Resonant Activation from the Zero-Voltage State of a Current-Biased Josephson Junction

Michel H. Devoret,[a] John M. Martinis, Daniel Esteve,[a] and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 26 July 1984)

The lifetime τ of the zero-voltage state of a current-biased Josephson junction in the thermal limit has been measured in the presence of a weak microwave perturbation. When the microwave frequency is close to the plasma frequency of the junction, the junction is "resonantly activated" out of the zero-voltage state, with a corresponding reduction in τ. The results are well explained by numerical simulations.

PACS numbers: 74.50.+r, 05.40.+j

The escape of a Brownian particle from a potential well is a process of long-standing importance as a model for the decay of a metastable state of a physical system.[1] One example of such a system is a current-biased Josephson[2] tunnel junction, which can be modeled as a particle constrained on a tilted washboard potential.[3] If the junction is biased below its critical current and is initially in a stationary (zero-voltage) state with the particle localized in a particular potential well, thermal fluctuations that modulate the angle of tilt of the washboard will eventually cause the particle to be activated out of the well.[4] For an underdamped junction, the particle then runs freely down the washboard, representing a nonzero-voltage state of the junction. The lifetime, τ, of the zero-voltage state is expected to decrease when one applies an additional alternating current that tilts the washboard sinusoidally. In this Letter, we describe an experiment in which a Josephson junction is activated out of the zero-voltage state by microwaves at a frequency close to the natural (plasma) frequency of the junction. This process, which we call "resonant activation," is applicable to the general case of the escape from a well by Brownian motion in the presence of a weak, oscillating force, and appears not to have been studied in detail previously.[5] We emphasize that this situation, in which the microwaves make only a small perturbation on the dynamics of the particle in the presence of thermal noise, is very different from the widely studied case[6,7] in which thermal fluctuations are small compared with a large microwave current that induces constant-voltage current steps.

A Josephson tunnel junction with critical current I_0 biased at a constant current I can be modeled as a particle moving in the one-dimensional potential[3] $U(\delta)=U_0(s\delta-\cos\delta)$. Here, $U_0=I_0\Phi_0/2\pi$, $s=I/I_0$, $\Phi_0=h/2e$, and δ, the phase difference across the junction, represents the position of the particle (see Fig. 1). In the zero-voltage state with $s<1$, the frequency of oscillation of the particle at the bottom of the well is the plasma frequency[2] $\omega_p=\omega_{p0}\times(1-s^2)^{1/4}$, where $\omega_{p0}=(2\pi I_0/C\Phi_0)^{1/2}$. We define $Q=RC\omega_p$, where, as we shall see, the shunt capacitance, C, and resistance, R, may contain frequency-dependent contributions from the bias circuitry in addition to their intrinsic values. The particle escapes from the well over the barrier of height[8] [see Fig. 1(b)]

$$\Delta U = 2U_0[(1-s^2)^{1/2} - s\cos^{-1}s] \approx (4\sqrt{2}/3)U_0(1-s)^{3/2} \qquad (1)$$

at a rate[1] which, in the thermal limit $k_BT >> \hbar\omega_p$, is given by

$$\tau^{-1} = \tau_a^{-1}\exp(-\Delta U/k_BT). \qquad (2)$$

The prefactor depends only weakly[9] on Q, and, for our present purpose, we can take $\tau_a^{-1}=\omega_p/2\pi$. If a small microwave current is applied, however, we shall see that τ^{-1} depends not only on ΔU and T, but also on ω_p and Q as well as on the microwave power P and frequency ω.

Nb-NbO$_x$-PbIn tunnel junctions were patterned photolithographically in a 10-μm$\times$10-μm cross-

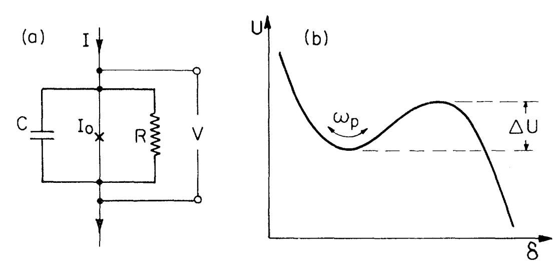

FIG. 1. (a) Resistively shunted junction model, and (b) U vs δ for a bias current below I_0.

strip geometry on Si chips. The junctions were of high quality, with a sharp rise in the quasiparticle tunneling current at the sum of the energy gaps (2.6 mV), a low subgap leakage current, and a critical current with a diffraction-pattern-like modulation in an external magnetic field. The junction capacitance was estimated to be 5 ± 2 pF. Each junction was attached to one end of a custom-made attenuating coaxial line (mount) that was connected to the room-temperature electronics via two coaxial lines equipped with radio-frequency and microwave filters. The measured attenuation of the filters was greater than 150 dB over the frequency range 0.1 to 12 GHz. A microwave current could be injected via a separate coaxial line, equipped with cold attenuators, capacitively coupled to the center conductor of the mount.

We first measured the lifetime of the zero-voltage state in the absence of microwaves to test the filtering and measurement schemes and to verify that our junction behaved according to the predictions of Eqs. (1) and (2). We ramped the bias current repeatedly and obtained a histogram of the current at which the junction switched to the free-running state.[8] From the histogram we computed $\tau(I)$, which we plot as $[\ln \tau_a^{-1}\tau(I)]^{2/3}$ vs I in Fig. 2. As is evident from Eqs. (1) and (2), such a plot should yield a straight line with slope $(4\sqrt{2}U_0/3k_\mathrm{B}T)^{2/3}/I_0$ that intersects the current axis at I_0. Thus, this procedure gives a direct measure of I_0 and T. [In practice, we correct for the approximation made in Eq. (1).] For the example shown in Fig. 2, we find $I_0 = 8.77 \pm 0.05$ μA and $T = 4.2 \pm 0.1$ K. The very good agreement between the temperature obtained from this plot and the bath temperature (4.2 K) provides compelling evidence that the junction is subjected only to Nyquist noise at the bath temperature.

Figure 2(a) also shows the modification of $\tau^{-1}(I)$ due to the application of a low-level microwave signal at two different frequencies. We see clearly defined resonances that we expect to peak when the microwave frequency, ω, is close to ω_p. To display these peaks more clearly, in Fig. 2(b) we show the smoothed curves obtained by subtracting the data in the absence of microwaves from those in the presence of microwaves. The arrow indicates the change in position of the maxima predicted from the scaling of the plasma frequency with $(1-s^2)^{1/4}$. We also studied the dependence of the plasma frequency on I_0 (by changing the magnetic field applied to the junction) and found that ω_{p0} scaled with $I_0^{1/2}$ as expected.

To determine *changes* in τ due to the microwaves with greater accuracy and over a greater range of microwave frequency, we measured τ directly at a fixed value of the bias current while the microwave frequency was swept through the plasma resonance. In this method, we applied a 10-kHz square-wave–modulated bias current, and measured the time that elapsed between the leading edge of the pulse and the junction switching event. The accuracy obtainable with this technique was typically 1% for an acquisition time of 1 sec. The inset in Fig. 3 shows an example of the expected exponential behavior of the switching events in the absence of microwaves. To study the effect of microwaves, we

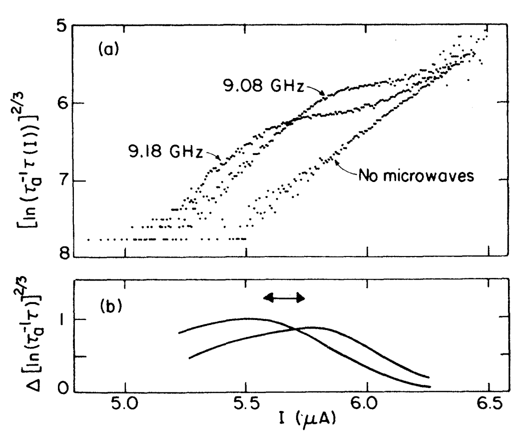

FIG. 2. (a) Dependence of $[\ln \tau_a^{-1}\tau(I)]^{2/3}$ on I in the absence of microwaves and at two microwave frequencies for a junction at 4.2 K with $I_0 = 8.77$ μA. (b) Smoothed curves obtained by subtracting the data in the absence of microwaves from the other two sets of data.

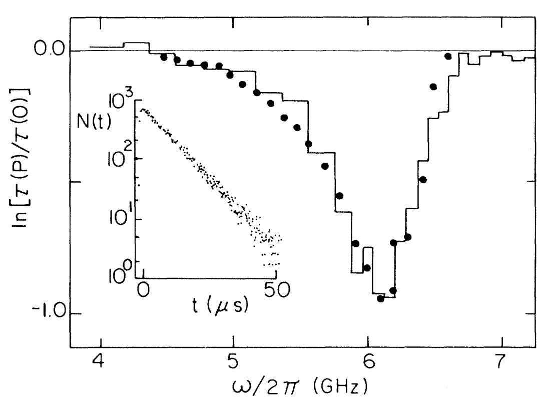

FIG. 3. Resonance in escape time vs microwave frequency. Dots represent measured values of $\ln[\tau(P)/\tau(0)]$ for a junction at 4.2 K with $I_0 = 4.64$ μA, $I = 3.07$ μA, and $\tau(0) = 8.4$ μs. Solid line represents results of numerical simulation. Inset shows exponential distribution of switching events for the same junction in the absence of microwaves.

injected microwave power into the junction at a level that varied between 10^{-18} and 10^{-14} W and swept the frequency over the range 0.1 to 12 GHz. Although we observed a sharp decrease in τ at the plasma frequency, the response exhibited a considerable amount of structure arising from resonances in the microwave injection circuit. We eliminated this problem by repeating the measurement at a slightly different value of I_0 so that the plasma frequency was shifted by 0.12 GHz. Thus, the two curves differed significantly only in the plasma resonance, and by combining them we obtained the response of the junction at the average of the plasma frequencies. An example is shown in Fig. 3, where we plot $\ln[\tau(P)/\tau(0)]$ vs ω.

We can obtain some insight into this resonant activation by considering the approximate model of a parabolic potential, with the same curvature as the real potential at the minimum, truncated at a height ΔU to give the same barrier height. For this simplified case, the stationary distribution of the junction phase difference can be calculated exactly. Setting the escape rate proportional to the probability that the particle has energy ΔU, we find the lifetime $\tau(P)$ in the presence of absorbed microwave power P in the limit $P \ll \omega_p k_B T$:

$$\ln\left(\frac{\tau(P)}{\tau(0)}\right) \approx -\left(\frac{\Delta U}{k_B T} - \frac{1}{2}\right)\frac{PQ^{-1}}{\omega_p k_B T} \times \frac{1}{[1-(\omega/\omega_p)^2]^2 + (\omega/Q\omega_p)^2}. \tag{3}$$

The last factor on the right-hand side of Eq. (3) is the response of a driven damped harmonic oscillator driven at frequency ω. This model predicts a resonance minimum at ω_p and a width at half height given by $\Delta\omega = \omega_p/Q$.

To take into account the anharmonic properties of the real potential we have performed a direct numerical simulation of the Brownian motion in the potential $U(\delta)$ in the presence of a weak, oscillating force. Our results indicate that the predictions for the harmonic oscillator model have to be modified as follows: (i) The resonance is asymmetric, (ii) the position of the minimum, ω_m, is slightly lower than ω_p, and (iii) the width $\Delta\omega$ is somewhat larger than ω_p/Q. Our best fit to the experimental data is shown in Fig. 3, with $\omega_p = 6.3$ GHz and $Q = 13$; the uncertainties were $\pm 2\%$ and $\pm 20\%$ for ω_p and Q, respectively. For these values of ω_p and Q, we find $\omega_m = 0.96\omega_p$ and $\Delta\omega = 1.25\omega_p/Q$.

The inferred values of $\omega_p/2\pi = 6.3$ GHz and $Q = 13$ lead to $C = 6.8$ pF and $R = Q/\omega_p C = 48\ \Omega$. This value of R is consistent with our estimates of the shunting impedance of the external circuitry, which we model as the characteristic impedance of the mount (measured to be about 20 Ω) in series with the stray inductance L_s of the leads connecting the junction to the mount. If we transform this circuit into a shunt resistance R in parallel with a shunt capacitance C', at 6.3 GHz we find that the measured values lead to $L_s \approx 0.6$ nH and $C' \approx -0.6$ pF. The value of L_s is consistent with our estimate based on the geometry of the leads, while the value of C' (which is, of course, included in our measured values of ω_p and Q) is too small to introduce a significant discrepancy between the measured value of the capacitance and our *a priori* estimate.

In summary, we have observed the new phenomenon of resonant activation in a current-biased Josephson junction. Apart from its inherent interest, the effect provides an *in situ* measurement of the junction parameters as modified by the complex impedance loading the junction. In the particular sample studied here, the external capacitive loading of the junction was unimportant, while the damping was completely dominated by the external resistance. This information would be very useful in experiments to investigate macroscopic quantum tunneling.[10] Work is currently under way to study the power and temperature dependence of the effect, and to extend the measurements into the quantum regime.

We thank Rob McGrath for helpful discussions on microwave engineering. This work was supported in part by the Commissiariat a l'Energie Atomique and by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U. S. Department of Energy under Contract No. DE-AC03-76SF00098.

[a]On leave from Service de Physique du Solide et de Resonance Magnetique, Centre d'Etudes Nucleáires de Saclay, F-91191 Gif-sur-Yvette Cedex, France.

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