Measurements of Flux-Flow and 1/*f* Noise in Superconductors

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Direct measurements of fluctuations associated with flux flow in superconductors are made with a superconducting quantum interference device. This method avoids the complicated problems concerned with voltage probe locations in earlier work, and allows an unambiguous determination of some characteristic noise power spectra in different current regimes. Models of fluxoid motion are proposed to account for the experimental results.

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Measurements of the flux-flow noise can provide information about the dynamics of fluxoid motion in superconductors.[1] Previous work on this subject has practically all followed the same traditional approach: After fluxoids had been set in motion under the influence of a transport current, noise voltages or power spectra were recorded at two potential contacts on the superconducting sample.[2] Despite many experimental and theoretical studies, this method has not yet yielded a consistent picture of the noise spectrum associated with fluxoid motion.[3] One complexity in these voltage measurements seems to arise from the fact that the noise power spectra are dependent on sample geometry,[4] especially the spacing between voltage probes. In this Letter we report measurements of the flux-flow noise by an entirely different method. With use of a superconducting quantum interference device (SQUID) to monitor *directly* the fluctuating component of flux flow, much of the complicated problems concerning voltage probe locations can be avoided, thereby allowing a simplified analysis of the measured noise spectra.

SQUID is the most sensitive magnetometer known to date; under proper shielding conditions, it can be arranged to respond mainly to the flux changes in its input detector loop. It seems natural that such a magnetometer should have been employed to investigate fluxoid motion; we are surprised that SQUID measurements have so far not been reported in the study of flux-flow noise. Samples used in our experiment are $In_{0.9}Pb_{0.1}$ alloy foils cold rolled to a thickness of 50 μm, and followed by annealing at 120 °C in air for 6 h. Typical samples are approximately 2 mm wide and 1.2 cm long. At the temperatures of our measurements, the κ value[5] is around 1.5. Magnetic field is generated by a small superconducting solenoid, usually run in a persistent mode for stability purposes. Current and voltage leads (8 mm apart) were attached to the superconducting strip for driving the fluxoids and for current-voltage (I-V) curve measurements. A rectangular Nb-wire superconducting detector loop of length L and width w was affixed on the sample surface by GE varnish; typical loop dimensions are $w = 0.2$ mm and $L = 4$ mm. Various dimensions of the detector loop have been used in the experiment to study its size dependence. The whole sample assembly was placed in a conventional cryostat with two concentric layers of Mumetal shield, and measurements were made in a screened room.

Measurements of the flux-flow noise were carried out by connecting the superconducting detector loop to a S.H.E. rf SQUID operated in the "fast" mode with no filter applied. The bandwidth of this instrument is from dc to 20 kHz. The SQUID output indicates *flux changes* in the detector loop due to fluxoid motion in the central portion of the superconductor sample beneath it. The absolute signal size is calibrated against a separate known flux source so that a change of a single flux quantum in the detector loop corresponds to an output signal of 2.2 mV with the "×10" setting used in our experiment. The orientation of the detector loop is such that fluxoids move across its width (normal to the length direction). The time-averaged dc component was not recorded. The ac component $\delta V(t)$ of the flux flow, which is believed to arise from the discreteness of the flux bundles and their interaction with the pinning centers as well as with one another, can be analyzed by using an oscilloscope, a spectrum analyzer, or an ac voltmeter connected to the SQUID output.

A representative ac voltmeter output as a function of the transport current is shown in Fig. 1. This broadband average ac voltage $\langle \delta V \rangle$ was obtained by use of a PAR 124A lockin amplifier as an ac voltmeter which measures within 12% accuracy of the root-mean-square ac voltage. Below the critical current I_c, fluxoids are not in motion and $\langle \delta V \rangle$ represents the intrinsic noise due to the SQUID and the measuring circuit. For a transport current between I_c and a somewhat loosely defined value I_Q, $\langle \delta V \rangle$ shows strong peaks, each with an abrupt

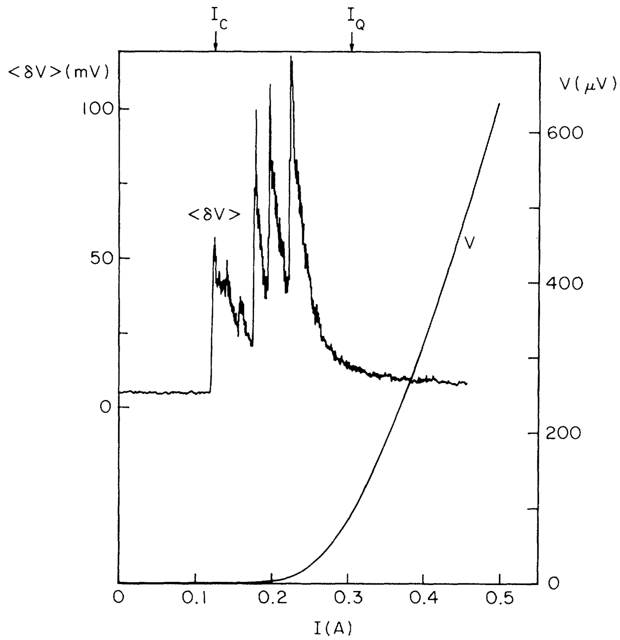

FIG. 1. A representative plot of $\langle \delta V \rangle$ vs transport current I, obtained with $H = 227.5$ Oe and $T = 3.672$ K. I_c marks the critical current at the onset of flux flow. I_Q separates the near-onset region from the quasilinear region. A V-I curve is included for comparison.

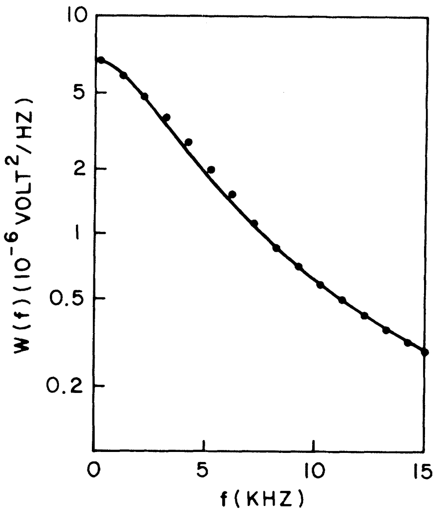

FIG. 2. Typical noise power spectral density in the quasilinear region, obtained with $I = 0.53$ A, $H = 325$ Oe, $T = 2.99$ K. Dots are data points and the curve is a fit of the form $W(f) = A[1 + (\pi f/f_c)^2]^{-1}$ with $f_c = 10.3$ kHz.

jump at the leading edge and followed by a rapid decrease. For current greater than I_Q, $\langle \delta V \rangle$ decreases smoothly to a low value. Also, for comparison sake, we have included in Fig. 1 the dc voltage measured with a Keithley nanovoltmeter connected directly to the potential leads of the sample. It is clear that $\langle \delta V \rangle$ measured via a SQUID gives a sharply defined value of I_c, indicating the onset of flux flow. We therefore call the interval between I_c and I_Q the "near onset" region. In contrast, the region with $I > I_Q$ is characterized by a smooth variation of $\langle \delta V \rangle$ and a nearly linear I-V relation ($d^2V/dI^2 \geqslant 0$); this is called the "quasilinear" region. Each region shows a generic noise power spectrum reflecting its flow characteristics as discussed below.

In the quasilinear region, the noise power spectral density $W(f)$ can be fitted very well by the form $A[1 + (\pi f/f_c)^2]^{-1}$; a typical curve is shown in Fig. 2. The characteristic frequency f_c determined from this fit can then be used to investigate the flux-flow phenomena when parameters such as temperature, magnetic field, current density, and detector loop dimensions are varied.[6] For a given magnetic field H and temperature T, f_c is found to vary linearly with $J - J_0$, where J is the current density and J_0 is the current density corresponding to I_c. Also, f_c at a constant value of $J - J_0$ increases with increasing H or T. It is interesting to note that f_c is *independent* of the width w of the detector loop but varies linearly with the detector length L.

Our observations do not support the shot-noise model suggested by van Gurp[1] based on voltage pulses resulting from the transit of independent flux bundles across the width of the specimen. To account for the results of the present experiment, we propose a picture[6] for the quasilinear region that a whole array of randomly distributed flux bundles moves in the superconductor extending from one side of the sample to the other.

A flux bundle entering (leaving) the detector loop gives rise to a sharp voltage increase (decrease) at the SQUID output. If an entering event is followed by a leaving event, a voltage pulse will be formed. These pulses have random widths because the leaving and entering events are uncorrelated. The characteristic frequency f_c is therefore indicative of the product of velocity and density of quasi-randomly distributed moving flux bundles. This picture is consistent with the time dependence of $\delta V(t)$ in the quasilinear region observed on an oscilloscope where a large fraction of the random voltage pulses actually shows a more or less constant amplitude (hence a low value of $\langle \delta V \rangle$). In fact, if we assume as a first approximation that those pulses with equal amplitude V_N are random-width square-wave pulses occurring at random time intervals, by calculation of the autocorrelation function and application of the Wiener-Khintchine theorem, the noise power density $W(f)$ can be shown[7] to take the form $A[1+(\pi f/f_c)^2]^{-1}$ with $A=2V_N^2/f_c$. The variations of A with temperature and magnetic field have also been studied in our experiment, and they will be reported elsewhere.[6] Our model is somewhat similar to the chain model[8] in which the fluxoids are assumed to move along the same path in a chain across the width of the specimen. Simultaneous operation of randomly distributed chains normal to the transport current direction will lead to a result also consistent with our data. It is reasonable that f_c is independent of w because the transit time of an isolated flux bundle does not appear in this model. The linear variation of f_c with L can be understood by considering the increase in f_c with increasing width of the moving pattern (or number of chains).

In the near-onset region, however, the noise power spectrum is more complicated and it is largely determined by the value of magnetic field and current. When the transport current I is higher than I_c and set at a value corresponding to one of the peak positions of $\langle \delta V\rangle$, the power spectrum exhibits $1/f$ noise; on the other hand, if the current I is set at values for the minima in $\langle \delta V\rangle$, the power spectra deviate appreciably from the $1/f$ behavior, as illustrated in Fig. 3(a).

In the sample under study there could be several kinds of pinning center such as bulk defects, grain boundaries, and surface irregularities. Each sharp jump in the $\langle \delta V\rangle$-I plot indicates that some flux bundles have just started to depin from one kind of pinning centers. As a result of interactions with these different kinds of pinning sites, flux bundles depinned at different current values therefore can move with different velocities and can also have different densities. The picture used before to describe the quasi-steady-state flow pattern in the quasilinear region is invalid here. This velocity spread also explains why $\langle \delta V\rangle$ can assume a large value in the near-onset region, as supported by a direct observation of the time dependence of $\delta V(t)$ on an oscilloscope where many pulses tend to overlap, giving rise to sizable variations in the measured amplitude of the ac component.

To understand the $1/f$ noise shown in Fig. 3(a), we consider a distribution of f_c (due to different flux bundle velocities or different densities, or both) in the spectrum as determined before, and the noise power spectral density can be calculated from

$$W(f)=\int g(\tau)\tau[1+(\pi f\tau)^2]^{-1}d\tau, \qquad (1)$$

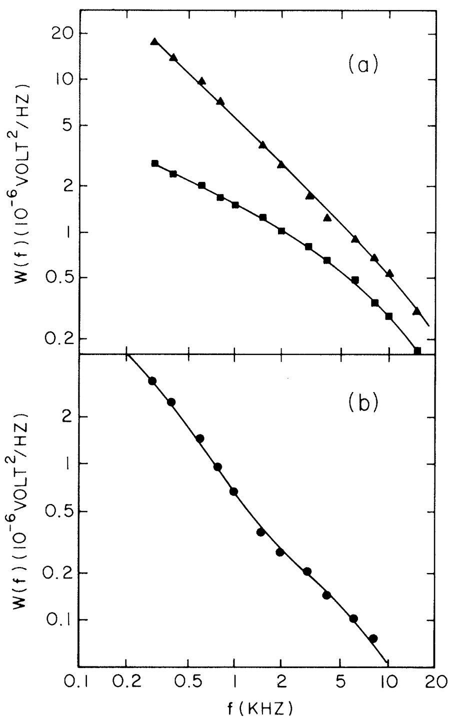

FIG. 3. (a) Typical noise power spectral density in the near-onset region; $H=52$ Oe, $T=4.12$ K. The upper curve shows $1/f$ noise, obtained with I set at a maximum of $\langle \delta V\rangle$. The lower curve was obtained with I at a minimum of $\langle \delta V\rangle$. Dots are data points, and lines are drawn to guide eyes. (b) $1/f$ noise and small deviations obtained with I set at a sharp $\langle \delta V\rangle$ maximum in the near-onset region. $H=182$ Oe, $T=4.03$ K. Dots are data points, and the curve is a fit obtained by use of Eqs. (1) and (2) with $\tau_1=5\times10^{-5}$ sec, $\tau_2=8\times10^{-5}$ sec, $\tau_3=6\times10^{-4}$ sec, and $\tau_4=4\times10^{-3}$ sec.

where $\tau=1/f_c$. If the distribution function $g(\tau)$ is assumed to take the form of $g(\tau)=\text{const}/\tau$ in the interval $\tau_1\leq\tau\leq\tau_2$, and $g(\tau)=0$ elsewhere, by an appropriate choice of the parameters τ_1 and τ_2, $W(f)$ can be fitted to a $1/f$ shape. This distribution function is physically plausible in view of the influence of the transport current and interactions with the pinning centers which determine the non-steady-state flow of fluxoids in the near-onset region. This concept can be further tested by an attempt to account for the small deviations actually observed within the frequency interval where the spectrum is essentially $1/f$, as illustrated in Fig. 3(b). In order to explain the observed small deviations in the middle of the $1/f$ noise spectrum, we have considered two ranges of distributions as follows:

$$g(\tau)=\begin{cases}\text{const}/\tau, & \text{for } \tau_1 \leqslant \tau \leqslant \tau_2 \text{ and } \tau_3 \leqslant \tau \leqslant \tau_4, \\ 0, & \text{otherwise.}\end{cases} \quad (2)$$

By choosing appropriate values for the τ_i's, the small deviations can be fitted quite satisfactorily by using Eqs. (1) and (2). The results are shown in Fig. 3(b) and demonstrate that the observed $1/f$ behavior as well as this modulation in the noise spectra are consistent with the velocity-distribution model suggested here for the near-onset region.

In conclusion, SQUID measurements of fluctuations in flux flow revealed distinct noise spectra in two different transport-current regimes. In the quasilinear region the observed spectra can be described by a nearly steady-state flow of a random array of flux bundles, whereas in the near-onset region velocity spreads due to interactions with different kinds of pinning centers give rise to a slightly modulated $1/f$ spectrum.

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