# Are Vacuum Bubbles a Cause of Major Disruptions in Tokamaks?

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The maximum amplitude of the $m=2$ tearing mode in tokamaks is found to increase rapidly with rising safety factor at the magnetic axis, $q(0)$. The resulting large magnetic islands encompass virtually the entire plasma cross section for $1.5 \lesssim q(0) \lesssim 1.8$ and are shown to be related to the *vacuum bubbles* formed during the saturation of the ideal $m=2$ kink mode. On the basis of these results, a mechansim for major distruptions is proposed along with a possible method for their elimination.

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Major disruptions in tokamak discharges are characterized by a rapid decrease in the toroidal current, leading to the termination of the discharge.[1-3] Kadomtsev and Pogutse first proposed that these disruptions were a consequence of the nonlinear development of ideal kink modes in a current-carrying plasma column; the highly convoluted final states have vacuum bubbles.[4] In later numerical studies it was shown that while such bubbles can develop in the case of a uniform current profile (no magnetic shear), rounded current profiles (with magnetic shear) strongly inhibit their formation.[5] The tearing mode $m=2$, $n=1$ (abbreviated 2/1, or $m/n$ in general) saturates benignly, the largest islands having widths $w$ of the order of $0.4a$, where $a$ is the minor radius.[6]

In full three-dimensional simulations of resistive tearing modes, the behavior of the modes is much more violent. The 2/1 mode can sometimes destabilize the 3/2, 5/3, and other modes,[7,8] culminating in a broad spectrum of magnetic turbulence which destroys the flux surfaces, and therefore confinement.[7] The growth of this turbulence has been proposed as a mechanism for major disruptions. On the other hand, while the 2/1 mode is almost universally observed prior to or during the disruption, it is less clear that the 3/2 and other modes are as ubiquitous. In the Joint Institute for Plasma Physics T-II tokamak the 3/2 mode is only seen during minor disruptions.[9] Some aspects of major disruptions have also not been explained in this scenario. In the T-4 tokamak, for example, Mirnov and Semenov[3] have observed that the disruption is only triggered after a predisruption broadens the current profile, raising $q(0)$ above approximately 1.5.

We show that when the central current profile is fairly flat and $1.8 \gtrsim q(0) \gtrsim 1.5$, where $q(r)=rB_z/RB_\theta(r)$, the $q=2$ magnetic islands grow to very large amplitude, encompassing essentially the entire plasma cross section. The formation of these large islands is shown to be related to the formation of vacuum bubbles during the nonlinear evolution of the ideal 2/1 kink mode. The rapid loss of energy which would result from the growth of these islands is proposed as a mechanism for major disruptions.

Our calculations are based on the reduced resistive magnetohydrodynamic (MHD) equations for helical perturbations in cylindrical geometry,[5]

$$d\nabla_\perp^2\phi/dt-\mu\nabla_\perp^4\phi=\hat{z}\times\nabla\psi\cdot\nabla J, \quad (1)$$

$$\nabla_\perp^2\psi=J-2n/m, \quad (2)$$

$$\partial\psi/\partial t=\hat{z}\times\nabla\psi\cdot\nabla\phi+\eta J, \quad (3)$$

where $d/dt=\partial/\partial t+\vec{v}\cdot\nabla$, $J$ is the axial current, and the velocity $\vec{v}$ and magnetic field $\vec{B}$ are given by $\vec{v}=\hat{z}\times\nabla\phi$ and $\vec{B}=\hat{z}+(nr/m)\hat{\theta}+\hat{z}\times\nabla\psi$, with $\phi$ and $\psi$ the stream function and helical flux function, respectively. The equations are written in normalized units: $t/\tau_A\rightarrow t$, $a\nabla_\perp\rightarrow\nabla_\perp$, $R\,\partial/\partial z\rightarrow\partial/\partial z$, and $\eta\tau_A c^2/4\pi a^2\rightarrow\eta$, where $c_A=(B_z^2/Mn)^{1/2}$ is the Alfvén velocity, $\tau_A=R/c_A$ is the Alfvén time, $\eta$ is the resistivity, $\mu$ is the viscosity, $a$ is the minor radius, $2\pi R$ is the periodicity length in $z$, and $n/m=\frac{1}{2}$ is the pitch of the perturbations.

We consider an equilibrium based on the standard $q$ profile,[7] $q(r)=q(0)[1+(r/r_0)^{2\lambda}]^{1/\lambda}$, with $r_0^{-2\lambda}\equiv[q(1)/q(0)]^\lambda-1$ and $\eta\sim J^{-1}$. Equations (1)-(3) are solved by expansion of the $\theta$ dependence in Fourier harmonics, use of a finite difference scheme (200 points) in the radial direction, and advancement of the equations in time by a semi-implicit scheme.[10] Because of difficulties which were previously encountered in simulating large magnetic islands,[6] we carefully checked our results for sensitivity to the number of poloidal harmonics (up to sixty are retained).

In Fig. 1 we show the time dependence of the island width $w$ for several values of $q(0)$ between 1.3 and 1.8 for $q(1)=3.4$, $\lambda=4$, and $\eta=50\mu=5\times10^{-5}$. For $q(0)=1.3$ the island grows and

 

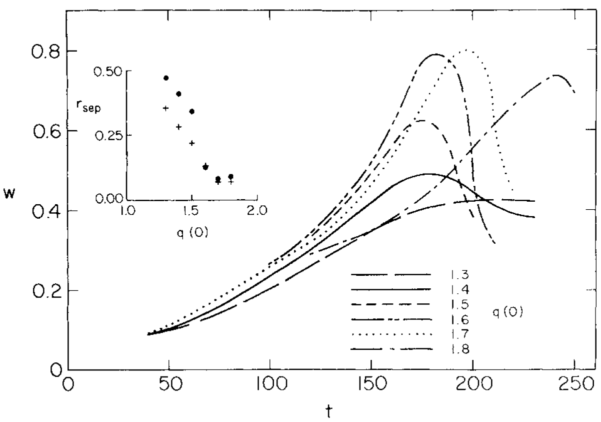


FIG. 1. The island width $w$ vs time $t$ for $q(0) = 1.3$–$1.8$. The inset shows the radius of the inner separatrix $r_{sep}$ at maximum island size vs $q(0)$.

saturates at moderate amplitude, $w \sim 0.4$, with essentially no overshoot. This behavior and that of the 2/1 tearing modes for $q(0) < 1.3$ are consistent with previous analytic models.[11] As $q(0)$ is increased further the maximum island size increases dramatically and strongly overshoots. This can perhaps be seen most dramatically in the inset of Fig. 1, where we plot the radius of the inner separatrix of the island (at the maximum $w$), $r_{sep}$, as a function of $q(0)$ (dots). For $1.6 \lesssim q(0) \lesssim 1.8$ the island essentially extends to the center of the plasma! The distance $r_{sep}$ is insensitive to $q(1)$. This rapid increase of $w$ with rising $q(0)$ does not follow from the quasilinear saturation model of the tearing mode. Indeed, in this model $w$ actually decreases for $q(0) > 1.5$.[11] This discrepancy should not be surprising as the quasilinear model should not be valid for large islands with $w \sim a$.

In Figs. 2(a) and 2(b) we show the contours of constant $\psi$ and $J$ at maximum island size ($t = 351$) for $q(0) = 1.7$ and $\eta = \mu = 5 \times 10^{-6}$ with other parameters as in Fig. 1. The island nearly encompasses the entire column while the current has been squashed into a narrow band across the middle of the column. Cuts of the current profile at $t = 0$ (solid line) and through the $O$ point (dotted line) and $X$ point (dashed line) of the island at $t = 351$ are shown in Fig. 2(c). The arrow denotes the location of the initial $q = 2$ surface. Note that although the resistivity in Fig. 2(a) is an order of magnitude smaller than in Fig. 1, the maximum island size for $q(0) = 1.7$ is nearly unchanged. For $q(0) = 1.7$ and $\eta = 5 \times 10^{-6}$, the time $\tau$ over which the island width doubles just prior to saturation is of the order of 100 compared to 60 for $\eta = 5 \times 10^{-5}$ so that $\tau$ is relatively insensitive to $\eta$ over this range, scaling as $\eta^{-0.2}$.

Our interpretation of the numerical results shown in Figs. 1 and 2 is that as $q(0)$ increases the difference in the pitch of the 2/1 tearing mode and the magnetic field lines near the center of the column decreases so that as the island grows inward the magnetic tension is not sufficient to stop the instability. In a peaked current distribution or when $q(0)$ is smaller, the magnetic tension rises rapidly as the island grows into a region where the local pitch of the field differs greatly from that of the mode and it stops the growth of the island. The simulations of large $m = 2$ islands presented here are not related to previous computations by Sykes and Wesson in which the $m = 2$ tearing mode suddenly increased in amplitude only after the magnetic island contacted the limiter.[12] At the time of limiter contact in their simulations $q(0) = 1$.

It is informative to compare the growth of large magnetic islands in the present calculation with previous investigations of bubble formation by the ideal kink mode. For a flat current profile of radius $r_0$, the growth rate of ideal modes is given by[13] $\gamma^2 = 2\Delta[1 - \Delta(m-1) - hm\Delta]n^2/m$, where $\Delta = m/nq - 1$ represents the mismatch of the pitch of the mode and that of the magnetic field in the current-carrying plasma and $h = r_0^{2m}/(1 - r_0^{2m})$ represents the stabilizing influence of the conducting wall at $r = 1$. For the same parameters as in Figs. 1 and 2 but with $\lambda \to \infty$, $r_0 = [q(0)/q(1)]^{1/2}$ and the ideal 2/1 mode is unstable for $2 \gtrsim q(0) \gtrsim 1.2$ with the maximum growth rate at $q(0) = 1.5$. We can compare the size of the "vacuum bubble" produced by such an ideal mode with the islands formed in Figs. 1 and 2 by integrating Eqs. (1)–(3) with $\lambda$ large. In Fig. 2(d) the $\psi$ contours are shown for $q(0) = 1.7$ at maximum island size ($t = 60$) with $\eta = \mu = 5 \times 10^{-5}$ and $\lambda = 20$. For this large value of $\lambda$ the mode is ideally unstable. The formation of magnetic islands by an ideal mode may seem surprising. Actually, the reconnection which forms the islands in this figure occurs in the "vacuum" (very high resistivity) region outside the current-carrying column (around the $q = 2$ surface) and is therefore not inconsistent with the usual constraint that the magnetic field topology is preserved for ideal modes. The "vacuum bubbles" ($J$ is small inside the island) in Fig. 2(d) are virtually identical to the magnetic islands of Fig. 2(a). The scaling of the minimum radius $r_{sep}$ of the vacuum bubbles with $q(0)$ is shown in the inset in Fig. 1 (crosses). The values of $r_{sep}$ for the resistive and ideal cases approach each other as $q(0)$ increases.

Our conclusion from this comparison of the saturation of resistive and ideal modes is that when the magnetic island of the tearing mode is very

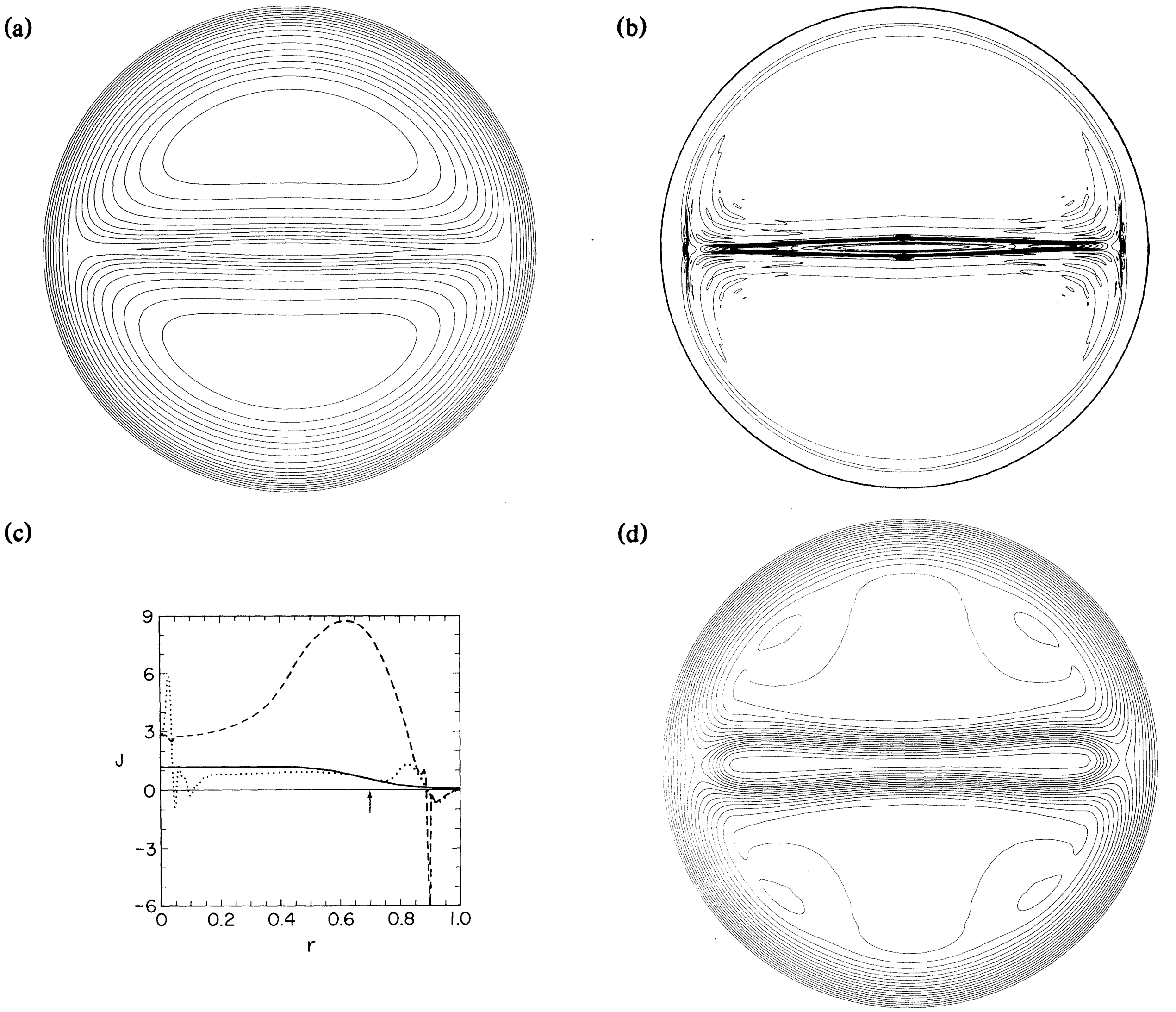


FIG. 2. The contours of constant (a) $\psi$ and (b) $J$ at maximum island size for $q(0)=1.7$. Cuts of the current profile across the $O$ point (dotted line) and $X$ point (dashed line) compared with the equilibrium (solid line) are shown in (c). The contours in (d) are the constant-$\psi$ contours at maximum amplitude of a mode which is linearly ideally unstable with $\lambda=20$ and other parameters as in (a).

large, the energetics of the saturation differs very little from that of the ideally unstable flat current profile. At large amplitude the mode does not distinguish between a flat or slightly rounded current profile and the saturation amplitude is the same for both. The time scale of the growth and saturation of the instability, however, does depend on the profile. The ideal modes grow and saturate on a time scale of order 50 Alfvén times compared to 200–300 Alfvén times for the tearing mode. We note finally that when the large islands of the resistive tearing mode have evolved beyond their maximum amplitude, they rapidly decrease and then grow again to a large amplitude in a cyclic manner [we have followed the case $q(0)=1.7$ through two full cycles]. The ideal modes exhibit a similar behavior, as has been noted previously.[5]

The strongest experimental evidence linking $q(0)\gtrsim 1.5$ with major disruptions comes from Mirnov and Semenov's detailed study of twenty disruptions on the T-4 tokamak.[3] In the time preceding the disruptions the current channel contracts as a result of the cooling of the edge plasma. The disruptions occur in two stages. In the first stage a predisruption associated with both $m=1$ and 2 modes causes a radial expansion of the temperature and current profiles, raising $q(0)$ from 1 to greater than 1.5. At this point the discharge sometimes recovers. The major disruption itself results from the rapid growth of an $m=2$ perturbation in this flattened current distribution which causes a second expansion of the column to the limiter on a time scale of 100 $\mu$sec ($\sim 10^3 \tau_A$). The usual theoretical model of disruptions based on the development of a broad spectrum of magnetic turbulence fits the observations of the predisruption where $q(0) \sim 1$ so that the formation of a single large island is not possible. The observation that the disruption itself is caused by an $m=2$ mode growing in a flattened current profile with $q(0) \gtrsim 1.5$ strongly supports our contention that the large islands formed in such profiles cause major disruptions.

The two-stage disruptions which were observed on the T-4 tokamak are also common on other tokamaks, including TFR,[14] PLT,[15] PDX,[15] and TFTR.[15] In TFR the rapid loss of plasma current during a major disruption does not begin until the second stage of the disruption.[14] Moreover, after the first phase of the disruption the discharge often recovers without significant loss of plasma current (a minor disruption). Thus, the second stage of the disruption is the essential feature of two-stage major disruptions.

Finally, if this mechanism for major disruptions has any validity, an obvious method to avoid disruptions is to strongly heat the center of the plasma column to drive $q(0)$ down towards unity. The resulting increase in the magnetic shear prevents the magnetic island (bubble) from expanding to the center of the plasma.

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