the line. If $h^{5/2}<<d<<h$, then $\tau_c\propto(hd^2)^{1/3}$, whereas if $h<<d<<h^{2/3}$, we find $\tau_c\simeq d$ independent, to leading order, of h.

In conclusion, we point out that if the observed $\tau\propto h^{2/3}$ lines are finite-time effects,[7] then we expect that as the field becomes smaller the line will crossover to $\tau\propto h^2$ since the *finite-time* contours are most probably analytic in h. On the other hand, in the weakly anisotropic mean-field system the line becomes more singular as $h\to 0$. Thus, checking the present work's predictions by transverse-irreversibility measurement might shed light on the important question whether the observed critical lines are related to the mean-field transitions or not. Finally, we stress that although we have discussed here a specific kind of random anisotropy, the general features of the crossover to Ising properties are expected to hold also for other types of randomly anisotropic interactions (e.g., dipolar anisotropy) or even local anisotropy fields as long as they randomly mix all spin directions.

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