Distinctive Signatures for Quantum Chromodynamics in Nuclear Physics

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(Received 15 December 1983)

For individual helicity amplitudes, asymptotic QCD gives results which differ from standard nuclear physics. For the deuteron electromagnetic form factors, QCD predicts $G_C = (Q^2/6M_d^2)G_Q$ at high Q^2. This gives a result for the polarization ratio p_x/p_{xz} which is dramatically different from predictions of several standard deuteron wave functions. Polarization-transfer measurements of the deuteron form factor in this Q^2 region are therefore a sensitive test of the validity of perturbative QCD for exclusive processes at these momentum transfers.

PACS numbers: 12.35.Eq, 13.40.Fn, 25.10.+s, 25.30.Bf

As QCD[1] is thought to be the fundamental theory of strong interactions, it is important to contrast the consequences of QCD with those of classical nuclear physics. The latter is a phenomenological theory written in terms of nucleons bound together by a finite number of different types of mesons, valid within certain limits. Often, the predictions of QCD can be matched by classical nuclear physics. It is important to search for situations where this cannot be the case, and to see for what parameter, or what momentum transfers, one theory begins to fail and the other to succeed.

In this note we focus on high-Q^2 elastic electron-deuteron scattering. The QCD result[2,3] that the form factor $A^{1/2}(Q^2)$ falls like Q^{-10} can be matched by classical nuclear physics, as may be seen below. However, we shall show that, with respect to the spin dependence of this process, the results from QCD and classical nuclear physics are not the same. In particular, QCD gives a unique relation between the charge and quadrupole form factors of the deuteron at high Q^2,

$$\lim_{Q^2 \to \infty} G_C = \tfrac{2}{3}\eta G_Q, \tag{1}$$

where $\eta = Q^2/4M_d^2$.

To get the Q^2 dependence of the form factors at high Q^2 it suffices to consider the deuteron as a collection of parallel-moving constituents,[2,3] six quarks in QCD, as in Fig. 1(a). One of the quarks absorbs a virtual photon of momentum q, with $Q^2 = -q^2 > 0$. To rebind the deuteron the momentum must be shared equally among the six quarks. We deal with a Q high enough to be much greater than the mean Fermi momentum of the quarks. The Fermi momentum distribution will determine how much deviation from equal sharing of momentum is allowed and so sets the scale of normalization but does not determine the asymptotic dependence of the amplitude on Q^2.

The Q^2 and spin dependence may be got from some simple rules.[4] We will use Fig. 1(a) as an example and comment later on why all the many other diagrams give the same result. We start by giving three rules[4] which can most easily be verified in the Breit frame.

The one-gluon rule [Fig. 1(b)]. This part of the larger diagram either is proportional to Q and conserves quark helicity or is proportional to m (a mass scale) and flips quark helicity according to whether the absorbed gluon is transverse (T) or longitudinal (L), respectively.

The two-gluon (or gluon-photon) rule [Fig. 1(c)]. If one gluon is absorbed and one emitted, the largest amplitude is constant in Q^2 (this includes the quark propagator but not the gluon propagators) and is the case where one gluon is transverse and the other longitudinal, and quark helicity is conserved. If both gluons are longitudinal the amplitude is $O(m/Q)$ with quark helicity flipped and if both gluons are transverse the amplitude is zero.

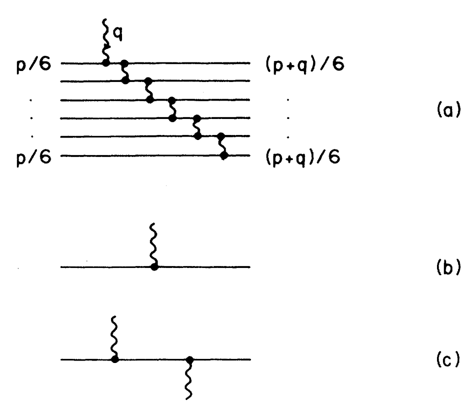

FIG. 1. Elastic *ed* scattering at high Q^2 with the momentum of the virtual photon shared equally among the deuteron's constituents. (b), (c) Subgraphs of (a).

The transverse gluon rule. Two quark lines connected by a transverse gluon have opposite helicities. This follows because the helicity direction of an absorbed transverse gluon is the same as the helicity direction of the quark that absorbs it; for an emitted transverse gluon the directions are opposite. There is no helicity correlation for quark lines connected by a longitudinal gluon.

The largest helicity amplitude for the deuteron falls like Q^{-10} compared with the leading amplitude for a single pointlike particle and is a helicity $0 \to 0$ transition.[5] To get this largest amplitude for Fig. 1(a), the bottom gluon must be T, and then L and T must alternate. Three pairs of quarks are connected by T gluons and so must have pairwise opposite helicities, and the total helicity of both the initial and final state is zero. Amplitudes with other initial and final helicities may be considered and are suppressed by powers of m/Q that can be determined by use of the rules given above.

To express the results we define[6] the matrix elements of the electromagnetic current J,

$$G_{I,\lambda\lambda'} = \langle d', \lambda' | \epsilon_I \cdot J | d, \lambda \rangle, \tag{2}$$

where ϵ_I, $I = t$ or L, is the polarization vector of the photon and d and λ are the momentum and helicity of the incoming deuteron. There are three independent G_I, and they are given in the Breit frame in terms of the charge, magnetic, and quadrupole form factors by

$$\begin{aligned} -(Q^2+4M_d^2)^{-1/2} G_{L,00} &= G_C + \tfrac{4}{3}\eta G_Q, \\ (Q^2+4M_d^2)^{-1/2} G_{L,+-} &= G_C - \tfrac{2}{3}\eta G_Q, \\ -(Q^2+4M_d^2)^{-1/2} G_{T,+0} &= \eta^{1/2} G_M. \end{aligned} \tag{3}$$

The analysis of Fig. 1(a) leads to, for high Q^2,

$$\begin{aligned} Q^{-1} G_{L,00} &= \text{const} \times Q^{-10}, \\ Q^{-1} G_{L,+-} &= \text{const} \times \eta^{-1} Q^{-10}, \\ Q^{-1} G_{T,+0} &= \text{const} \times \eta^{-1/2} Q^{-10}, \end{aligned} \tag{4}$$

so that G_C falls like Q^{-10}, G_M and G_Q fall like Q^{-12}, and there is a leading-order cancellation in $G_{L,+-}$ giving Eq. (1).

Diagrams like Fig. 2(a), where more than two gluon or photon lines attach to a given quark, require an extension of the rules given above (e.g., if n gluon and photon lines attach, the largest amplitudes go like Q^{2-n}, they have an odd total number of T gluons and photons which have alternate-sign helicities if all are considered incoming, and the conserved quark helicity has the same sign as the majority of T gluons and photons) but result in no

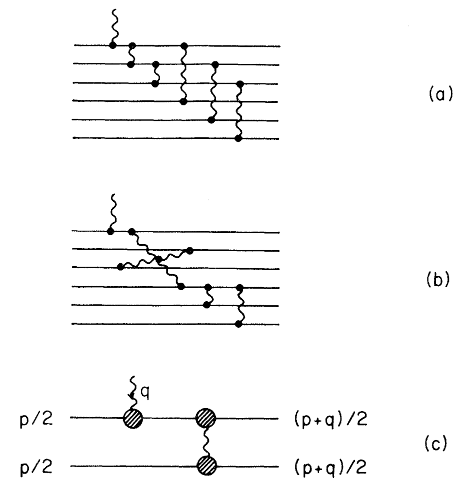

FIG. 2. (a),(b) Additional graphs for ed elastic scattering. (c) High-Q^2 ed elastic scattering in a neutron-proton model.

change to the power counting. The same conclusion holds if the quartic gluon coupling is involved [Fig. 2(b)]; the cubic gluon coupling does not enter in the collinear approximation.

Treating the deuteron as two collinear nucleons can give the high-Q^2 limit of the form factors from classical nuclear physics [Fig. 2(c)]. Allowing only vector-meson exchanges with only γ_μ couplings gives the same helicity results as QCD. Also, if we put a dipole form factor at the γNN vertex and a monopole form factor at each meson-N-N vertex, then a Q^{-10} falloff for the leading form factor follows.[7,8] Repeating the exercise for scalar and pseudoscalar meson exchanges also gives a Q^{-10} leading falloff. However, these exchanges flip the fermion spin to leading order in m/Q so that the helicity predictions are not the same. In particular, the helicity amplitude $G_{L,\pm}$, from which the result (1) is obtained, is dominated by scalar, pseudoscalar, and anomalously coupled vector-meson exchange at high Q^2. The ratio G_C/G_Q will then depend on the relative size of the scalar, pseudoscalar, and vector coupling constants and asymptotic meson-N-N form factors.

Perturbative QCD predictions are certainly valid at sufficiently high Q^2, but it is not clear that they are valid for exclusive processes in the Q^2 range of 1–4 $(\mathrm{GeV}/c)^2$, as has been strongly claimed in the literature.[3,5] Recently, Isgur and Llewellyn Smith[9] have given simple estimates which cast doubt on the validity of these calculations at low momentum transfers. In this Letter we do not take a stand on whether or not calculations of the type given here are valid for Q^2 as low as 1–2 $(\mathrm{GeV}/c)^2$. Instead, we emphasize that a stringent and possibly clean *test* of their validity comes from examining G_C and G_Q separately. This leads us to recall, now as a QCD test, one of the methods that has been suggested[10] for separating the G_C and G_Q form factors, namely, measuring the polarization ratio p_x/p_{xz}. The vector polarization p_x of the outgoing deuteron need not be zero if the initial electron is polarized, and p_{xz} is a tensor polarization. Since we are taking ratios of form factors, if the causes of the nonleading corrections are similar, they may be expected to cancel out. We have[10]

$$I_0 p_x = -\tfrac{4}{3}[\eta(1+\eta)]^{1/2} \times G_M(G_C + \tfrac{1}{3}\eta G_Q)\tan\tfrac{1}{2}\theta, \tag{5}$$

$$I_0 p_{xz} = -2\eta[\eta + \eta^2 \sin^2\tfrac{1}{2}\theta]\tfrac{1}{2}\theta]^{1/2} \times G_M G_Q \sec\tfrac{1}{2}\theta,$$

where $I_0 = A + B\tan^2\theta/2$. Thus,

$$\lim_{Q^2\to\infty} \frac{p_x}{p_{xz}} = \frac{2}{3}\frac{G_c + \frac{1}{3}\eta G_Q}{\eta G_Q} = \frac{2}{3}, \tag{6}$$

where the last equality used Eq. (1).

A plot of p_x/p_{xz} for several classical nuclear physics models of the deuteron wave function is shown in Fig. 3. The Q^2 on this plot are not "asymptotic," but are in the range where agreement with QCD is claimed for $A^{1/2}(Q^2)$. There is a near unanimity among the classical nuclear physics models, the exception being the Lomon-Feshbach model which sets the wave function equal to zero inside radial separation 0.8 fm. The classical nuclear physics predictions differ in both sign and magnitude from the asymptotic QCD prediction. If we suppose that the form factors G_C and G_Q have their asymptotic ratio and take account of the kinematic factors given in Eq. (5), the QCD p_x/p_{xz} ratio is reduced for $\theta < 180°$ (Fig. 3) but the difference from classical nuclear physics is still dramatic.

In summary, we have pointed out that while overall results from QCD and classical nuclear physics can be rather similar, individual helicity amplitudes can be quite different. We have given one example of this as a prediction of a definite ratio for two of the deuteron's electromagnetic form factors. The predictions from classical nuclear physics are quite different from perturbative QCD in the range $Q^2 \gtrsim 1$ $(\mathrm{GeV}/c)^2$. There are strong statements in the literature[3,5] that asymptotic QCD behavior is already seen in the spin-averaged form factor for Q^2 this low; it would be dramatic to see also asymptotic

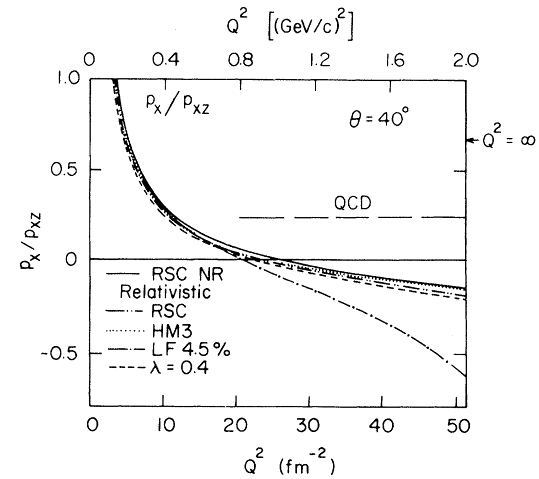

FIG. 3. The polarization ratio p_x/p_{xz} calculated for $\theta = 40°$ with the use of deuteron wave functions with relativistic and nonrelativistic impulse approximations (see Ref. 9 for details) compared with the QCD predictions from Eqs. (5) with $G_C = \frac{2}{3}\eta G_Q$. The $Q^2 \to \infty$ QCD result is also indicated.

QCD behavior in spin-nonaveraged form factors. Measurements of the relevant polarization may be quite feasible with polarized electron beams and an additional analyzing scattering and, if not done before the forthcoming SURA (Southeastern Universities Research Association) machine is ready, will add further to that facility's interest.

We wish to thank S. J. Brodsky and H. C. von Baeyer for helpful comments.

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