Single- and Double-Octupole Excitations in ^{148}Gd

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In the two-valence-neutron nucleus ^{148}Gd, in-beam ^{148}Sm$(\alpha,4n)$ and $(^3$He$,3n)$ measurements have identified nine $\nu^2\times$octupole levels with $3<I<14$. The measured half-life of the $(\nu f_{7/2}^2\times3^-)_{9^-}$ state gives $B(E3,9^-\to6^+)=55(6)B_{\mathrm{W}}$. A high-lying 12^+ level, which decays by $E3$ to the 9^- state, as well as a 14^+ level are shown to be two-phonon octupole excitations of stretched $\nu^2\times3^-\times3^-$ character. All these results can be quantitatively derived from the experimental data for the coupling of one neutron to the core octupole observed in ^{147}Gd.

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The study of particle-phonon coupling phenomena in nuclei provides the basic understanding for the vibrational anharmonicities, which in turn are of crucial importance for the properties of two-phonon excitations. Of particular interest are cases where the coupling strength between valence particle and core phonon is weak, which in general is only fulfilled for particle-octupole vibration coupling.[1,2] The only results available so far were from the region around ^{208}Pb where the 2.6-MeV 3^- octupole state is the lowest-lying core excitation and well separated from other states.

Some time ago it was recognized[3] that also ^{146}Gd has a 3^- first excited state which lies at 1.6 MeV, 1 MeV lower than in ^{208}Pb, and a number[4,5] of studies have provided first results on particle-octupole coupling in this region. An important feature of the ^{146}Gd 3^- phonon is its dominant $h_{11/2}d_{5/2}^{-1}$ proton particle-hole component, which becomes evident[4] in the $\pi h_{11/2}\times3^-$ septuplet of ^{147}Tb. The experiments have located the $\frac{15}{2}^+$ and $\frac{17}{2}^+$ septuplet members which are separated by 0.8 MeV. This large splitting can be understood as an effect of the exclusion principle where the $\pi h_{11/2}$ particle in the phonon interferes with the $h_{11/2}$ valence proton. Of the $\nu f_{7/2}\times3^-$ septuplet in ^{147}Gd one knew[3] only the $\frac{13}{2}^+$ member which occurs as low as 1 MeV as a result of interaction with the close-lying $\nu i_{13/2}$ single-particle state. Also the first example of a nuclear two-phonon octupole excitation has been observed[6] in ^{147}Gd, at 2.6 MeV. It is the stretched $(\nu f_{7/2}\times3^-\times3^-)_{19/2^-}$ configuration and decays by $E3$ to the 1-MeV $\frac{13}{2}^-$ one-phonon septuplet member. Five of the remaining six members of the one-phonon multiplet were observed in a recent experiment,[7] all within <180 keV of the 1.58-MeV core phonon energy. These results show that here the $\nu f_{7/2}\times3^-$ coupling is weak, comparable to ^{209}Bi, where the splitting[2] of the $\pi h_{9/2}\times3^-$ septuplet is <250 keV.

In such a situation it should also be possible to recognize in the energy spectrum of the two-particle nucleus the octupole multiplets with two valence particles coupled to the core phonon. In ^{148}Gd such excitations should occur not far from the yrast line, since the $f_{7/2}$ neutron orbital is well separated from the above-lying high-j neutron single-particle states. Although it probably is not feasible to observe all twenty members of the $\nu f_{7/2}^2\times3^-$ multiplet we will show below that a number of these states are populated in (α,xn) reactions. In these experiments we have also observed two further two-phonon octupole states, which involve the stretched coupling of valence particles and core phonons, analogous to the above mentioned $\frac{19}{2}^-$ state in ^{147}Gd. All these results can be analyzed in a quantitative manner with use of the empirical one-particle–phonon coupling data.

The $(\alpha,4n)$ and $(^3$He$,3n)$ reactions were used to study the ^{147}Gd energy levels. The experiments included γ-ray excitation function and angular distribution measurements as well as four-parameter $\gamma\gamma$ coincidence studies. Also conversion electron spectra were measured. These data established the ^{148}Gd high-spin states up to 6-MeV excitation and $I^\pi=18^+$ shown in Fig. 1, where the γ-ray deexcitation is given only for the levels of interest in the present note. The levels indicated to the right can be characterized as multiparticle shell-model excitations involving the two valence neutrons coupled either to negative-parity proton particle-hole or to the $(\pi h_{11/2}^2 j_0^{-2})_{10^+}$ core excitations. These levels will be discussed in a forthcoming article together with a more complete account of the experimental

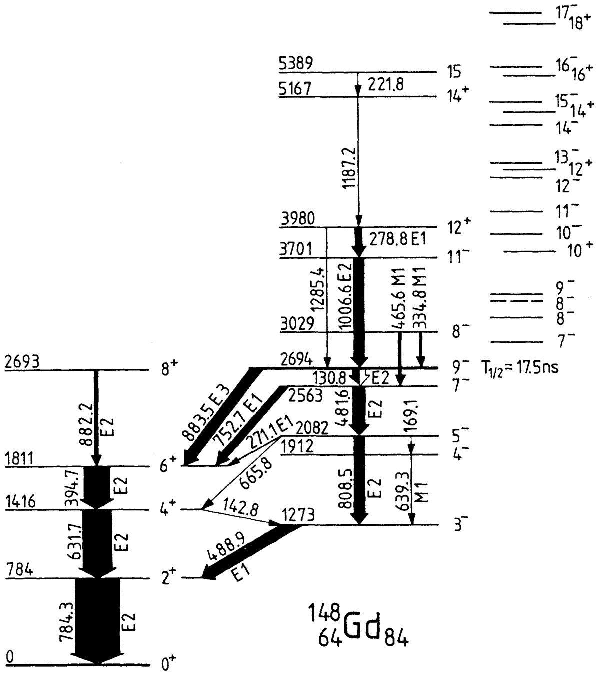

FIG. 1. Two-neutron and $\nu^2\times$ octupole levels of ^{148}Gd as populated in $(\alpha,4n)$ at 51 MeV. Other observed high-spin states are shown to the right. Multipolarities are given when derived from conversion-electron data.

results. In the present note we consider the pure two-valence-neutron states (shown to the left) and the couplings of these states to the ^{146}Gd octupole phonon (center). Of the former type four excitations were observed. The levels up to 6^+ are of predominant $\nu f_{7/2}^2$ character, and a firm $\nu f_{7/2}h_{9/2}$ assignment is established[8] for the 2.693-MeV 8^+ level. Ten octupole states were identified including the $T_{1/2}=17.5(10)$-ns 9^- isomer at 2.694 MeV which decays to the 6^+ state through an $E3$ transition with $55(6)B_{\mathrm{W}}$. The lower-lying negative-parity states must be octupole levels since other excitations with negative parity will not occur below 2.6 MeV. The octupole nature of the 8^-, 11^-, and 12^+ levels is deduced from γ-ray branching ratios which strongly suggest that these levels cannot be of the multiparticle character of the levels shown to the right. The $E3$ decay of the 12^+ state strongly supports the octupole assignment. Furthermore, we note that only two $(\pi^{+1}\pi^{-1})_{8^-}$ shell-model states are expected around 3 MeV, and the decay branchings clearly characterize the third state, at 3.029 MeV, as an octupole state. The highly selective decay of the two high-lying 14^+ and 15 states also suggests octupole nature, but the upper one has no parity assigned yet and is therefore not considered in the discussion below.

For calculating the $\nu f_{7/2}^2\times 3^-$ energies in ^{148}Gd we first consider the coupling of one $f_{7/2}$ neutron to the core octupole as observed[7] in ^{147}Gd. This one-particle×phonon spectrum also specifies the coupling of two $f_{7/2}$ neutrons to the 3^- phonon. From the ^{147}Gd spectrum of Fig. 2 it is apparent that the $\frac{13}{2}^+$ septuplet member strongly interacts with the $\nu i_{13/2}$ single-particle state. In all calculations we therefore diagonalize this interaction. To determine the coupling matrix element $m(i_{13/2},f_{7/2}\times 3^-)$ one must know the $\nu f_{7/2}$ to $\nu i_{13/2}$ single-particle energy separation which earlier[6] was

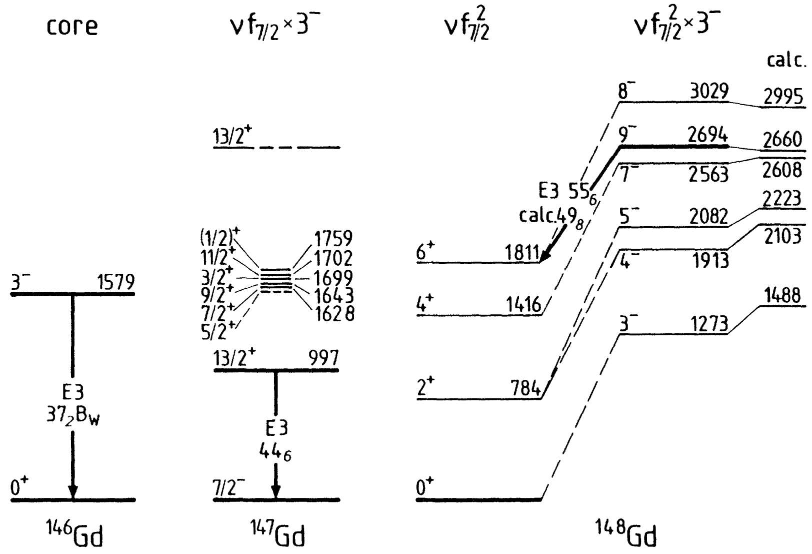

FIG. 2. Observed $\nu f_{7/2}^2\times 3^-$ octupole excitations in ^{148}Gd compared with calculated results.

derived from indirect spectroscopic evidence to be ~ 2.1 MeV. The observed 1.0-MeV $\frac{13}{2}^+$ energy of ^{147}Gd is then reproduced with $m = -0.8$ MeV, which is quite similar to the analogous $m(j_{15/2}, h_{9/2}\times 3^-) = -0.88$-MeV coupling strength deduced[2] from more complete experimental data for ^{209}Pb.

In contrast to the one-particle case of ^{147}Gd, where the $i_{13/2}$ excitation affects only one multiplet member, the analogous mixing in the two-neutron case of ^{148}Gd involves the $3^- < I < 9^-$ members of the $\nu f_{7/2} i_{13/2}$ two-neutron multiplet which lie above the ^{148}Gd yrast line and have not been observed. For the calculation we assume that the unperturbed fi states with $4^- < I^\pi < 9^-$ lie 2.1 MeV above the $(f_{7/2}^2)_{6^+}$ level. The fi 3^- and 10^- states are assumed to lie lower in energy by 600 and 100 keV, respectively, which are the estimated residual interactions for these couplings. These assumptions fully specify the diagonal energy for calculation of the $(f\times 3, i)$ interactions in the $f^2\times 3^-$ multiplet. The anharmonicities arising from the $\nu f_{7/2}\times 3^-$ couplings with $\frac{1}{2} < j < \frac{11}{2}$ are known[7] to be much smaller (Fig. 2). They are treated as a perturbation and added to the diagonal energies.

For each I of the $f_{7/2}^2 \times 3^-$ multiplet, we diagonalized the interaction matrix within the basis states $[(f^2)_{J=0,2,4,6}\times 3^-]_I$ and $(fi)_I$, where in each case the appropriate geometrical factor,

$$[2(2J+1)(2j+1)]^{1/2}\begin{Bmatrix} \frac{7}{2} & \frac{7}{2} & J \\ 3 & I & j \end{Bmatrix},$$

is taken into account for the off-diagonal $i_{13/2}$ coupling matrix element $m(j=\frac{13}{2})$, as well as for the perturbation contributions $(j < \frac{13}{2})$ to each diagonal element. The present analysis does not provide a mechanism for direct coupling of the different $f_J^2\times 3^-$ submultiplets; the mixing of the states is mediated exclusively through the pertinent fi level. Within each of the $f_J^2\times 3^-$ groups, characteristic anharmonicities result, which are related to the relative orientation of the angular momentum vectors of the particles.

In Fig. 2 the calculated energies of the octupole states are compared with experiment. We note that their energies are well reproduced, and also that the calculated relative energy shifts within each J group agree excellently with experiment in the two cases where more than one group member is known.

To the $9^- \to 6^+ E3$ strength two components contribute, viz., the core-octupole $E3$ and the $\nu i_{13/2} \to \nu f_{7/2}$ single-particle $E3$ transition. The latter can be extracted from the measured $44(6)B_{\rm W}$ of the 997-keV $E3$ transition[3] in ^{147}Gd. With the composition of the state as specified above, and the $37(2)B_{\rm W}$ core-octupole strength,[3,6] one derives $B(E3, \nu i_{13/2} \to \nu f_{7/2}) = 8.5(4.5)B_{\rm W}$. The $B(E3)$ value in ^{148}Gd is calculated as

$$B^{\rm calc}(E3, 9^- \to 6^+) = \{\alpha(\tfrac{20}{13})^{1/2}[B(E3, i\to f)]^{1/2} + \beta[B(E3, 3^-\to 0^+)]^{1/2}\}^2 = 49(8)B_{\rm W},$$

where the 9^- state has the composition $\alpha|fi\rangle + \beta|f_6^2\times 3^-\rangle$. Also this result is in good agreement with the measured $B^{\rm exp}(E3, 9^-\to 6^+) = 55(6)B_{\rm W}$.

The 11^- level at 3.701 MeV is assigned as a stretched one-phonon octupole excitation built on the $(\nu f_{7/2} h_{9/2})_{8^+}$ state. The coupling of the $(\nu h_{9/2}\times 3^-)_{13/2^+}$ level to the $i_{13/2}$ single-neutron state will be negligible since the $\nu i_{13/2} \to \nu h_{9/2}$ $E3$ transition involves a spin-flip. The $h_{9/2}$ neutron therefore acts as a spectator, and the 11^- octupole state should be completely analogous to the $\nu f_{7/2}\times 3^-$ excitation in ^{147}Gd at 997 keV. The observed 1008-keV 8^+ to 11^- energy separation in ^{148}Gd is in close agreement with this expectation. Here we have tacitly assumed that the attractive residual interactions for the fully aligned $(\nu h_{9/2} f_{7/2})_{8^+}$ and $(\nu h_{9/2} i_{13/2})_{11^-}$ singlet couplings are equal.

We assign the 12^+ and 14^+ levels at 3.980 and 5.167 MeV as the stretched two-phonon octupole excitations built on the aligned $(\nu f_{7/2}^2)_{6^+}$ and $(\nu f_{7/2} h_{9/2})_{8^+}$ two-neutron states. No other 12^+ level is expected below 5 MeV except for the $(\nu f_{7/2}^2)_{2^+}(\pi h_{11/2}^2)_{10^+}$ level which is observed at 4.500 MeV (Fig. 1). The 1285-keV $E3$ transition to the 9^- state provides independent proof for the proposed two-phonon assignment. In the case of the 14^+ state the energy argument cannot be quite as strong, but the highly selective decay suggests also a two-phonon character. The energies of these double-octupole states can be predicted from the experimental information[6] on the $(\nu f_{7/2}\times 3^- \times 3^-)_{19/2^-}$ level in ^{147}Gd. Figure 3 gives a synopsis of the three observed double-octupole states. The anharmonicities for the ^{147}Gd $\frac{19}{2}^-$ excitation have been discussed[6] in a recent article, where it was shown that in addition to the coupling with the $i_{13/2}$ neutron a second contribution is significant which arises from Pauli blocking of the dominant $\pi h_{11/2} d_{5/2}^{-1}$ amplitude in the core phonon. This effect causes an upwards shift of 0.41 MeV for the 6^+

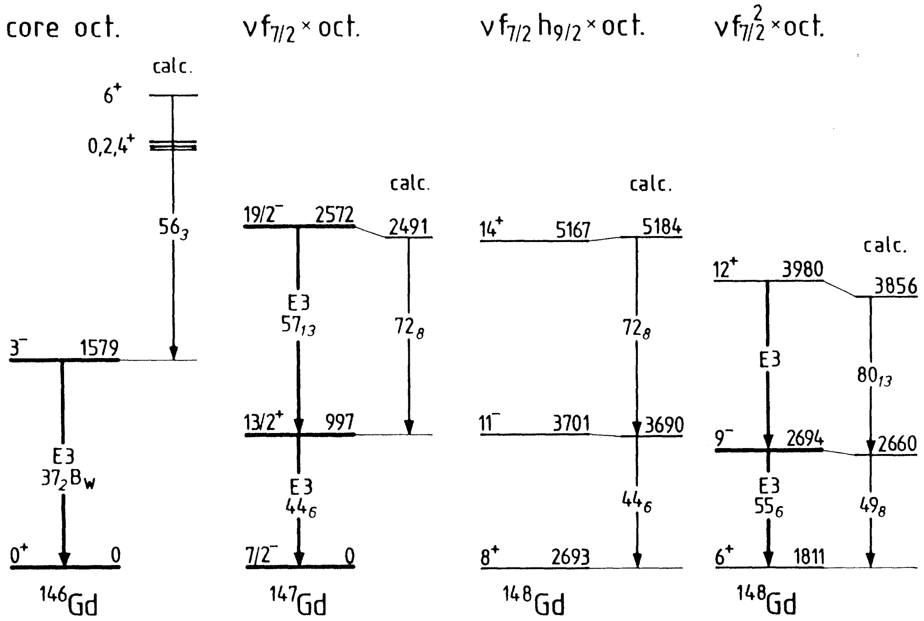

FIG. 3. Single- and double-octupole excitations in Gd nuclei with 82 to 84 neutrons compared with calculated results.

two-phonon core states. The calculated $\frac{19}{2}^-$ energy in Fig. 3 results from diagonalization of the interaction of the $(\nu i \times 3^-)_{19/2}$ and $(\nu f \times 3^- \times 3^-)_{19/2}$ states, where the 0.41-MeV energy is added to the diagonal two-phonon energy. The Pauli blocking also reduces[6] the two- to one-phonon $B(E3)$ value, which becomes $(2\times 37-18)B_\mathrm{W}$. With this value, and the $(\nu i \rightarrow \nu f)$ $E3$ transition strength given above, the $\frac{19}{2}^- \rightarrow \frac{13}{2}^+$ $B(E3)$ value is calculated as $72(8)B_\mathrm{W}$.

In the calculation of the 14^+ two-phonon state in ^{148}Gd we again ignore the presence of the $h_{9/2}$ neutron which does not couple to the phonon. The 14^+ energy therefore is evaluated by the same method as the $\frac{19}{2}^+$ two-phonon state in ^{147}Gd.

In the $(\nu f_{7/2}^2 \times 3^- \times 3^-)_{12^+}$ state, the presence of two $f_{7/2}$ neutrons causes nonstretched contributions. The configurations present in that 12^+ level are $f_6^2\times 3\times 3$, $fi_{10}\times 3$, $fi_9\times 3$, and i^2. We diagonalize the interaction with the fi states within this 4×4 matrix, where again the +0.41-MeV Pauli blocking shift is added to the $f^2\times 3\times 3$ diagonal energy. Similarly as in the calculation above, the pertinent contributions from the $(f\times 3)_{j<13/2}$ couplings are also included by perturbation. Results are given in Fig. 3. Both calculated two-phonon level energies are in nice agreement with experiment.

In conclusion, we have identified six low-lying $\nu f_{7/2}^2 \times 3^-$ octupole levels in ^{148}Gd and have shown that their energies are well predicted from the anharmonicities of the $\nu f_{7/2}\times 3^-$ septuplet known in ^{147}Gd. To our knowledge this is the first case where the coupling of two valence particles to the core octupole phonon could be studied in such detail and analyzed quantitatively within a particle-vibration coupling picture. A double-octupole state with $I^\pi = 12^+$ was identified by observation of the stretched $E3$-$E3$ cascade decay to the $(f_{7/2}^2)_{6^+}$ two-neutron state. The energy of this 12^+ level, and of a second double-octupole state with $I^\pi = 14^+$, can be derived with good accuracy by use of angular momentum recoupling and the measured octupole energies in ^{147}Gd.

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