Implications of Nonspectator *B*-Meson Decays and *B*-Lifetime Measurements

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The nonspectator contributions to the *B*-meson decays are calculated with use of the measured ratio τ_{D^+}/τ_{D^0} and the assumption $f_B^2/f_D^2 = m_D/m_B$. We find $\tau_{B^+}/\tau_{B^0} = 1.4$ to 1.8. From the measured values of *B*-meson lifetime, the semileptonic branching ratio, the ratio $\Gamma(b \to ue\nu)/\Gamma(b \to ce\nu)$, and the ϵ parameter, the constraints on Kobayashi-Maskawa angles are evaluated and their implications for m_t, $|\epsilon'/\epsilon|$, B^0-$\bar{B}^0$ mixing, and $K \to \pi\nu\bar{\nu}$ are presented.

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The experimental[1] determination of the B lifetime has generated considerable interest.[2-5] That measurement along with the anticipated discovery of the t quark could, at least in principle, yield essentially a unique determination of the Kobayashi-Maskawa[6] (KM) parameters. This is expected to lead to an important new framework for testing the standard electroweak model especially as regards *CP* nonconservation. For that strategy to work some of the key quantities must be experimentally measured and theoretically calculated to the required precision. With that in mind, in this work, we incorporate the nonspectator (NS) contributions to the *B*-meson lifetime which have been ignored in the recent discussions. We find that the NS contributions to the B lifetime are such that

$$\tau_{B^+}/\tau_{B^0} = 1.4\text{–}1.8, \tag{1}$$

where τ_{B^+} and τ_{B^0} are the $B^\pm$ and $B^0(\bar{B}^0)$ lifetimes, respectively. In addition to the experimental value (i.e., the weighted mean) for the B lifetime,[1]

$$\tau_B = (1.4 \pm 0.4) \times 10^{12} \text{ sec}, \tag{2a}$$

the following experimental results[7] are included:

$$\Gamma(b \to ue\nu)/\Gamma(b \to ce\nu) < 0.05, \tag{2b}$$

$$\Gamma(B \to e\nu x)/\Gamma(B \to \text{all}) = (11.6 \pm 0.5)\%, \tag{2c}$$

$$\epsilon = (2.227 \pm 0.08) \times 10^{-3}, \tag{2d}$$

$$\tau_{D^+}/\tau_{D^0} = 2.2^{+0.9}_{-0.6}. \tag{2e}$$

We study the resulting constraints on the KM parameters (mixing angles θ_2, θ_3, and the *CP* phase δ) and pursue the implications for the B^0-$\bar{B}^0$ mixing parameter (i.e., $\Delta M_{B^0}/\Gamma_{B^0}$), for ϵ', and for $K \to \pi\nu\bar{\nu}$ which are important tests of the standard model.

The NS decays of neutral D mesons are assumed to arise via the annihilation graph whose contribution is evaluated by use of the one-gluon emission model.[8] We will attempt to minimize the model dependence by using the now available experimental information (2e) on τ_{D^+}/τ_{D^0} to constrain the most sensitive parameter (i.e., f_D/m_u) that enters in such a calculation.[9] We recall that in that model the contribution ($\Gamma_{\text{NS}D}$) of NS decay (e.g., $D^0 \to s + \bar{d} + \text{gluon}$) via the annihilation graph is given by[8]

$$\Gamma_{\text{NS}D} = G_F^2 a_8^+ \alpha_s (m_D^5/648\pi^2)(f_D^2/m_u^2), \tag{3}$$

where $a_8^+ = (f_+ + f_-)^2/4$, $f_\pm$ being the usual coefficients that incorporate QCD renormalization effects on the weak Lagrangian. Thus, $\tau_{D^+}/\tau_{D^0} = (\Gamma_{SD} + \Gamma_{\text{NS}D})/\Gamma_{SD}$, Γ_{SD} being the neutral- or charged-D decay width via the spectator graph.[10] The experimental value (2e) then yields $f_D/m_u \simeq 2.0 \pm 0.5$.

For $B^0(\bar{B}^0)$ decays there are two types of NS contributions. First there is the three-body decay (e.g., $B^0 \to \bar{c} + u + \text{gluon}$). As a result of the large charm-quark mass one also has two-body modes (e.g., $B^0 \to \bar{c}u$) via the annihilation graph. For the three-body mode we have

$$\Gamma_{\text{NS}B}^{(3)} = G_F^2 a_8^+ \alpha_s \frac{m_B^5}{648\pi^2} \frac{f_B^2}{m_d^2} P_c |U_{bc}^* U_{ud}|^2, \tag{4}$$

where the U's stand for the KM angles in the usual notation and P_c is the phase-space correction that depends on m_c/m_B.[11] The numerical value of this decay width is, of course, controlled sensitively by f_B/m_d. We assume that

$$f_B^2/f_D^2 = m_D/m_B, \tag{5}$$

or since[8] $f_B^2 = 12|\psi_B(0)|^2/m_B$, Eq. (5) implies that $|\psi_B(0)|^2 = |\psi_D(0)|^2$. This is reasonable since in the nonrelativistic approximation (that we are using), $|\psi_{B,D}(0)|^2$ depends on the reduced mass $\simeq m_u = m_d$. Furthermore, in the same approximation and with the one-gluon exchange potential, for the hyperfine partners of K, D, and B mesons one has

$$(M_{K^*}^2 - M_K^2) \simeq (M_{D^*}^2 - M_D^2) \simeq (M_{B^*}^2 - M_B^2).$$

The existing data on $D(D^*)$ and $K(K^*)$ support this very well (better than 1%).[12]

Now all B decays can be cast in the obvious generic form

$$\Gamma_{B\to X} = \Gamma_B^0 (A_u |U_{bu}|^2 + A_c |U_{bc}|^2), \qquad (6)$$

where $\Gamma_B^0 = G_F^2 m_B^5/192\pi^3$ and A_u, A_c are functions of α_s, f_B, and phase space. So, since $\Gamma_{B\to X}$ is completely specified by A_u, A_c we introduce the shorthand notation $\Gamma_{B\to X}:(A_u, A_c)$ which should be used in conjunction with (6). Thus, for example, for $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, and Λ (the QCD scale) $=0.1$ GeV one has

$$\Gamma_B^e:(0.584, 0.290); \quad \Gamma_{SB}:(4.237, 1.955), \qquad (7)$$

where Γ_B^e, Γ_{SB} stand for the electronic and the total decay widths via the spectator graph. Therefore,

$$(0.584/4.237) = 14.1\% \leq R_{SB}^e \equiv \Gamma_B^e/\Gamma_{SB} \leq (0.290/1.955) = 15.5\%. \qquad (8)$$

Thus if one considers only the spectator decays the semileptonic branching ratio (R_{SB}^e) ranges from 14.1% to 15.5%[13] which is too high compared to the world average of $(11.6 \pm 0.5)\%$ given in Eq. (2c). Furthermore, since to an excellent approximation the semielectronic decays necessarily have to proceed via the spectator graph the largeness of the theoretical semielectronic branching ratio (7) compared to experiment (2c) indicates that the nonleptonic B decay width of the spectator model is an underestimate and must be augmented. Once the annihilation mechanism (described above) resulting in two- and three-body decays is included then (for the stated parameters and with $f_B/m_d = 1.2$)

$$\Gamma_{B^+}^{\rm tot} \equiv (\Gamma_{SB} + \Gamma_{{\rm NS}B^+}^{(3)} + \Gamma_{{\rm NS}B^+}^{(2)}):(4.615, 1.955), \qquad (9)$$

$$\Gamma_{B^0}^{\rm tot} \equiv (\Gamma_{SB} + \Gamma_{{\rm NS}B^0}^{(3)} + \Gamma_{{\rm NS}B^0}^{(2)}):(5.695, 2.965). \qquad (10)$$

Using (9) and (10) one finds for the semielectronic branching ratio of B^+ and B^0

$$12.8\% \leq R_{B^+}^e \leq 15.4\%;$$

$$9.5\% \leq R_{B^0}^e \leq 10.2\%. \qquad (11)$$

So, the average semielectronic branching ratio becomes

$$11.2\% \leq \bar{R}_B^e \equiv (R_{B^+}^e + R_{B^0}^e)/2 \leq 12.8\%, \qquad (12)$$

which is now completely compatible with experiment (2c). We thus find that treating the annihilation graph via the model of Ref. 8 as a phenomenological tool to constrain f_D/m_u and extrapolating f_B/m_d via Eq. (5) gives a valid description of decays of mesons containing c and b quarks.

Now we proceed to calculate the constraints on the KM parameters and the implications for the standard model. For the theoretical expression for the total decay width we take[14,15] $\bar{\Gamma}_B^{\rm tot} = (\Gamma_{B^0}^{\rm tot} + \Gamma_{B^+}^{\rm tot})/2$, and we demand that $\bar{\tau}_B = \bar{\Gamma}_B^{-1}$ be compatible (i.e., within 1σ) with the experimental result (2a). Similarly, using the standard theoretical expressions for the other three physical quantities [and again demanding that they stay within 1σ of the experimental values (2b)–(2d)] we search for constraints on the KM angles θ_2, θ_3, and δ as a function of the top-quark mass (m_t). The resulting lower and upper limits on these parameters as a function of m_t are shown in Fig. 1. The curves

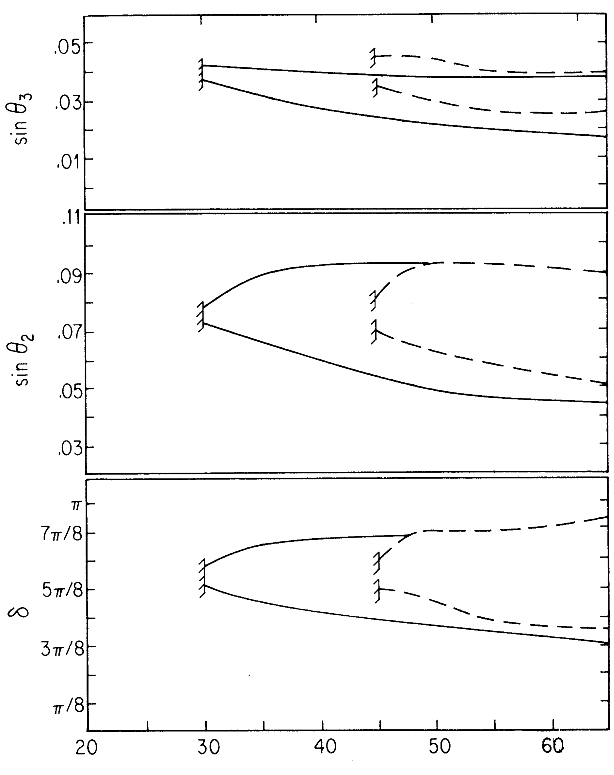

FIG. 1. Lower and upper bounds on KM parameters (Ref. 15). Dashed lines are for $B_k = 0.33$; solid lines are for $B_k = 0.50$.

show the limits for two values of B_k,[16] i.e., $B_k = 0.33$ and 0.50. We recall that $B_k = 0.33$ is the value calculated by use of SU(3) and current algebra.[17] I estimate the uncertainty in this number to be less than 50%.[18]

I have limited our considerations to $m_t < 65$ GeV since the constraints on the KM parameters (and consequently on the physical quantities to be discussed below) become less stringent and therefore less interesting if one allows for $m_t \gtrsim m_W$. We find that $m_t > 45$ GeV for $B_k = 0.33$ and $m_t > 30$ GeV for $B_k = 0.50$. This is of course similar to the findings of Ginsparg, Glashow, and Wise.[2] For the range of $30 \lesssim m_t \lesssim 65$ GeV we find that the KM parameters must lie in the narrow domain such that $0.015 \leqslant \theta_3 \leqslant 0.045$, $0.045 \leqslant \theta_2 \leqslant 0.095$, and $3\pi/8 \leqslant \delta \leqslant 15\pi/16$. In particular, for the range of $30 \leqslant m_t \leqslant 40$ GeV where very preliminary indications for the top quark from collider experiments are reported, one finds that the allowed domain of KM parameters is even more severely restricted: $0.025 \leqslant \theta_3 \leqslant 0.045$; $0.06 \leqslant \theta_2 \leqslant 0.095$; $\pi/2 \leqslant \delta \leqslant 13\pi/16$.[19] It is especially interesting that the phase δ is quite large ($\sin\delta > 0.2$ for $m_t \leqslant 65$ GeV and $\sin\delta > 0.5$ for $m_t < 40$ GeV). This means that the *CP*-nonconserving effects in $B^0(\overline{B}^0)$ and $B^{\pm}$ decays could be quite substantial.[20]

Figure 2 shows the allowed region for the ratio τ_{B^+}/τ_{B^0} versus the ratio τ_{D^+}/τ_{D^0} as implied by the above model for the annihilation graph and the constraints of Eq. (2). From Fig. 2 we note that $1.4 \leqslant \tau_{B^+}/\tau_{B^0} \leqslant 1.8$.

Another interesting parameter that depends sensitively on f_B is ΔM_B relevant to B^0-$\overline{B}^0$ mixing. Indeed, the quantity of direct experimental relevance is the number of same-sign dileptons divided by all dileptons that are obtained in e^+e^- annihilation via the production and subsequent decays of $B^0, \overline{B}^0$. This is given by $R_B \simeq [(\Delta M_B/\Gamma_B^{\text{tot}})^2/2]$ which increases as[4] $f_B^4 |U_{tb}^2 U_{td}^{*2}|$.[2] Using the experimental information (2) on B, D, and K decays and Eq. (5) for f_B we calculate R_B. The results are shown in Fig. 3. For $m_t \leqslant 40$ GeV, R_B ranges from 8% to 30%.[21]

Let us now briefly discuss the implications of the present calculations for ϵ'/ϵ whose importance for testing the standard model was first emphasized by Gilman and Wise.[22] Recently Gilman and Hagelin[4] have demonstrated that with the assumption of the bag-model calculation of certain "penguin" operators, the lower bound on ϵ'/ϵ is proportional to $s_2c_2s_3s_\delta$. In the present model with the resulting constraints I have evaluated the lower bound on ϵ'/ϵ as a function of m_t. The result is shown in Fig. 3. We find that $\epsilon'/\epsilon \geqslant 0.01$ for $m_t \leqslant 40$ GeV and $\epsilon'/\epsilon \geqslant 0.007$ for $m_t \leqslant 65$ GeV.[21] We recall that the existing experimental limit is $|\epsilon'/\epsilon| < 0.02$, and an experiment in progress could reduce this by about a factor of 5.[23]

Finally, we pursue the implications for the reaction $K \to \pi\nu\bar{\nu}$ due to the constraints obtained on KM parameters. Figure 3 shows the lower and upper bound on $B(K \to \pi\nu\bar{\nu})$ as a function of m_t. The dependence on m_t is rather inappreciable and

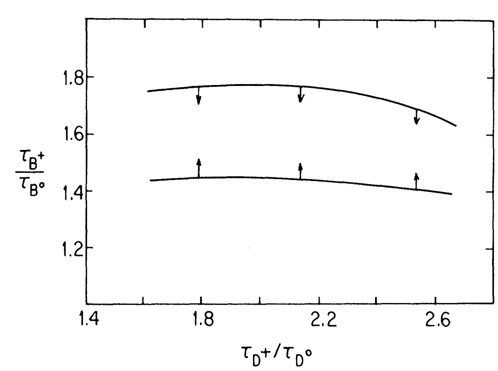

FIG. 2. Lower and upper bounds for τ_{B^+}/τ_{B^0} vs τ_{D^+}/τ_{D^0} (Ref. 15).

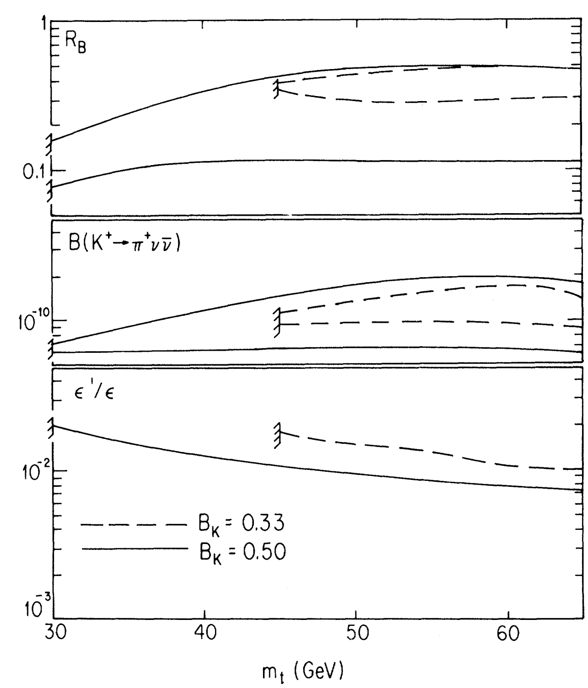

FIG. 3. Lower bound on ϵ'/ϵ, and lower and upper bounds on $B(K^+ \to \pi^+\nu\bar{\nu})$ and on $R_B \equiv$ (number of dileptons of the same sign)/(number of dileptons of the opposite sign) due to B-$\overline{B}$ mixing (Ref. 15). Note that $B(K^+ \to \pi^+\nu\bar{\nu}) = \Gamma(K^+ \to \pi^+\nu\bar{\nu})/\Gamma(K^+ \to \pi^0 e^+\nu_e)$.

$B(K \to \pi\nu\bar{\nu})$ ranges from $(0.6 \text{ to } 2)\times 10^{-10}$.[21] Thus, in the standard model the rate for this reaction is now severely constrained and the reaction can therefore serve as a viable probe of new phenomena.

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[8]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 4, 962(E) (1980).

[9]See also J. P. Leveille, Michigan University Report No. UM-HE81-18 (unpublished); L. Maiani, J. Phys. (Paris), Colloq. **43**, C3-631 (1982), and references therein.

[10]We caution the reader that alternative explanations for the D^+-D^0 lifetime difference exist. See, e.g., P. Rosen, Phys. Rev. Lett. **44**, 4 (1980).

[11]For mass effects see J. Cortes, X. Pham, and A. Tounsi, Phys. Rev. D **25**, 188 (1982).

[12]See also Maiani, Ref. 9.

[13]On varying m_b and m_c over the range $4.4 < m_b < 4.7$ GeV and $1.4 < m_c < 4.6$ GeV we find $13.6 \leq R_{SB}^{\xi} \leq 15.5\%$.

[14]In the numerical calculation of the total decay widths all (i.e., Cabbibo suppressed and allowed) tree graph spectator and nonspectator decays are included. The masses of u,d,s quarks occurring in the final state are set equal to zero.

[15]The bounds on KM angles and other physical quantities have been obtained by demanding that the theoretical expressions remain within 1σ of the experimental values given in (2). These bounds also include an arbitrary variation of m_b, m_c, and f_B over the range $4.4 < m_b < 4.7$ GeV, $1.4 < m_c < 1.6$ GeV, and $0.9 \lesssim f_B/m_d \lesssim 1.4$.

[16]B_k is the ratio of the matrix element $\langle K^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|\bar{K}^0\rangle$ evaluated in any given model divided by its value using vacuum saturation, e.g., $B_k = 0.33$ in the model of J. F. Donoghue, E. Golowich, and B. R. Holstein [Phys. Lett. **119B**, 412 (1982)].

[17]Donoghue, Golowich, and Holstein, Ref. 16.

[18]$B_k \lesssim 0.25$ requires $m_t \gtrsim 65$ GeV which we are not considering in this paper. For $0.25 \leq B_k \leq 0.33$ the bounds on KM angles are not too different from those for $B_k = 0.33$.

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[21]It may be useful to point out that our approach is different from that of Ref. 4 in that not only are we including the effects of the annihilation graphs but also all the experimental constraints of Eq. (2) are simultaneously imposed and further $m_t < 65$ GeV is assumed. As a result, while we are in rough agreement with Ref. 4 on ϵ'/ϵ, our bounds on θ_2, R_B, and $B(K \to \pi\nu\bar{\nu})$ are somewhat different.

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