Isospin-Symmetry Breaking in Electroweak Theories

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The breaking of isospin symmetry in the electroweak theory with a dynamical Higgs sector is analyzed. The natural size of the breaking in various amplitudes is estimated, given that the breaking is very large in the fermion mass matrix. Special attention is paid to the parameter $\rho \equiv M_W^2/M_Z^2\cos^2\theta_w$. Hypercolor models are then investigated. There it is noted that $\rho-1$ will naturally exhibit a sensitive, linear dependence on the fermion-doublet mass splittings.

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The observation of the charged and neutral weak bosons[1,2] reinforces our belief in the standard model of color-electroweak interactions. The final missing link is an understanding of the mechanism of electroweak symmetry breaking. It is attractive to assume that the breaking arises spontaneously, and within this framework at least two distinct possibilities emerge. One is that of the light, elementary Higgs field whose weak self-interactions are arranged to give it a nonzero vacuum expectation value. A problem with this scheme is that truly elementary Higgs fields offer no understanding of fermion masses which are merely parametrized by the Yukawa couplings of the Higgs field to the fermions. Another possibility is that the spontaneous symmetry breakdown is due to new matter and strong forces at a mass scale around 1 TeV. In hypercolor models, for example, the Higgs sector is a set of bound states and resonances formed from new, strongly interacting fermions.[3] If new matter is involved, the masses of ordinary fermions must come from some direct interaction between the ordinary fermions and the new matter. In hypercolor models this must look like a four-fermion interaction or, at a deeper level, perhaps arise from the exchange of very massive ($\gtrsim 10$ TeV) extended-hypercolor bosons. While no realistic model of this sort has been constructed, it remains an attractive idea and one that at least offers the possibility of a deeper understanding of fermion masses.

In either of these cases, the observed strong isospin nonconservation in the fermion mass matrix must be built into, if not explained by, the Lagrangian. It is important then to ask how this breaking infects other sectors of the theory through quantum corrections. In particular, one may expect corrections to the gauge-boson mass matrix and the prediction

$$\rho = M_W^2/M_Z^2\cos^2\theta_w = 1, \tag{1}$$

which is known to hold experimentally to within a few percent.[4] The relation $\rho=1$ will be satisfied if the global symmetry of the Higgs sector is $SU(2)_L \otimes SU(2)_R$, which then spontaneously breaks to $SU(2)_{L+R}$.[5] The explicit breaking of the $SU(2)_R$, required to get the right fermion mass matrix, will necessarily give corrections to this relation. We briefly review this problem in the elementary Higgs case and then discuss these corrections in the presence of a heavy, strongly interacting Higgs sector and, in particular, in hypercolor models. There we find a qualitatively new effect and discuss its experimental significance.

In the elementary-Higgs model the SU(2) symmetry necessary to produce $\rho=1$ arises as an accidental global symmetry of the potential of a complex scalar doublet Φ. The potential $V=V(\Phi^\dagger\Phi)$ is $O(4)\approx SU(2)\otimes SU(2)$ symmetric. When $\langle\phi^0\rangle \neq 0$, this O(4) breaks to $O(3)\approx SU(2)_{L+R}$. The original O(4) symmetry is larger than the $SU(2)_L\otimes U(1)_Y$ symmetry needed to couple the Higgs sector consistently to the gauged weak-interaction sector, and this extra symmetry is the origin of the relation $\rho=1$.

It is clear that both gauge interactions and isospin breaking in the fermion mass matrix violate the SU(2) symmetry. As expected, radiative gauge-boson effects produce a contribution of order α_w to $\Delta\rho \equiv \rho-1$.[6]

To see the consequence of isospin breaking in the fermion mass matrix we write out the Yukawa couplings for a generic family of fermions:

$$\mathscr{L}_Y = y_u(u_L^\dagger d_L^\dagger)\begin{pmatrix}\Phi^0\\ \Phi^-\end{pmatrix}u_R + y_d(u_L^\dagger d_L^\dagger)\begin{pmatrix}-\Phi^{-*}\\ \Phi^{0*}\end{pmatrix}d_R + \text{H.c.}, \tag{2}$$

where u denotes charge-$\frac{2}{3}$ quarks and d charge-$(-\frac{1}{3})$ quarks. For $y_u=y_d=y$ this takes the $SU(2)_L$

$\otimes$ $SU(2)_R$ symmetric form $y q_L^\dagger M q_R$, where M is the 2×2 matrix field of the σ model. In the case $y_u \neq y_d$ the contribution of fermion loops to the gauge-boson two-point function gives[7]

$$\Delta\rho = \xi \frac{1}{16\pi}\left(\frac{\alpha_w}{M_W^2}\right)\left(\frac{2m_u^2 m_d^2}{m_u^2 - m_d^2}\ln\frac{m_d^2}{m_u^2} + m_u^2 + m_d^2\right), \tag{3}$$

where ξ counts colors.

This expression vanishes quadratically as $\Delta m \equiv m_u - m_d$ goes to zero. The quadratic dependence appears since at least two mass insertions are required by helicity conservation. The experimental constraint[4]

$$\Delta\rho \lesssim 0.05 \tag{4}$$

then requires the mass of a heavy quark with a massless partner to be less than about 400 GeV. One may also derive Eq. (3) by considering the fermion loop contribution to the Goldstone-boson propagator.[8]

The above discussion can be extended to a strongly interacting Higgs sector by utilizing the gauged nonlinear sigma model to describe the theory at $E < 1$ TeV.[9] Corrections to the leading low-energy behavior can then be summarized in the form of operators of increasing dimension that exhibit the $SU(2)_L \otimes SU(2)_R$ symmetry of the Higgs sector together with $SU(2)_R$-breaking effects. By estimating the natural size of these operators, the sensitivity of low-energy measurements, such as the ρ parameter, to the 1-TeV dynamics and to the breaking of $SU(2)_R$ to U(1) can be completely described.[10]

The coupling of the Goldstone fields to the gauge fields is given by

$$\mathscr{L}_{NL} = (f^2/4)\,\mathrm{Tr}(D_\mu U)^\dagger (D^\mu U), \tag{5}$$

where $f \equiv (2M_W/g) = 250$ GeV, $U \equiv M/f$ obeys the nonlinear constraint $UU^\dagger = U^\dagger U = 1$, and

$$D_\mu U = \partial_\mu U + ig\vec{\tau}\cdot\vec{W}_\mu U/2 - ig' B_\mu U \tau_3/2.$$

The fermion masses are again described by the Yukawa coupling $\mathscr{L}_y$ [Eq. (2)].

Among the new operators consistent with $SU(2)_L \otimes U(1)$ symmetry, there is only one with the same dimension as $\mathscr{L}_{NL}$ (dimension two) with U dimensionless. It can be written in the form

$$\mathscr{L}_2 = (af^2/4)[\mathrm{Tr}(\tau_3 U^\dagger D_\mu U)]^2, \tag{6}$$

where a is a dimensionless parameter. This operator is the only one, apart from $\mathscr{L}_{NL}$, that contributes to the gauge-boson propagators at $q^2 = 0$, and therefore it completely determines $\Delta\rho$ as measured in low-energy neutrino scattering experiments. A simple computation reveals that $\Delta\rho = -2a$. A minimal size for a can be estimated by noting that $\mathscr{L}_2$ will be induced by radiative corrections involving the U(1) gauge field. It is found[10] that this source of $SU(2)_R$ breaking gives rise to an a of order $(\alpha_w/\pi)\tan^2\theta_w$, a value well within the experimental bound.

The $SU(2)_R$ breaking in the Yukawa interactions [Eq. (2)] will also induce $\mathscr{L}_2$. The lowest-order computation is the same as in the elementary-Higgs theory, leading to Eq. (3). This analysis can be extended to higher orders in the loop expansion and to higher-dimension operators. The result, if we note that the loop expansion rapidly breaks down because of the strong Higgs interactions, is the following: If there is no other source of $SU(2)_R$ symmetry breaking, beyond the Yukawa coupling $\mathscr{L}_y$ and the U(1) gauge field coupling, then even in the presence of a strongly interacting Higgs sector with a 1-TeV mass scale, no quantum-induced corrections to $\mathscr{L}_2$ and $\Delta\rho$ are generated that exceed the elementary-Higgs result [Eq. (3)]. This analysis will be presented in detail and extended to higher-dimension operators and other physical effects in a forthcoming paper.[11]

We now analyze the case where the electroweak symmetry breaking is due to new strongly interacting fermions—hyperfermions (T). The SU(2) symmetry that guarantees $\rho = 1$ at the tree level is the diagonal subgroup of the $SU(2)_L \otimes SU(2)_R$ chiral symmetry group of the hyperfermion sector.

Without further interactions the chiral symmetry of the ordinary fermions is unbroken—they are massless. The most economical way to give them mass is to couple them to hyperfermions via an effective four-fermion interaction of the form

$$\mathscr{L}_{fT} = (G_1/2)\bar{f}_L[\bar{T}T + i\vec{\tau}\cdot(i\bar{T}\gamma_5\vec{\tau}T)]f_R + (G_2/2)\bar{f}_L[\bar{T}T + i\tau\cdot(i\bar{T}\gamma_5\vec{\tau}T)]\tau_3 f_R + \mathrm{H.c.} \tag{7}$$

Since isospin nonconservation in the strong hypercolor interaction would give a large $[O(\alpha_{HC})]$ contribution to $\Delta\rho$ we assume that the only source of isospin nonconservation, other than the $U(1)_Y$ gauge field coupling, is the coupling $\mathscr{L}_{fT}$. Then a simple one-loop estimate gives

$$\Delta m = m_u - m_d \simeq (G_2/8\pi^2)\Lambda_{HC}^3, \tag{8}$$

where Λ_{HC} is the scale at which the hypercolor interaction becomes strong. It can be helpful to think of m_u and m_d as arising in a two-step process. First, the hypercolor dynamics leads to the effective Yukawa coupling [Eq. (2)] along with the nonlinear Goldstone Lagrangian [Eq. (5)]. The vacuum value $U=1$ then gives Eq. (8), with the identification $y_u - y_d \simeq G_2 \Lambda_{\rm HC}^2/8\pi^2$ and $f \approx \Lambda_{\rm HC}$.

We now describe the new contributions to the operator $\mathscr{L}_2$ and to $\Delta\rho$ that arise in hypercolor theories. The first observation is that there will naturally exist a variety of four-fermion interactions in addition to $\mathscr{L}_{fT}$, with coupling strengths on the order of G_1 and G_2. These will, for example, be induced by iterations of the interaction $\mathscr{L}_{fT}$. If the integrations are not cut off at energies below the unitarity bound, $\simeq G_1^{-1}, G_2^{-1}$, these new interactions will naturally be of the same strength as $\mathscr{L}_{fT}$. It might be, of course, that the integrations are damped below the unitarity bound. In extended hypercolor (EHC) models, for example, $\mathscr{L}_{fT}$ could be the result of an EHC boson exchange with a small dimensionless coupling constant. The higher-order interactions will then be small. It is natural in these models, however, that the tree-level exchange of an EHC boson also contributes to the additional four-fermion interactions. Unless some suppression mechanism is present, this contribution will be of strength comparable to $\mathscr{L}_{fT}$.

Among the four-fermion interactions to be expected from these considerations are the following, involving only right-handed fermions:

$$\mathscr{L}'_{fT} = G'_2 \bar{f}_R \gamma^\mu \tau_3 T_R \bar{T}_R \gamma_\mu \tau_3 f_R + \text{H.c.}, \tag{9}$$

$$\mathscr{L}_{TT} = G_3 \bar{T}_R \gamma^\mu \tau_3 T_R \bar{T}_R \gamma_\mu \tau_3 T_R. \tag{10}$$

These two have been exhibited simply because they act as a new source of $SU(2)_R$ symmetry breaking and because they contribute most directly to $\Delta\rho$.

To estimate the contribution of either $\mathscr{L}'_{fT}$ or $\mathscr{L}_{TT}$ to $\Delta\rho$, it is simplest to imagine turning off the U(1) gauge coupling g'. In this limit, $\theta_w = 0$ and ρ is simply M_W^2/M_Z^2. Turning g' back on will then give higher-order corrections to the dominant contributions.

Consider first the interaction $\mathscr{L}'_{fT}$. It can contribute to the W and Z masses through the graph shown in Fig. 1. Because $\mathscr{L}'_{fT}$ involves only right-handed fermions, there can be two τ_3's in the trace around the figure-eight loop and therefore a nonvanishing contribution to $\Delta\rho$. With $g'=0$, however, the right-handed fermions do not couple to the gauge bosons without the helicity flip provided by mass insertions on the fermion lines. With these necessary insertions on the light-fermion lines the contribution to $\Delta\rho$ will not be more important than the one-loop result [Eq. (3)]. To estimate its size, it is again useful to proceed in two steps. Integrating out the hypercolor fields will produce a new, $SU(2)_R$-breaking, effective coupling of the gauge field to right-handed fermions. Appropriate factors of U will also be present to produce a gauge-invariant operator.[11] If we call the dimensionless coupling associated with this new interaction $y' \equiv G'_2 \Lambda_{\rm HC}^2/8\pi^2$, the final integration over the ordinary fermion loop will produce a contribution to $\Delta\rho$ of order $\xi(y'/8\pi^2)(m^2/f^2)$. We expect y' to be of the same order as y_u and y_d, and therefore this is a smaller contribution to $\Delta\rho$ than Eq. (3) unless fermion masses become on the order of M_W or larger.

It is the interaction $\mathscr{L}_{TT}$ that potentially gives the larger contribution to $\Delta\rho$. Since it is the product of two $SU(2)_R$-violating currents, the figure-eight trace in Fig. 2 will contribute directly to $\Delta\rho$. Furthermore, since the hyperfermion mass is expected to be of order $\Lambda_{\rm HC}$ ($\simeq 1$ TeV), no price is paid to convert the right-handed hyperfermion to the left-handed one necessary to couple to the gauge fields. An estimate analogous to the one above then gives

$$\Delta\rho \simeq y_3/8\pi^2, \tag{11}$$

where $y_3 \equiv G_3 \Lambda_{\rm HC}^2/8\pi^2$. (A color factor could also be included.) To see the significance of this result, we rewrite it in terms of mass splittings. With use of Eq. (8), and for $y_3 \simeq y_u - y_d$,

$$\Delta\rho \simeq \frac{y_3}{y_u - y_d} \frac{\Delta m}{8\pi^2 f} \simeq \frac{1}{8\pi^2} \frac{\Delta m}{f}. \tag{12}$$

This is linear in the ordinary fermion mass splittings.[12]

The numerical factors in this expression are only rough estimates. Nevertheless, it is perhaps worth-

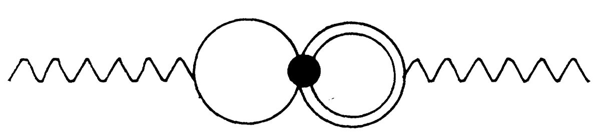

FIG. 1. Gauge-boson self-energy diagram from interaction $\mathscr{L}'_{fT}$. Single lines are light fermions while double lines represent hyperfermions.

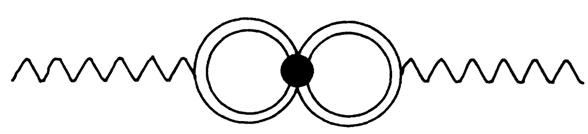

FIG. 2. Gauge-boson self-energy diagram from interaction $\mathscr{L}_{TT}$.

while to compare its size to the result [Eq. (3)] and to the experimental bound. For $m_u >> m_d$ and for $\xi = 3$, the expression in Eq. (3) becomes $(3/16\pi^2) \times (\Delta m^2/f^2)$. Clearly, for small enough Δm, the linear expression [Eq. (12)] will dominate the quadratic one. If the numerical factor in Eq. (12) can be trusted, the two expressions become comparable when Δm is of order f, corresponding to a $\Delta\rho$ on the order of a few percent. This lies just within the current experimental bound. If this bound can be reduced, then the linear expression [Eq. (12)] will play the dominant role in constraining the mass splitting Δm.

Although the linear dependence is obtained indirectly, through the effect of the four-hyperfermion operator $\mathscr{L}_{TT}$, we have argued that it is likely to be a generic property of hypercolor theories. It is, in fact, not unlikely that this kind of result is a general feature of any theory in which electroweak symmetry breaking is due to some new dynamics with a mass scale of a few teraelectronvolts. There must always be some $SU(2)_R$-violating interactions to give the correct fermion mass matrix and these could well feed back into $\Delta\rho$ in the manner discussed here.

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