Vacuum-Field Rabi Splittings in Microwave Absorption by Rydberg Atoms in a Cavity

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The absorption spectrum of a system of N atoms interacting with a single mode of the quantized radiation field is exactly calculated. Such a spectrum shows vacuum-field Rabi splittings, and thus microwave absorption by Rydberg atoms in a cavity should be a useful way to observe these splittings.

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Rydberg atoms have provided one with the testing ground[1–3] for many of the predictions of the quantum electrodynamic effects in atoms contained in cavities. In particular, spontaneous emission from a single atom interacting with a single mode of the cavity has been shown to possess several remarkable features—for example, Kleppner and coworkers[3] have shown how one can inhibit spontaneous emission by selecting a suitable mode of the cavity. Sanchez-Mondragon, Narozhny, and Eberly have shown that the spontaneous emission spectra show a doublet structure which arises from vacuum-field Rabi oscillations.[4] The cooperative effects[5] in spontaneous emission make the structure of these spectra very complicated and thus might lead to difficulities in the observation of such spectra.

In this Letter I present an exact result for the absorption spectra of a system of N atoms contained in a cavity. The atoms interact with a single mode of the quantized radiation field[6] in the vacuum state. We are thus calculating the susceptibility of the system consisting of atom and field mode. The perturbation is provided by the external field acting on the system. The absorption spectra show a doublet structure. Such a doublet structure, in the case of exact resonance, corresponds to the vacuum-field Rabi oscillations. The results of the present investigation show that microwave absorption experiments on Rydberg atoms in a cavity should be useful in observing the fundamental effects of radiation-matter interaction, such as vacuum-field Rabi oscillations. The results of the present study also imply that the susceptibility of a quantum mechanical system contained in a cavity will undergo resonant modifications due to the interaction of the system with the modes of the cavity even if the cavity field is in the vacuum state.

The Hamiltonian[6] for a system of N two-level atoms of frequency ω_0 interacting with a single-mode radiation field of frequency ω is

$$H=\hbar\omega_0\sum_i S_i^z+\hbar\omega a^\dagger a+\sum_i(\hbar g S_i^+ a+\text{H.c.}). \tag{1}$$

Here g is the coupling constant between the atom and the cavity mode which is represented by annihilation and creation operators a and $a^\dagger$. Each two-level atom is characterized by the spin-$\frac{1}{2}$ operators $S_i^\pm$ and S_i^z. In terms of the collective variables $\vec{S}=\sum_i\vec{S}_i$, (1) becomes

$$H=\hbar\omega_0 S^z+\hbar\omega a^\dagger a+(\hbar g S^+ a+\text{H.c.}). \tag{2}$$

This system is initially in the ground state $|\psi_0\rangle$ with energy E_0:

$$|\psi_0\rangle=\left|\frac{N}{2},-\frac{N}{2};0\right\rangle,\quad E_0=-\frac{N}{2}\hbar\omega_0. \tag{3}$$

Here $|0\rangle$ represents the vacuum of the radiation field and $|S,M\rangle$ represents the eigenstate of S^2 and S^z. We will now calculate the quantum mechanical susceptibility of the system (1). Note that this is quite different from what one normally does as the vacuum state of the field is included as a part of the system. The Hamiltonian (2) gives the coherent interaction between the radiation field and the atom, i.e., it describes the exchange of energy between the atom and field. An important source of dissipation from the cavity is the leakage of photons at some rate κ. The leakage rate must not be too large, otherwise the incoherent process will dominate and the vacuum-field Rabi oscillations will not be seen. Therefore we will work in the limit $g\sqrt{N}>>\kappa$. The density matrix ρ for the combined atom-field system obeys

$$\frac{\partial\rho}{\partial t}=-\frac{i}{\hbar}[H,\rho]-\kappa(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a). \tag{4}$$

In terms of the eigenstates $|\psi_{\{i\}}\rangle$ (with energy $E_{\{i\}}$), the off-diagonal elements of ρ satisfy

$$\frac{\partial \rho_{\{i\}\{j\}}}{\partial t} = -\frac{i}{\hbar}(E_{\{i\}} - E_{\{j\}})\rho_{\{i\},\{j\}} - \kappa \sum_{\{k\},\{l\}} \Lambda_{\{i\}\{j\},\{k\}\{l\}}\rho_{\{k\},\{l\}}, \tag{5}$$

where the Λ's can be obtained from the structure of the incoherent term in (4). In what follows we do not need their explicit form. In view of the condition $g\sqrt{N} >> \kappa$ we can make the secular approximation[7] which leads to

$$\partial\rho_{\{i\}\{j\}}/\partial t = -(i/\hbar)(E_{\{i\}} - E_{\{j\}})\rho_{\{i\}\{j\}} - \Gamma_{\{i\}\{j\}}\rho_{\{i\}\{j\}}. \tag{6}$$

The diagonal elements of ρ in secular approximation satisfy the Pauli-type master equation.[7] The susceptibility can now be calculated by considering the interaction of the system with an external field of frequency Ω. Using Eq. (6) one can show[8] that the susceptibility tensor $\chi_{\alpha\beta}$, in terms of the eigenfunctions $|\psi_{\{i\}}\rangle$ and the eigenvalue $E_{\{i\}}$ of (2), is given by

$$\chi_{\alpha\beta}(\Omega) = \sum_{\{i\}} \frac{\langle\psi_0|(\vec{d}_\alpha S^+ + \text{H.c.})|\psi_{\{i\}}\rangle\langle\psi_{\{i\}}|\vec{d}_\beta S^+ + \text{H.c.}|\psi_0\rangle}{(E_{\{i\}} - E_0 - \hbar\Omega - i\hbar\Gamma_{\{i\},0})} + \text{terms with } \Omega \to -\Omega \tag{7}$$

$$= \vec{d}^*_\alpha \vec{d}_\beta N \sum_{\{i\}} \frac{|\langle N/2, -N/2+1;0|\psi_{\{i\}}\rangle|^2}{(E_{\{i\}} - E_0 - \hbar\Omega - i\hbar\Gamma_{\{i\},0})} + \text{terms with } \Omega \to -\Omega, \tag{8}$$

where $\vec{d}_\alpha$ represents the αth component of the dipole matrix elements $\vec{d}_{12}$. Thus the susceptibility can be calculated from the eigenfunctions[9] of H, which have been extensively studied. The eigenfunctions of H are classified by the eigenvalues of S^2; $S^z + a^\dagger a = C$. Thus in the evaluation of (8) we only need the eigenfunctions corresponding to $S = N/2$, $C = -N/2+1$. These eigenfunctions can be shown to have the simple structure[10]

$$\psi_+^{(S,C)} = \cos\theta|N/2, -N/2+1;0\rangle + \sin\theta|N/2, -N/2;1\rangle,$$
$$\psi_-^{(S,C)} = -\sin\theta|N/2, -N/2+1;0\rangle + \cos\theta|N/2, -N/2;1\rangle, \tag{9}$$

$$E_\pm^{(S,C)} = \hbar\omega_0(-N/2+1) - \hbar\Delta/2 \pm \tfrac{1}{2}\hbar(\Delta^2 + 4Ng^2)^{1/2}, \quad \tan\theta = \frac{-\Delta + (\Delta^2+4Ng^2)^{1/2}}{2q\sqrt{N}}, \quad \Delta = \omega_0 - \omega. \tag{10}$$

On substituting (9) and (10) in (8) and simplifying we find, for example, that the imaginary part of the susceptibility is given by[11]

$$\chi_{\alpha\beta}(\Omega) = (\pi N \vec{d}^*_\alpha \vec{d}_\beta/\hbar)\chi(\Omega),$$

$$\text{Im}\chi(\Omega) = \cos^2\theta \frac{\Gamma_-/\pi}{\Gamma_-^2 + \{\Omega - \omega_0 + \Delta/2 - \frac{1}{2}(\Delta^2+4Ng^2)^{1/2}\}^2} + \sin^2\theta \frac{\Gamma_+/\pi}{\Gamma_+^2 + \{\Omega - \omega_0 + \Delta/2 + \frac{1}{2}(\Delta^2+4Ng^2)^{1/2}\}^2}. \tag{11}$$

Note that the result (11) for the susceptibility holds to all powers in g as well as for an arbitrary number of atoms. The absorption spectrum which is proportional to $\text{Im}\chi(\Omega)$ follows a simple scaling law as far as the number of atoms is concerned, i.e., $g \to g\sqrt{N}$. The absorption spectrum has a doublet structure with peaks at

$$\Omega = \omega_0 + \Delta/2 \pm \tfrac{1}{2}(\Delta^2 + 4Ng^2)^{1/2}, \tag{12}$$

with weight factors depending on Δ, $g\sqrt{N}$. For large detunings, the usual peak at ω_0 is recovered. For the resonant case $\Delta = 0$, i.e., for the cavity mode on resonance with the atomic frequency, the peaks occur at

$$\Omega = \omega_0 \pm g\sqrt{N}, \tag{13}$$

with equal weights. The absorption spectra[12,13] show the characteristic vacuum-field Rabi splittings. The peaks can be resolved provided that the instrumental width is much smaller than $2g\sqrt{N}$. The simple structure of the susceptibility (11), which holds *both for single-atom and multiatom situations,* suggests that microwave absorption by Rydberg atoms in a cavity can be a simple and useful method of studying the vacuum-field Rabi oscillations.[14]

In conclusion I have shown how the susceptibility of a system interacting with the vacuum of the quantized radiation field carries useful information on the vacuum-field Rabi oscillations in a system which may or may not be cooperative. One expects similar effects in the nonlinear susceptibilities of such systems.

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[10]For simplicity the coupling constant g has been chosen real by a choice of the phase of the dipole matrix element.

[11]Here $\Gamma_\pm$ are the damping factors—Γ_+, for example, represents the damping in the equation of motion for the off-diagonal element $\langle\psi_0|\rho|\psi_\mp^{S,C}\rangle$. The explicit forms of $\Gamma_\pm$ are not important for the argument of this paper.

[12]The simplicity of the absorption spectra is worth noticing. The emission spectra (see Agarwal, Ref. 5), for example, for the case of two initially excited atoms, consist of lines $\omega-\omega_0=\pm g\sqrt{2},\ \pm(\sqrt{3}\pm1)\sqrt{2}g$ at resonance.

[13]In the dressed-atom language (Ref. 7), the absorption spectra probe the dressed states of a system of N atoms. The "dressing" here is being done by the vacuum of the cavity field.

[14]Our study shows that it is not only simpler to have more atoms in the cavity but also advantageous as then the splitting of the two peaks is large.