Monte Carlo Renormalization Group for SU(3) Lattice Gauge Theory

R. Gupta
Physics Department, Northeastern University, Boston, Massachusetts 02115

and

G. Guralnik
Los Alamos National Laboratory, Los Alamos, New Mexico 87545

and

A. Patel
California Institute of Technology, Pasadena, California 91125

and

T. Warnock
Cray Research Inc., Los Alamos, New Mexico 87545

and

C. Zemach
Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 16 July 1984)

We present the first results for the nonperturbative β function along the Wilson axis for the SU(3) pure gauge theory using the Monte Carlo renormalization-group method. The results show that there is no asymptotic scaling for the coupling in the fundamental representation $K_F < 6.1$. The renormalized action generated by the $\sqrt{3}$ block transformation is also estimated for use in future calculations.

PACS numbers: 11.15.Ha, 11.10.Gh, 11.30.Jw

After the initial qualitative success of Monte Carlo simulations in deducing the long-distance properties of QCD from first principles, the focus has shifted to detailed comparison of the numerical results with the experimental data. To extract the continuum properties, one needs to know how a particle mass, measured in units of the lattice spacing a, changes with the coupling g_{bare}, i.e., how the physical quantities scale. For an asymptotically free field theory, this corresponds to knowing the β function. The two-loop perturbative result has been naively used so far, even though the simulations have been done at $g_{\text{bare}} \sim 1$. There is little reason to believe that at such large couplings corrections due to higher-order terms, irrelevant lattice operators, and nearby phase structure in the extended coupling constant space are negligible. With future Monte Carlo calculations in mind, we have calculated the nonperturbative β function at $K_F = 6.25$, 6.5, and 7.0 for the simple plaquette action. We also estimate the renormalized action generated by the $\sqrt{3}$ block transformation in a six-coupling space.

The Monte Carlo renormalization-group (MCRG) method,[1] as applied to SU(3), is identical to the SU(2) calculation and is described in detail by Patel and co-workers.[2,3] The geometry of the $\sqrt{3}$ block transformation is explained by Patel and co-workers[2,3] and Cordery, Gupta, and Novothy.[4] The method to find the renormalized action is discussed by Patel[3] and Gupta and co-workers.[5] We direct the readers unfamiliar with these methods or those interested in the details to these references.

The operators (with the corresponding couplings defined in parentheses) used in the matching of the block expectation values and in the improved action were the simple plaquette U_p in the **3** (K_F), **6** (K_6), **8** (K_A), **10**, **15**, and **15′** representations, and the rectangular, L-shaped, and twisted six-link operators designated as U_{6p} (K_{6p}), U_{6l} (K_{6l}), and U_{6t} (K_{6t}), respectively. The SU(3) action in the $[K_F, K_6, K_A, K_{6p}, K_{6l}, K_{6t}]$ space is defined to be

$$S = \mathrm{Re}\{K_F \sum \mathrm{Tr}\, U_p + K_{6p} \sum \mathrm{Tr}\, U_{6p} + K_{6l} \sum \mathrm{Tr}\, U_{6l} + K_{6t} \sum \mathrm{Tr}\, U_{6t} + K_6 \sum [\tfrac{3}{2}(\mathrm{Tr}\, U_p)^2 - \tfrac{1}{2}\mathrm{Tr}\, U_p] + K_A \sum (\tfrac{9}{8}|\mathrm{Tr}\, U_p|^2 - \tfrac{1}{8})\}. \quad (1)$$

Here the higher representations have been constructed from U_p, all the traces are normalized to unity, and the sums are over all sites and positive orientations of the loops. For the simple plaquette action $K_F \equiv 6g^{-2}$, the asymptotic scaling is defined by the two-loop perturbative β function

$$\frac{\partial(g^{-2})}{\partial(\ln a)} = -\frac{11}{8\pi^2} - \frac{51}{64\pi^4}g^2 + \ldots . \quad (2)$$

The quantity we calculate using MCRG is

$$\Delta K_F = -[\Delta(6g^{-2})/\Delta(\ln a)]\ln\sqrt{3}, \quad (3)$$

i.e., the discrete β function at K_F evaluated for a scale change of $\sqrt{3}$.

In Wilson's MCRG method, ΔK_F is calculated by matching the long-distance behavior of the two theories defined by K_F and $K_F - \Delta K_F$. In this matching process, the theory starting at K_F has been renormalized one more time than the theory starting at $K_F - \Delta K_F$, so that the two starting theories differ by the scale factor $b = \sqrt{3}$ of the renormalization-group transformation. After matching, the two theories flow together along the renormalized trajectory. To control the finite-size effects, the comparison of the block expectation values is made on the same physical size lattice, i.e., the starting lattice at K_F is chosen to be b times larger. Even then residual finite-size effects are present as a result of incomplete convergence of actions at the level of comparison. To provide conviction that the two theories have converged, the starting lattice should be large enough and the starting action close to the renormalized trajectory to observe matching at a few different levels—three in our calculation.

The advantages of the "$\sqrt{3}$ block transformation" are that it has a small scale factor, it uses the maximum (six out of seven) number of degrees of freedom in the construction of each block link, and it does not require gauge fixing. The body diagonals of the four positively oriented three-dimensional (3D) cubes, associated with a given point on a 4D hypercubic lattice, form the orthogonal basis vectors for the block lattice. The block link is constructed from the six topologically equivalent paths connecting the nearest-neighbor block sites. Their sum Σ has the same gauge transformation properties as the individual paths. The block link variable U^1 is defined as the projection of Σ back onto the group manifold, i.e., the matrix which maximizes $\mathrm{Re\,Tr}(\Sigma^\dagger U^1)$. This definition preserves local gauge invariance. To construct U^1 we note that by polar decomposition $\Sigma = U \times VDe^{i\phi}V^\dagger$, where D is a positive definite diagonal matrix and U and V are SU(3) matrices. Now the diagonal matrix $X = V^\dagger U^\dagger U^1 V$ that maximizes $\mathrm{Re\,Tr}(De^{-i\phi}X)$ can be expressed in terms of a single phase.[3] This phase was solved for numerically by the Newton-Raphson method. We found that for $K_F > 6.0$, the phase ϕ is almost zero and U^1 is approximately the same as U. At $K_F = 6.0$, the block expectation values were smaller by $\sim (1\text{–}2)\%$ when U was used as the block link variable instead of U^1. The difference decreased as K_F was increased. The subtlety is not important in the calculation of the β function since the same approximation is used for both the lattices.

The update of the pure gauge theory was done with a twenty-hit Metropolis algorithm. The first 500 sweeps were discarded to ensure thermalization. Thereafter, the block lattices were constructed every fifteenth (tenth) sweep on the 9^4 $[(3\sqrt{3})^4]$ lattice. The different values of couplings and the statistics are shown in Table I. We exploit-

TABLE I. The values of ΔK_F at different levels of matching for different values of the couplings. The statistics for each run are given in parentheses. ΔK_F was determined by linear interpolation and the errors are based on a 1σ fit. Also shown is the value of ΔK_F corresponding to asymptotic scaling.

9^4 lattice K_F (conf.)	$(3\sqrt{3})^4$ lattice K_F (conf.)	ΔK_F for matching on 3^4	$(\sqrt{3})^4$	1^4	Two-loop ΔK_F
7.0 (593)	6.51 (1650)	0.43	0.50	0.48	0.484
	6.54 (1800)	±0.005	±0.015	±0.02	
6.5 (593)	6.03 (1650)	0.43	0.46	0.45	0.487
	6.06 (1650)	±0.005	±0.005	±0.01	
6.25 (680)	5.82 (1650)	0.41	0.42	0.40	0.488
	5.85 (1650)	±0.005	±0.005	±0.01	

ed the freedom in the choice of the block site to gain a large factor in the statistics. This was done by summing over all possible constructions of the 3^4 (9 such), $(\sqrt{3})^4$ (81 such), and 1^4 (729 such) block lattices.

The results for ΔK_F are presented in Table I and Fig. 1. The detailed matching at $K_F = 6.25$ is shown in Table II. The significant feature is the dip in the β function at $K_F = 6.25$. This is a likely consequence of the repellent phase structure in the extended $[K_F, K_A]$ plane. The flow from $g_{\text{bare}} = 0$ to $g_{\text{bare}} = \infty$ slows down on the weak-coupling side of the crossover region, and accelerates once it is past the phase structure. The same feature was observed for the SU(2) theory.[2,3] Our result at $K_F = 7.0$ is consistent with asymptotic scaling and we can clearly see that no asymptotic scaling occurs for $K_F < 6.1$. This confirms earlier conclusions based on the calculations of the string tension and 0^{++} glueball mass and the deconfining transition temperature.[6] In calculating ΔK_F, we have effectively held the string tension fixed (matched Wilson loops). There is a large difference in the strong-coupling "β functions" obtained by holding the string tension and the 0^{++} glueball mass fixed (see Fig. 1). A universal β function can be defined only when all physical quantities scale in the same fashion. This highlights the need to determine the point of onset of scaling (i.e., when all the mass ratios become independent of K_F), in order to obtain reliable continuum results.

The multicoupling renormalized action $\{K^{A^1}\}$ was calculated by comparing the expectation values Ω_i on the once blocked $(3\sqrt{3})^4$ lattice with those on a $(3\sqrt{3})^4$ lattice updated with coupling $\{K^B\}$. To first order,

$$\langle \Omega_i \rangle_B \sim \langle \Omega_i \rangle_{A^1} - \langle :\Omega_i \Omega_j: \rangle_{A^1} (K_j^{A^1} - K_j^B), \quad (4)$$

and

$$\langle \Omega_i \rangle_{A^1} \sim \langle \Omega_i \rangle_B + \langle :\Omega_i \Omega_j: \rangle_B (K_j^{A^1} - K_j^B), \quad (5)$$

where $\langle :\Omega_i \Omega_j: \rangle$ is the connected correlation function. The values of $\{K^B\}$ were selected on basis of the consistency check, Eqs. (4) and (5). The two independent estimates of $\{K_j^{A^1}\}$ were compared to ensure the validity of the linear extrapolation, and we used the mean $\{K_j^{A^1}\}$, which is correct to second order. As can be seen from Table III, the results are very much dependent on the number of operators kept. Also, the contributions of the six-link loops and the higher representations are not small. By comparing these results with the SU(2) results,[2,3,5] we find that the various operators have

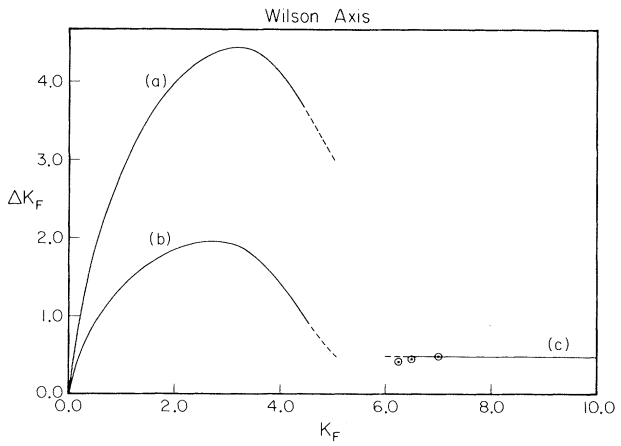

FIG. 1. Comparison of the measured ΔK_F with the strong-coupling (a,b) and the two-loop perturbative result (c). The strong-coupling results were obtained by keeping the string tension (a) and the 0^{++} glueball mass (b) fixed. The statistical errors are smaller than the size of the dots.

TABLE II. The matching of the block expectation values for $K_F = 6.25$. The statistical errors in the parentheses were estimated by binning the data into sets of 50 (34) configurations for the $(3\sqrt{3})^4$ (9^4) lattices, since the maximum autocorrelation length, defined by the autocorrelation coefficient being 0.1, was 10 on the 1^4 lattices.

Lattice Size	Operator	$(3\sqrt{3})^4$ lattice K_F=5.82	9^4 lattice K_F=6.25	$(3\sqrt{3})^4$ lattice K_F=5.85
9^4	plaq		.61825(06)	
	6p		.41640(09)	
	6l		.45111(09)	
	6t		.41017(11)	
	6		.34442(07)	
	8		.30703(07)	
$(3\sqrt{3})^4$	plaq	.5745(2)	.5173(2)	.5785(2)
	6p	.3592(3)	.2834(3)	.3645(3)
	6l	.3942(2)	.3519(2)	.3994(2)
	6t	.3496(3)	.3314(3)	.3553(3)
	6	.2953(2)	.2288(2)	.2994(2)
	8	.2593(2)	.1946(2)	.2633(2)
3^4	plaq	.4360(5)	.4415(5)	.4449(5)
	6p	.2018(6)	.2072(7)	.2105(6)
	6l	.2649(6)	.2691(6)	.2741(6)
	6t	.2445(6)	.2454(7)	.2538(6)
	6	.1583(4)	.1607(5)	.1651(4)
	8	.1296(4)	.1315(4)	.1358(4)
$(\sqrt{3})^4$	plaq	.3268(14)	.3326(17)	.3423(14)
	6p	.1179(12)	.1246(16)	.1301(12)
	6l	.1688(13)	.1766(17)	.1826(14)
	6t	.1566(14)	.1664(20)	.1703(15)
	6	.0844(07)	.0861(09)	.0930(08)
	8	.0648(06)	.0660(08)	.0720(06)
1^4	plaq	.3660(32)	.3873(46)	.3860(33)
	6p	.3270(17)	.3450(32)	.3365(16)
	6l	.2634(23)	.2824(33)	.2793(24)
	6t	.2346(33)	.2640(68)	.2509(33)
	6	.1332(21)	.1453(32)	.1458(24)
	8	.1205(19)	.1302(29)	.1310(21)

TABLE III. Projection of the renormalized SU(3) action for different starting actions along the Wilson axis. The errors in parentheses were estimated from the fluctuations of the results for different $\{K^B\}$ points.

Projection of the improved action for SU(3) in the $[K_F, K_{6p}, K_{6l}, K_{6t}, K_6, K_A]$ space					
Couplings	$K_F=6.25$	$K_F=6.5$		$K_F=7.0$	
K_F	7.31(7)	7.85(7)	4.8(1)	8.91(5)	5.3(1)
K_{6p}/K_F	−0.018(1)	−0.023(1)	−0.123(4)	−0.030(2)	−0.126(2)
K_{6l}/K_F			0.044(2)		0.047(2)
K_{6t}/K_F			0.094(2)		0.089(2)
K_6/K_F	−0.086(15)	−0.096(8)	−0.12(2)	−0.091(3)	−0.03(2)
K_A/K_F	−0.142(8)	−0.125(13)	−0.14(2)	−0.116(6)	−0.21(2)

the same signs and the qualitative behavior is very similar. The SU(2) calculation showed that it is mostly K_{6p} that is necessary to get rid of the leading irrelevant operator.[2,3] Hence, we strongly recommend that improved-action Monte Carlo calculations be done by means of the four-coupling action estimated in Table III.

We thank Cray Research Inc., the C Division of Los Alamos, and Geoffrey Fox for generous support. We especially thank W. Dissly and G. Spix of Cray Research Inc. for their help. This work was supported in part by the National Science Foundation under Grants No. PHY-83-05734 and No. PHY-82-15249 and by the Department of Energy under Contracts No. DE-AC02-76-ER03130.A012 and No. DE-AC03-81-ER40050.

[1]K. Wilson, in *Recent Developments in Gauge Theories,* edited by G. 't Hooft *et al.*, NATO Advanced Study Institute Series B, Vol. 59 (Plenum, New York, 1980); S. K. Ma, Phys. Rev. Lett. **37**, 461 (1976); L. P. Kadanoff, Rev. Mod. Phys. **49**, 267 (1977); R. H. Swendsen, Phys. Rev. Lett. **42**, 859 (1979), and in *Phase Transitions: Cargese 1980,* edited by M. Lévy *et al.*, NATO Advanced Study Institute Series B, Vol. 72 (Plenum, New York, 1982), and references therein; S. H. Shenker and J. Tobochnik, Phys. Rev. B **22**, 4462 (1980).

[2]A. Patel, R. Cordery, R. Gupta, and M. Novotny, Phys. Rev. Lett. **53**, 527 (1984); A. Patel and R. Gupta, California Institute of Technology Report No. CALT-68-1142, 1984 (unpublished).

[3]A. Patel, Ph.D. thesis, California Institute of Technology, 1984 (unpublished).

[4]R. Cordery, R. Gupta, and M. Novotny, Phys. Lett. **B128**, 425 (1983).

[5]R. Gupta and A. Patel, Phys. Rev. Lett. **53**, 531 (1984); R. H. Swendsen, in *Real Space Renormalization,* edited by T. W. Burkhardt and J. M. J. van Leeuwen, Topics in Current Physics, Vol. 30 (Springer-Verlag, New York, 1983).

[6]R. Gupta and A. Patel, Phys. Lett. **138B**, 296 (1984); D. Barkai, K. J. M. Moriarty, and C. Rebbi, Phys. Rev. D **30**, 1293 (1984); S. W. Otto and J. D. Stack, Phys. Rev. Lett. **52**, 2328 (1984); A. D. Kennedy, J. Kuti, S. Meyer, and B. J. Pendleton, Institute of Theoretical Physics, University of California, Santa Barbara, Report No. NSF-ITP-84-61, 1984 (unpublished).