Friction and Particle-Hole Pairs

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(Received 3 May 1984)

We investigate the dissipative motion of a particle coupled to a normal Fermi fluid, where particle-hole pairs are the relevant low-energy excitations. It is shown how to derive an effective action by use of linear-response theory and integrating out of the fermions' degrees of freedom. Its properties are discussed in detail, with particular emphasis on the similarities and connections with other descriptions of friction widely used in the literature.

PACS numbers: 74.50.+r, 05.30.Fk, 72.15.Qm, 72.70.+m

A question which recently has raised much interest is the effect induced by dissipation on quantum phenomena.[1,2] The standard starting point is a phenomenological Hamiltonian in which the environment is simulated by an appropriately chosen set of harmonic oscillators.[2] This makes the quantum treatment of the system particularly simple, and general arguments show that it should be adequate to describe the low-energy behavior of a wide class of environments.[2] In particular, a microscopic Hamiltonian derived for Josephson junctions[3] has closely related properties.

In this work we will analyze the case in which the environment is a gas (or liquid) of fermions, and the relevant low-energy excitations are particle-hole pairs. Many such systems are known to induce friction in the classical limit,[4,5] and we will investigate to which extent the quantum results obtained for harmonic oscillators[6-10] are also valid in this situation. While many physical realizations are possible, we will mostly deal with the case of a heavy charged particle interacting with the valence electrons of a metal, either inside or at the surface,[4,11,12] so we will also allow the fermions to interact between themselves. For normal Fermi systems, the Landau theory prevents a qualitative change of the low-energy excitations, even in the presence of interactions.

That these systems do induce friction and velocity-dependent forces on an external particle can be proven by rigorously deriving the Fokker-Planck equation.[4,5] A more straightforward way is, however, to calculate the energy dissipated per unit time by a charge moving with constant velocity. Using linear response theory,[13] we have (for the electron gas)

$$\frac{dE}{dt}=Q^2\int \vec{v}\cdot\vec{q}\,V_q\,\mathrm{Im}[\epsilon^{-1}(\vec{q}\cdot\vec{v},q)]\,d^3q, \qquad (1)$$

where $\vec{v}$ is the velocity, V_q the Coulomb potential, Q the charge of the particle, and ϵ^{-1} the dielectric constant of the electron gas. At low velocities we can write

$$dE/dt=\eta v^2, \qquad (2)$$

where

$$\eta=\tfrac{1}{3}Q^2\int q^2V_q\frac{\partial}{\partial\omega}\{\mathrm{Im}[\epsilon^{-1}(\omega,q)]\}|_{\omega=0}d^3q, \qquad (3)$$

in agreement with the result expected for a dissipative medium with friction coefficient η.[14]

In order to make connection with the treatment of quantum effects for the case of harmonic oscillators, we will use the path-integral formalism and derive an effective action which only involves the coordinates of the external particle. The Lagrangian of the system is (in imaginary times)

$$L=\tfrac{1}{2}M\dot{R}^2+L_0+Q\int V(R-r)\rho(r)\,d^3r, \qquad (4)$$

where L_0 describes the fermion environment, V is the interaction potential between the external particle and the bath, and $\rho(r)$ stands for the fluctuations in the charge density of the fermions. We assume the system to be neutral, so that the mean value of $\rho(r)$ is zero.

We now need to integrate out the fermion degrees of freedom for each possible path of the external particle, $R(t)$. That is, we have to compute

$$Z(R(t))=\mathrm{Tr}\langle\exp[-S_{\mathrm{int}}(R(t),r_i)]\rangle, \qquad (5)$$

where r_i stands for the set of fermion coordinates, and we are dealing only with the coupling term in the Lagrangian (4). Unlike the case of harmonic oscillators, the integral implied in (5) is not Gaussian, and cannot be performed exactly, even for noninteracting fermions. We can, however, expand (5) in terms of the coupling between the particle and the environment, $Q(t)$, which, for convenience, we will assume to be time dependent. With

Fourier transformation of (4), we can write

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial Q(t)\,\partial Q(t')} = -\int \exp\{i\vec{q}\cdot[\vec{R}(t)-\vec{R}(t')]\}\, V_q^2 \langle \rho_q(t)\rho_{-q}(t')\rangle\, d^3q, \tag{6}$$

where the average is to be taken over the fermions' degrees of freedom, and the correlation function is a time-ordered product. With neglect of higher-order derivatives, Z is given by

$$Z \simeq \exp\left[-Q^2\int dt\int dt'\int d^3q\, \exp\{i\vec{q}\cdot[\vec{R}(t)-\vec{R}(t')]\}\, V_q^2\langle \rho_q(t)\rho_{-q}(t')\rangle\right] \tag{7}$$

which is the desired effective action within the linear-response approximation.

It is illustrative to consider (7) in terms of Feynman diagrams. The expansion of $\log(Z)$ in powers of the coupling constant is equivalent to a cumulant expansion in which all self-energy insertions with two external legs are included, as shown in Fig. 1(a); this result is exact if (5) were a Gaussian integral, as with harmonic oscillators. On the other hand, we are neglecting "two-electron-hole" diagrams like the ones in Fig. 1(b), which amounts, as mentioned above, to work within linear response. Finally, the density-density correlation function in (7) can include effects due to interactions among the fermions, as schematically depicted in the renormalized bubbles in Fig. 1(a).

Macroscopic friction is related to the low-energy behavior of the environment,[1,2] which, in turn, determines the long-time features of the effective action (7). Taking the Fourier transform of $\langle\rho_q(t)\rho_{-q}(t')\rangle$, and expanding it for low energies, we have

$$\langle \rho_q\rho_{-q}\rangle_\omega \simeq |\omega| f(q) + \ldots . \tag{8}$$

Then, the effective action in (7) can be written as

$$\begin{aligned} S_{\text{eff}}(R(t)) &= Q^2\int dt\int dt'\int d^3q\; V_q^2 f(q)\exp\{i\vec{q}\cdot[\vec{R}(t)-\vec{R}(t')]\}(t-t')^{-2} \\ &= Q^2\int dt\int dt'\, g(R(t)-R(t'))(t-t')^{-2}, \end{aligned} \tag{9}$$

where the function g is the Fourier transform of $V_q^2 f(q)$. This action is very close to the one analyzed in Refs. 1 and 2, and includes, as a particular case, the action derived in Ref. 3. Moreover, for small values of $R(t)-R(t')$, the first term in (9) which depends on the position of the particle is of the form

$$S(R(t)) \simeq \int dt\int dt'\,\eta\,|R(t)-R(t')|^2|t-T'|^{-2}, \tag{10}$$

where η can be identified as the friction coefficient of the system. In the particular case of the electron gas, where V_q is the Coulomb interaction, it coincides with the definition given in (3), as expected.

So far, the arguments have been very general, and only require the validity of the linear-response approximation, and the expansion (8). Linear response should be widely applicable, as discussed in detail in Ref. 2. Formula (8) is related to the amount of phase space available for the creation of particle-hole pairs, i.e., is a kinematical constraint, independent of the details of the interaction between the external particle and the environment.[15] This means that dissipation is a general feature of fermion environments. On the other hand, standard coupling to phonons cannot have the same long-time behavior, because momentum conservation prevents phonon emission at velocities smaller than the velocity of sound.

We will now analyze the possible effects due to the deviations of $g(R(t)-R(t'))$ from the quadratic form given in (10). For the electron gas, and with use of the RPA dielectric function, it can be shown that the expansion leading to (10) is only

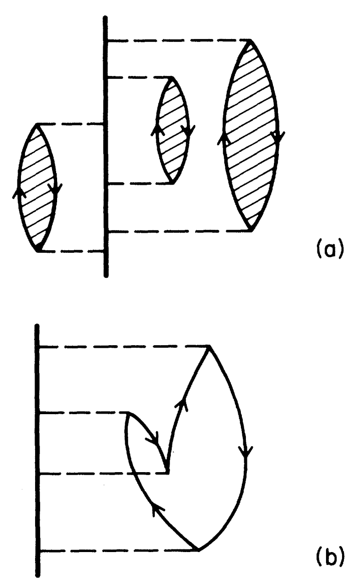

FIG. 1. (a) Some diagrams included in the effective action discussed in the text [Eq. (7)]. Shading stands for renormalized bubbles, including effects of interactions between fermions. (b) Diagrams not included in the effective action.

valid for distances small compared with k_F^{-1}, i.e., microscopic distances. In general, g cannot be factorized into a function of $R(t)$ and a function of $R(t')$, and the action (9) cannot be simulated by a finite number of phonon branches coupled linearly to the external particle; in terms of possible representations of such an action by a set of harmonic oscillators, it means that a nondegenerate set of modes $\{n\}$ with a unique relationship between the energy ω_n and the quantum index n does not suffice, because it leads to a factorizable action. The same can be argued for any finite number of branches; in this case, the function $g(R,R')$ is a sum of factorizable terms. The only possible realization of this action in terms of oscillators requires an infinite number of degenerate modes for each energy, completely filling the lower part of the ω-q phase space.

The results on the localization transition, in Refs. 6–8, and the loss of quantum coherence, in Ref. 9, have been derived assuming that the external particle stays most of the time in a deep potential well, with occasional excursions to neighboring wells. Then, the only relevant quantity is $g(R_i - R_{i+1})$, where R_i stands for the average position of the particle in well i, and R_{i+1} is the corresponding value for well $i+1$. This number should replace the quantity $\eta(R_{i+1} - R_i)^2$ derived from expression (10). Otherwise, the analysis to be carried out is identical. For practical applications, this implies that the critical value of the friction coefficient which would induce a localization transition is higher than what is expected from a quadratic action, because physically more realistic actions, like (9), are bounded at long distances. In the case of a two-level system, a different analysis, valid also in the strong-coupling cases, can be used to give bounds on the effective friction coefficient as a function of the coupling potential.[16,17]

Finally, it is interesting to note that the argument given so far can be reversed, and we can analyze the possible forms of an effective action which induces friction effects. Given a term in the action

$$S_{\text{int}}(R(t)) = \int dt \int dt' \, A(R(t) - R(t'), t - t'), \tag{11}$$

we can obtain the energy dissipated by a particle moving with constant velocity v. The action along such a trajectory is

$$S_{\text{int}}(v, t_0) = \int_0^{t_0} dt \int_0^{t_0} dt' \int dq \int d\omega \exp[i\omega(t-t')] \exp[\vec{q} \cdot \vec{v}(t-t')] A(q,\omega). \tag{12}$$

We know, from the Hamilton-Jacobi equation, that $\partial s/\partial t + H = 0$, and $H = \eta v^2 t_0$; therefore

$$S_{\text{int}}(v, t_0) = \eta v^2 t_0^2. \tag{13}$$

Taking in Eq. (12) the limit of small velocities and large times, it means that we can identify[18]

$$\eta = \lim_{\substack{t-t' \to \infty \\ R-R' \to 0}} \left[(t-t')^2 \frac{\partial^2 A(R-R', t-t')}{\partial (R-R')^2} \right], \tag{14}$$

so that classical friction is related to the long-time and short-distance behavior of the effective action, which is the limit exactly described by the harmonic oscillator model discussed in depth in Ref. 2. Although the argument presented here is far from rigorous, it suggests that most of the physics associated with dissipative environments is included in the action written in Eq. (10), irrespective of their internal structure.

In conclusion, we have used linear-response theory to derive an effective action which describes the motion of an external particle coupled to a normal Fermi fluid. It has been shown that these environments can provide a microscopic basis for the phenomenological "harmonic oscillator" model widely used in the literature. Hence, the results obtained so far may be relevant for a wide variety of topics. As possible examples, it is interesting to mention quantum diffusion of particles[19] or interstitials[20] inside metals, or at the surface,[21] $1/f$ noise and defects in disordered metals,[22] power laws and infrared singularities in the dielectric relaxation, and nuclear spin relaxation rates of a large number of systems in condensed matter.[23,24]

I am thankful for many useful discussions to V. Ambegaokar, A. Caldeira, A. Schmid, and G. Schön. This research was supported in part by the National Science Foundation under Grant No. PHY77-27084, supplemented by funds from the National Aeronautics and Space Administration.

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