Nonlinear Self-Contraction of Electron Waves

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We present laboratory evidence of modulationally unstable electron wave packets which can be described by a nonlinear geometrical optics theory. Growth times for self-contraction are found to be much faster than ion response times and the bursts do not appear to be related to Zakharov Langmuir-wave collapse.

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A variety of phenomena can explain localization of wave energy in nonlinear dispersive systems. These include modulational instabilities,[1] fluctuation-induced localization,[2] and soliton generation.[3] Langmuir-wave bursts observed both in space[4] and in laboratory plasmas have been attributed to slow ion-time-scale Zakharov Langmuir-wave collapse. The Zakharov equations model a modulational instability that couples electron wave envelopes to ion waves and quasineutral density depressions.[1] On the other hand, wave bursts observed in deep water wave experiments,[5] and in ion beam-plasma systems[6] have been identified as nonlinear self-contraction of wave packets,[7] a description unrelated to the Zakharov formulation. In this Letter we present the first experimental evidence of a Langmuir modulational plasma instability analogous to the deep-water-wave case.[5] This instability occurs on electron time scales that are faster than Zakharov ion response times by several orders of magnitude, which suggests that turbulent plasmas may be dominated at early times by this faster process, and then later by the well known quasineutral effects.

The data in this experiment can be described by a nonlinear geometrical optics (NLGO) theory pioneered by Ostrovskii[7] and extended to a nonlinear Schrödinger equation (NLSE) description[8] which can evolve into soliton solutions. For a nonlinear dispersive system to be modulationally unstable, the key requirement of the NLGO model is the Lighthill criterion,[7] namely, that the product of the linear dispersion $(\partial^2\omega/\partial k^2)$ and the nonlinear amplitude (a) correction to the carrier frequency $\partial\omega/\partial a^2$ be negative.

The experiment was carried out in a triple-plasma device[9] which allowed two counterstreaming electron beams to be injected, one from each source chamber, into the target region where they collide. Each beam energy could be varied up to $E_b \sim 350$ eV with fractional beam density up to $n_b/n_e \lesssim 0.01$, so that the target plasma was driven into a state of strong turbulence. Target plasma with a plasma density of $n_e \leqslant 10^8$ cm^{-3} and neutral pressure $p_0 \lesssim 6\times10^{-5}$ Torr is generated by beam impact ionization of neutral argon gas. Transient bursts were detected with electrostatic wire probes (length, 0.2 cm; diameter, 0.1 cm) and capacitive probes electrostatically shielded up to the tip, with frequency calibration obtained by use of a coaxial line source. Transient data for floating potential and electron saturation current were recorded on storage oscilloscope screens and then photographed.

Typical spectrum analyzer data for electrostatic wave power versus frequency show a broad fundamental peak near the plasma frequency ($\omega_{pe}/2\pi \approx 60$ MHz, $\delta\omega/\omega_{pe} \sim 15\%$) and narrower bands near 15 and 3 MHz. These came from frequency mixing over the broad spread of the fundamental, and were the initial modulation source for the carrier envelopes. Each electron beam drove unstable growing beam-plasma modes, with nonlinear interactions resulting in a finite parallel-beam thermal spread ($v_e/v_b \lesssim 0.1$, $\Delta v_b/v_e \sim 0.2$ at the interaction region, where v_e is the thermal velocity and v_b is the beam velocity).[9] The unstable beam mode has a dispersion relation[10] for which the maximum growth rate occurs at group velocity substantially slower than the beam velocity (i.e., $v_g = \partial\omega/\partial k < v_b$). In addition, there is negative dispersion ($v_g' \equiv \partial^2\omega/\partial k^2 < 0$) in the region, which enables us to apply the NLGO approach to modulated wave packets on this branch. Note that both slow group velocity and negative dispersion are contrary to what one expects for a cold-beam–cold-plasma wave at[11]

$$\omega \sim \vec{k}\cdot\vec{v}_b\{1-\tfrac{1}{2}[1-\tfrac{1}{2}(n_b/2n_0)^{1/3}]\} \lesssim \omega_{pe}.$$

The usual Zakharov model does not treat this type of dispersion relation, although beam waves can parametrically couple to Langmuir waves that are within its purview.

The major effect of nonlinear dispersion on a wave packet with slowly varying amplitude and phase characterized by wave number k and frequency ω is the dependence of phase velocity or frequency on the envelope amplitude a. The lowest-order nonvanishing frequency correction is $\omega(k,a) = \omega_0(k) + \alpha a^2$, where $\alpha = (\partial\omega/\partial a^2)_{a=0}$ and $\omega_0(k)$ is the dispersion relation in the linear approximation. If we let the wave become slightly nonstationary so that k and a vary in space as $\eta = \eta_0 + \eta_1 \exp(i\kappa x - i\nu t)$ ($\nu << \omega$, $\kappa << k_0$), the evolution equation for k and the energy-conservation equation define a plane-wave system. $\mathrm{Im}\nu \sim \kappa \times (-\alpha a^2 v_g')^{1/2}$ corresponds to a weakly modulated wave train that will break down into packets of decreasing spatial extent (increasing k). This was first noted by Ostrovskii.[7] The Lighthill criterion $[\omega - \omega_0(k)]v_q' \leq 0$ corresponds to the elliptic partial differential equations which describe the wave-packet self-contraction.[7]

A physical interpretation is that a slightly modulated carrier wave has a higher frequency at larger amplitude, and hence a higher phase velocity (ω/k). The wave number of carrier waves in regions of decreasing (increasing) phase velocity increases (decreases) as the carrier waves pile up (spread out) on the leading (trailing) side of a packet. When significant gradients in amplitude and wave number evolve, the inclusion of second derivatives in the fundamental equations for the NLGO approach or adiabatic approximation of Whitham[7] gives rise to a single NLSE for the complex envelope amplitude.[8] Our NLSE system $(\partial\omega/\partial a^2 > 0,\ v_g' < 0)$ has been shown to be longitudinally unstable by Karpmann[8] over a range of $\kappa_\parallel$ with maximum growth rate at $\kappa_\parallel^2 = -2\alpha a^2/v_g'$, $\kappa_\perp = 0$, which leads to the approximate growth rate $\mathrm{Im}\nu/\omega_0 \sim \alpha a_0^2/\omega_0$.

Figure 1 shows a burst evolving in space and time with a carrier frequency near 60 MHz. Data are given for three axial locations at 2-cm intervals with plasma density $n_e \sim 5\times 10^7\ \mathrm{cm}^{-3}$. Note that the burst increases in height as its width decreases and that the carrier modulation becomes more apparent. Additional and probably unrelated quasirepetitive structures are observed to recur ("burst") near twice the frequency of the ion acoustic noise peak in a hierarchy of time scales which are similar to those found by Christiansen, Jain, and Stenflo.[12]

The initial assumption of nonlinear amplitude dependence of the carrier frequency $[\omega = \omega_0(k) + \alpha a^m$, where $m = 2]$ is tested in Fig. 2, a graph of carrier frequency versus modulation-envelope amplitude for many bursts. These data give a value for $m = 1.7 \pm 0.5$, and positive α. Selected points from the data of Gurnett *et al.*,[4] which were attributed to Langmuir-soliton collapse, are also graphed and suggest that an alternative nonquasineutral explanation to soliton collapse cannot be ruled out by the evidence.

The bursts propagate mostly in the longitudinal direction for at least 10 cm with velocity near the electron thermal velocity $[(0.3\text{–}3)v_e]$ and are weakly dependent on the beam velocity. Waves of this group velocity can exist on the finite-beam-temperature dispersion relation of O'Neil and Malmberg[10] corresponding to a group velocity $\partial\omega/\partial k \approx 0.13 v_b$ which is consistent with the v_b dependence and is of the order of magnitude of the wave burst velocities. An approximately constant product of packet height×width is predicted by both the NLGO[7] and NLSE models.[8] Figure 3 is a graph of normalized amplitude perturbation $\delta a = a(z)a_0(0)/a_0(z)a(0)$, where a is the nonlinearly evolved packet maximum amplitude and a_0 is the minimum amplitude of the data, versus the reciprocal full width at half maximum, $[W(z)/W(0)]^{-1}$. Good agreement between the trends of data and theory is evident. The burst evolution is characterized by amplitude increase (δa) between two locations (Δz) rather than time.

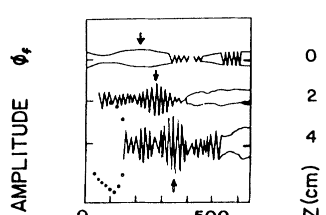

FIG. 1. Simultaneous oscilloscope traces of floating potential vs time at three axial positions (Δz). Propagation and localization occur for the electron wave packet indicated by the arrows. Dots on the bottom trace denote successive maxima and minima. Only the envelope is shown where the carrier wave details were obscure.

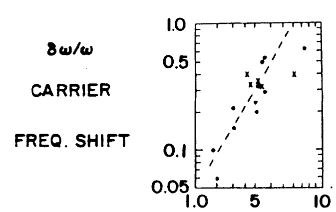

FIG. 2. Normalized frequency shift $\delta\omega/\omega$ of the laboratory carrier wave (dots) showing a nonlinear increase of $\delta\omega/\omega$ with burst amplitude. Selected points from the satellite data of Gurnett *et al.* (Ref. 4) that have previously been attributed to Zakharov Langmuir-soliton collapse (crosses) also fall in this range.

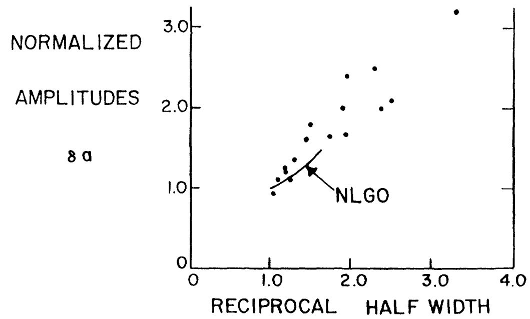

FIG. 3. Normalized packet amplitude δa vs reciprocal half-width $[W(z)/W(0)]^{-1}$, graphed with each point representing a two-position measurement of an electron wave burst. Constant height×width is a line with slope equal to 1 through the origin, and is consistent with NLGO (indicated) and NLSE soliton evolution.

The evolution growth rate can be directly estimated from Fig. 1 as $\mathrm{Im}\nu/\omega_0 \sim 0.1$–$0.3$, or indirectly from a growth distance Δz by $v_g/\Delta z \omega_0 \sim \mathrm{Im}\nu/\omega_0 \sim 0.01$–$0.5$ ($f_{pe} \sim 50$–250 MHz), and compares favorably with theoretical estimates using $\mathrm{Im}\nu/\omega_0 \sim \delta\omega(a^2)/\omega_0 \sim 0.06$–$0.5$ from the nonlinear carrier frequency deviation shown in Fig. 2. Note that the rapid growth rate observed here is much faster than Zakharov model predictions. Thus the model used to describe previous space and laboratory data is not consistent with the results presented here.

Density fluctuations of $\delta n_e/n_e \sim 20\%$ are esitmated from a wire probe biased into electron saturation and are consistent with potential fluctuations of $e\delta\phi/T_e \sim 20\%$ estimated from a capacitive probe. Both measurements are consistent with a criterion for electrostatic strong turbulence if $e\delta\phi/T_e \sim E^2/8\pi n_e T_e >> k^2/k_D$. For our experiment $k^2/k_D^2 \sim 8\times 10^{-3}$, where wave number was measured by correlation methods.

The following features of the electron wave bursts observed in this experiment fit the NLGO theory: (1) carrier wave frequency dependence, (2) dispersion relation for carrier waves, (3) constant height×width for burst envelopes, and (4) fast growth rate on electron time scales. To be a candidate for the NLGO self-contraction many nonlinear dispersive systems in plasmas and elsewhere can fulfill the Lighthill criterion, $(\partial^2\omega/\partial k^2)\partial\omega/\partial a^2 < 0$. Data which may fit this description have been reported in a tokamak plasma[13] and also as evidence for fast transverse filamentation and collapse in a magnetized plasma column.[12]

We believe that these data are the first laboratory evidence for longitudinal self-contraction of electron wave packets. Electrostatic probes biased into ion saturation do not detect ion currents on these fast time scales so that there is no apparent ion dynamical participation in this strongly turbulent plasma. These waves initially self-contract in accordance with a NLGO model, with group velocities on the order of the electron thermal velocity, and must be due to nonneutral electron density fluctuations only. The physical origin of the nonlinear frequency deviation $\omega = \omega_0 + \alpha a^2$ is not known. Some possibilities are electron bunching,[14] movement of the wave parameters on the dispersion curve, or ponderomotive force effects on electrons. Further evolution into envelope solitons could be described by a NLSE equation and the data have approximately constant envelope height×width as predicted by theory. However, collision data are required to definitely characterize these bursts as solitons.

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