Implications of a Class of Grand-Unified Theories for Large-Scale Structure in the Universe

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We consider a class of grand-unified theories in which cosmologically significant axion and neutrino energy densities arise naturally. To obtain large-scale structure we consider (1) an inflationary scenario, (2) inflation followed by string production, and (3) a noninflationary scenario with density fluctuations caused solely by strings. We show that inflation may be compatible with the recent observational indications that $\Omega < 1$ on the scale of superclusters, particularly if strings are present.

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Axions with mass on the order of 10^{-3}–10^{-4} eV have been suggested as candidates for the dark matter in galactic halos.[1,2] It has also been shown that axions with a cosmologically significant energy density provide an important component in the mechanism for generating structure in the universe on scales up to $10^{15} M_\odot$.[3,4] In this picture, axions, being gravitationally unstable on all scales, will cluster first, providing the seed potential wells for galaxy formation so that the galaxy distribution on scales up to $\sim 10^{15} M_\odot$ clusters would naturally follow the axion mass distribution. Observational support for such a relationship is discussed by Blumenthal *et al.*[5] They point out that the ratio of dark to luminous mass is roughly constant up to the scale of rich galaxy clusters.

An SO(10) grand-unified theory (GUT) framework which leads to the production of cosmologically significant axions has been given.[6] In this Letter, we first argue that within this class of models (and suitable extensions thereof such as E_6), a cosmologically significant neutrino mass is obtained naturally. We then proceed to discuss some cosmological implications of this result for the formation of structure in the universe within the context of three different scenarios: (1) an inflationary scenario, (2) an inflationary scenario followed by string production, and (3) a noninflationary scenario with density fluctuations produced solely by strings.

As an example of a grand-unified theory which gives $\Omega_a \simeq \Omega_\nu$, consider the following SO(10) model[6] [the global U(1) Peccei-Quinn symmetry[7] is not explicitly exhibited]:

$$\mathrm{SO}(10) \xrightarrow[M_x \sim 10^{15}\,\mathrm{GeV}]{} \mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \xrightarrow[f_a \sim 10^{12}\,\mathrm{GeV}]{} \mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1) \xrightarrow[M_W \sim 100\,\mathrm{GeV}]{} \mathrm{SU}(3) \otimes \mathrm{U}(1)_{\mathrm{em}}. \tag{1}$$

Both the global U(1) symmetry and the local $B-L$ symmetry are broken at a scale of order 10^{12} GeV. (Note that the value of the intermediate scale is not put in by hand, but is determined from the renormalization-group equations of the gauge couplings.) From the results of Ref. 1, it follows that $\Omega_a \simeq 0.1$–1.

Let us now consider neutrino masses in this model. The breaking of $B-L$ at scale f_a, caused by a **126**-plet of Higgs fields, induces a Majorana mass term for the right-handed neutrino ν_{Ri} of order $h_i f_a$, where h_i denotes the Yukawa coupling of the ith generation. The breaking of SU(2) ⊗ U(1) to $\mathrm{U}(1)_{\mathrm{em}}$ is achieved by a Higgs decaplet and gives rise to Dirac mass terms $m_{\nu_i}^{(D)} = m_{u_i}$ (where u_i denotes $u, c, t, \ldots$) linking the left- and right-handed neutrinos. Moreover, it can be shown that an effective Majorana mass term for the left-handed neutrino ν_{Li} of order $c_i \simeq h_i (\lambda_1/\lambda_2) \langle\phi_{10}\rangle^2/f_a$ is also induced.[8] Here λ_1 denotes the quartic Higgs coupling between the **126** and the **10**, λ_2 is the quartic self-coupling of **126**, and $\langle\phi_{10}\rangle$ is the vacuum-expectation value of the **10**. With $f_a \simeq 10^{12}$ GeV, λ_1/λ_2 of order unity, and $h_i \sim O(g^2)$ [where g denotes the SO(10) gauge coupling], c_i is in the electronvolt range. Diagonalization of the neutrino mass matrix (neglecting, for simplicity, mixings between generations) yields the eigenvalues

$$(m_{\nu_i})_{\text{heavy}} \simeq h_i f_a,$$
$$(m_{\nu_i})_{\text{light}} \simeq c_i - m_{ui}^2/(m_{\nu i})_{\text{heavy}}. \tag{2}$$

It follows from Eq. (2) that electronvolt neutrino masses arise naturally in the class of models under discussion. Indeed, as a result of the presence of the c_i term in the mass matrix, the light neutrino of each generation can have a mass in the electronvolt range. Thus, neutrinos can contribute significantly to the dark matter in the universe.

We now discuss the implications of significant axion and neutrino energy densities for the evolution of structure in the universe. Two mechanisms for producing density fluctuations in the early universe have been extensively discussed, viz., inflation[9] and strings.[10] Recently, it was pointed out[11] that one could obtain another scenario in which inflation is followed by string production.

The inflationary phase is associated with the transition from SO(10) to $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. It can be implemented by generalizing the arguments of Shafi and Vilenkin[12] where the SU(5) model is discussed. The breaking of $B-L$ and the U(1) symmetry can occur during, or at the end of, the inflationary era. The spectrum of density fluctuations produced in this scenario is essentially of the Harrison-Zel'dovich[9] type.

According to recent observations,[13] the value for Ω obtained on scales up to $\sim 10^{15} M_\odot$ is $\simeq 0.2 \pm 0.1$, considerably less than unity, the value predicted by the new inflationary cosmology. As a reasonable upper limit for Ω_{sc} of superclusters,[14] we may take $\Omega_{\text{sc}} \lesssim 0.5$. Therefore, since axions and baryons cluster on scales smaller than rich clusters and superclusters, their contribution to Ω must be $\lesssim 0.5$. The balance of the total Ω in the universe must therefore be in the mass density of a neutrino component which is not traced by the galaxy distribution if we are to have $\Omega = 1$.

We must therefore require that the neutrinos be light enough so that they will not cluster on scales below $\sim 10^{16} M_\odot$. In order to arrange this, especially since the neutrino Jeans mass drops significantly between the redshift z_{nr} when the neutrinos become nonrelativisitic and the present time, we invoke neutrino phase-space limits using the arguments of Tremaine and Gunn[15] in reverse to get an upper limit on m_ν. These authors find that for neutrinos to be able to cluster on the scale of rich clusters, their mass must be greater than $\sim 4 h_{50}^{-1/2}$ eV (where h_{50} is the Hubble constant in units of 50 km $\text{s}^{-1}\,\text{Mpc}^{-1}$).

The neutrino contribution to Ω is $\Omega_\nu = 4.56 \times 10^{-2} [m_\nu/(1 \text{ (eV)}] N_f h_{50}^{-2} T_{2.8}^3$ where N_f is the number of neutrino flavors of approximately equal mass and $T_{2.8}$ is the present temperature of the cosmic blackbody radiation in units of 2.8 K. We require Ω_ν to be $\gtrsim 0.5$ so that the total $\Omega = 1$. For this, one needs at least three flavors of neutrinos, each of approximately 3–4 eV. As discussed above, this situation is readily obtained in the SO(10) model. [If the neutrino clustering is inefficient (see discussion by Bond, Szalay and White)[16], m_ν could be larger and N_f smaller.]

The maximum neutrino Jeans mass for three neutrinos of roughly equal mass is[17] $M_{J\nu}^* = 2.7 \times 10^{18} [m_\nu/(1 \text{ eV})]^{-2} M_\odot$, which for $N_f = 3$ and $m_\nu \simeq 3.6$ eV gives $M_{J\nu}^* \simeq 2 \times 10^{17} M_\odot$. The corresponding spatial scale at present for pancaking structure would be ~ 150 Mpc. It is interesting to note that this scale may correspond to the tentative "superpancaking" scale proposed recently by Dekel[18] in order to attempt to account for the correlation function of clustering of superclusters.[19] Structure on this scale would have to correspond to density perturbations $\delta \equiv \delta\rho/\rho$ just becoming nonlinear $(\delta = 0.5\text{–}1)$ at the present time.

The spectrum of perturbations in a universe dominated by axions and neutrinos is readily estimated by adopting the arguments previously given for a baryon-neutrino universe.[20] It is convenient to define $\xi \equiv \Omega_a/(\Omega_a + \Omega_\nu)$ such that $\xi \leq \frac{1}{2}$. (We assume, for simplicity, that $\Omega_b << \Omega_a, \Omega_\nu$.)

For $z < z_{\text{eq}} \simeq 0.93 \times 10^4 (1-\xi)^{-1} \Omega_\nu h_{50}^2 T_{2.8}^{-4}$ the neutrino Jeans mass decreases as $(1+z)^{3/2}$. (Here z_{eq} is the redshift corresponding to equal matter and radiation densities in the universe.) Neutrino perturbations on scales below $M_{J\nu}^*$ are erased at $z \simeq z_{\text{eq}}$. The axion perturbations, however, grow like

$$\delta\rho_a/\rho_a \equiv \delta_a \propto t^\alpha \propto (1+z)^{-3\alpha/2}, \tag{3}$$

where $\alpha = [(1+24\xi)^{1/2} - 1]/6$. (The growing-mode solution is similar to that obtained for the baryon-neutrino hybrid scenario after decoupling.[20]) Thus,

$$\delta_a(z) \simeq \delta_a(z_{\text{eq}}) \left(\frac{1+z_{\text{eq}}}{1+z} \right)^{3\alpha/2}. \tag{4}$$

This continues until $z \simeq z_M$ when the neutrino Jeans mass becomes $\simeq M$,

$$(1+z_M) \simeq (M/M_{J\nu}^*)^{2/3} (1+z_{\text{eq}}). \tag{5}$$

For $z < z_M$ the overall density fluctuation is

$\delta\rho/\rho \propto t^{2/3} \propto (1+z)^{-1}$. Thus,

$$\frac{\delta\rho}{\rho}(z < z_M) \simeq \xi\delta_a(z_M)\left[\frac{1+z_M}{1+z}\right] \simeq \xi\delta_a(z_{\rm eq})\left[\frac{1+z_{\rm eq}}{1+z}\right]\left[\frac{M}{M^*_{J\nu}}\right]^{(2/3-\alpha)}. \quad (6)$$

As a rough approximation, $\delta_a(z_{\rm eq}) \approx$ const when $M < M^*_{J\nu}$ for a Zel'dovich spectrum. (See, however, Ref. 21.) This gives

$$\delta\rho/\rho \propto M^{(2/3-\alpha)} \quad (M < M^*_{J\nu}) \quad \text{(inflation alone)}, \quad (7)$$

which is an *increasing* function of M since $\alpha < \frac{2}{3}$. For $M > M^*_{J\nu}$, the neutrino perturbations are not damped and $\delta\rho/\rho \propto M^{-2/3}$.

From this discussion it appears that even in the most optimistic case where $\xi = \frac{1}{2}$, we have $\alpha = 0.43$, so that the scales between the present neutrino Jeans mass and $M^*_{J\nu}$ may not collapse before $M^*_{J\nu}$ does. We thus run into the timing problems which are becoming well known for the neutrino pancaking scenario. In particular, it is hard to envision the development of quasars[22] and substructure[23] with such a model, although the situation here is not as difficult as that with pure neutrino pancakes as a result of the presence of axions,[21] as we discuss below.

The presence of strings, which provide an additional source of density fluctuations, can eliminate the above difficulty.[24] Assume that topologically stable strings, with mass per unit length characterized by a superheavy (GUT) scale, appear at or near the end of the inflationary phase. A specific example showing how this could occur is shown in Ref. 11. In the present case this is readily achieved either by the appending of a new spontaneously broken global U(1) symmetry to the SO(10) model or by use of an E_6 model. As a result of the presence of strings, and, in particular of closed loops,[25] $\delta_a(z_{\rm eq}) \propto M^{-1/3}$ for $M < M^*_{J\nu}$. Substitution in Eq. (6) then gives

$$\delta\rho/\rho \propto M^{(1/3-\alpha)} \quad (M < M^*_{J\nu}) \quad \text{(string loops)}, \quad (8)$$

as compared with the results of Eq. (7) when loops are not present.

Using Eq. (8) with $\xi = \frac{1}{2}$ and $\alpha = 0.43$, we find $\delta\rho/\rho \propto M^{-0.1}$. Therefore, if $\delta\rho/\rho \sim O(1)$ on scales $\sim (10^{16}\text{–}10^{17})M_\odot$ at $z = 0$ as suggested by Dekel,[8] scales $\sim 10^{10} M_\odot$ went nonlinear at $z \simeq 4$, corresponding to the epoch of quasar formation. Thus, in the presence of axions and neutrinos, an inflationary scenario supplemented by strings (or wall-string systems[24]) appears to offer a better prospect of explaining the observed large-scale structure in the universe than one without strings. Of course, more detailed numerical calculations and clustering simulations should be performed to test this conclusion. In fact, growth of axion perturbations during the radiation era[26] will have the effect of increasing α to $\alpha_{\rm eff} = \alpha + \epsilon$. This effect may be enough to make the spectrum in the case of inflation without strings flat at low M. In the string-inflation scenario, this effect eases the requirement on Ω_a needed for an acceptable $\alpha_{\rm eff}$, making $\Omega_a < 0.5$ (as indicated by the observations) acceptable.

Finally, let us discuss the scenario in which we dispense with inflation and density fluctuations are produced solely by strings. In this case, since the density parameter Ω need not be unity, ξ can be greater than $\frac{1}{2}$ and α can be > 0.434. (Of course, we need have only one ν flavor in the electronvolt mass range to get Dekel's[18] scale.) In particular for $\Omega_a >> \Omega_\nu$, $\alpha = \frac{2}{3}$. A natural extension of SO(10) which gives the desired strings[25] is provided by the following breaking of E_6 [once again the global U(1) Peccei-Quinn symmetry is broken at the same scale as $B-L$]:

$$E_6 \xrightarrow[\sim 10^{16}\,\text{GeV}]{} SO(10) \otimes Z_2 \rightarrow SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes Z_2 \rightarrow SU(3) \otimes SU(2)_L \otimes U(1) \otimes Z_2. \quad (9)$$

For E_6 symmetry breaking at a scale $\eta \sim 10^{16}$ GeV, the energy per unit length of the strings formed is $\mu \sim \eta^2 \simeq 10^{32}$ GeV^2. With this value of μ, it follows from the discussion of Ref. 25 that in this scenario neutrino perturbations would be on the verge of becoming nonlinear at the "superpancake" scale at the present time, as suggested by observations.[18,19]

To conclude, significant axion and neutrino energy densities arise naturally in a class of grand-unified theories. An axion-neutrino–dominated universe model for the formation of large-scale structure may avoid the problems associated with the pure neutrino-dominated pancake models. These models also allow for structure on scales greater than that given by the pure hierarchical clustering models of galaxy formation, which may be desirable in view of some recent analyses suggesting the clustering of clusters. Finally, the prediction of the new inflationary cosmology that Ω be unity can be reconciled with the observation $\Omega_{sc} < 1$ in this framework, particularly if string loops (or string-wall systems) are present.[26]

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