# Magnetic Instabilities in Accelerating Plasma Surfaces

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The existence of an interchange instability strictly associated with electron inertia is demonstrated. This is characterized by a growth rate significantly larger than the usual ion-inertial Rayleigh-Taylor rate and by self-generated magnetic fields localized around the accelerating plasma surface. This novel instability may be partially responsible for the observed magnetic fields in ablatively accelerated laser plasmas.

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Recent consideration of electron inertial effects has led to the discovery of a magnetic surface plasma wave.[1] This mode is thought to be intimately connected with the spontaneous generation of magnetic fields that are observed in laser-produced plasmas.[2] As various phenomena, e.g., surface energy transport,[3] flux limitation,[3,4] and anomalously fast plasma blowoff,[3] have been attributed to these magnetic fields, the mechanisms involved in their production are of considerable interest.[5] We describe one such mechanism within the context of the familiar interchange instability but which is modified by the inclusion of electron dynamics in an accelerating plasma shell.

A plasma undergoing some type of bulk acceleration is a feature of many confinement concepts. The abundant free energy involved can drive instabilities that are of the interchange variety. Previous studies of these interchange modes have exclusively relied on ion inertia with electron inertia considered only through a phenomenological collision term, e.g., the so-called resistive interchange instabilities.[6] A more complete inclusion of the electron dynamics has not previously been considered. In this Letter, we deal with this aspect of the Rayleigh-Taylor problem and demonstrate the existence of another interchange-type instability exclusively associated with electron inertia. This instability is found to be characterized by a growth rate far exceeding the usual, ion-inertial interchange rate and by the production of dc magnetic fields localized around the plasma surface. While the case of ablatively accelerated plasmas is of primary interest here, the development presented is sufficiently general for application to other accelerating systems, e.g., the magnetic pinch configuration.

First, consider the equilibrium properties of a rigid, field-free plasma slab of width $L$ undergoing an acceleration that arises from a difference in external pressures acting on the two sides of the slab. In the case of laser implosion, the external pressure is due to the ablation of the layer and in pinches it is the magnetic pressure which is due to the flow of the current on the outer surface of the slab. We assume that the outside density is negligibly small compared to the slab density. If the pressure force on the left side of the slab ($z \leqslant z_0$) is $P_{0-}$ and on the right ($z \geqslant z_0 + L$) it is $P_{0+}$, the plasma slab will accelerate in the $z$ direction with value $g$ given by[7]

$$g = \frac{P_{0-} - P_{0+}}{n_0(m_e + m_i)L}, \tag{1}$$

where $n_0$ is the average particle number density, and $m_e$ ($m_i$) is the electron (ion) mass. In a comoving reference frame, the electron pressure profile $P_{0e}(z)$ inside the slab must satisfy

$$\partial_z P_{0e}(z) \cong -(P_{0-} - P_{0+})/L = -n_0 m_e g_e, \tag{2}$$

where the ion pressure is ignored and pressure balance at the two surfaces is assumed, i.e., $P_{0e}(z_0) = P_{0-}$ and $P_{0e}(z_0 + L) = P_{0+}$. In addition, the equilibrium behavior of the plasma is taken to be ideal, $P_{0e}(z) = n_0(z)\theta_0(z)$, where $\theta_0$ is the electron temperature and $\partial_z \ln\theta_0 / \partial_z \ln n_0 << 1$ is assumed.

To describe the departure of the system from this equilibrium state, we use the equations of electron number, momentum, and energy conservation without heat flow, together with Maxwell's equations in their linearized forms:

$$i\omega\delta n_e + \nabla\cdot(n_0\delta\vec{v}_e) = 0, \tag{3}$$

$$(i\omega + \nu_e)m_e\delta\vec{v}_e + m_e\vec{g}_e\,\delta n_e/n_0 = -e\,\delta\vec{E} - (1/n_0)\nabla\delta P_e, \tag{4}$$

$$i\omega n_0^{-\gamma}\delta P_e - i\omega\gamma n_0^{-\gamma}\theta_0\delta n_e + \delta v_{ez}\partial_z(P_{0e}n_0^{-\gamma}) = 0, \tag{5}$$

$$\nabla\times\delta\vec{B} \cong -4\pi e(n_0/c)\delta\vec{v}_e, \tag{6}$$

$$\nabla\times\delta\vec{E} = -(i\omega/c)\delta\vec{B}, \tag{7}$$

$$\nabla\cdot\delta\vec{B} = 0, \tag{8}$$

where the zero subscripts denote equilibrium values, the $\delta$ prefix designates perturbed quantities, $\nu_e$ is an electron-ion collision frequency, $\gamma$ is an adiabatic exponent, and the replacement $\partial_t\delta \rightarrow i\omega\delta$ has been performed. We have chosen again a comoving reference frame so that $\vec{v}_{0e}=0$ is taken for the unperturbed electron velocity, and the perturbed ion velocity (pressure) is ignored relative to the perturbed electron velocity $\delta\vec{v}_e$ ($\delta P_e$). For low-frequency phenomena, $\omega^2 << \omega_{pe}^2 = 4\pi e^2 n_0/m_e$, neglect of the displacement current in Eq. (6) can be justified *a posteriori* and $\delta n_e$ is seen to vanish by inspection of Eqs. (3) and (6).

Substituting Ampere's law, Eq. (6), into Eq. (4), taking the curl of Eq. (4), and using Eq. (7) gives

$$-\nabla\times[(\nu_e+i\omega)(m_e c/4\pi n_0 e)\nabla\times\delta\vec{B}]-(ie\omega/c)\delta\vec{B} = -[\nabla(1/n_0)\times\nabla\delta P_e], \tag{9}$$

where the right-hand side is recognized as a magnetic field source term. Under the assumption that $\nu_e(z)$ varies as $n_0(z)\theta_0^{-3/2}(z)$, Eq. (9) may be rewritten as

$$\left(\frac{\nu_e+i\omega}{i\omega}\right)\frac{c^2}{\omega_{pe}^2}\nabla^2\delta\vec{B}-\delta\vec{B}+\frac{c^2}{\omega_{pe}^2}\left(\partial_z \ln n_0-\frac{3}{2}\frac{\nu_e}{i\omega}\partial_z\ln\theta_0\right)\hat{z}\times\nabla\times\delta\vec{B}-\frac{ic}{e\omega n_0}\partial_z \ln n_0\nabla\delta P_e\times\hat{z}=0, \tag{10}$$

where Eq. (8) has been used. From Eqs. (2) and (5),

$$\delta P_e = (-c/4\pi n_0 e i\omega)\nabla\times\delta\vec{B}\cdot\hat{z}[m_e n_0 g_e(1-\gamma)-\gamma n_0\theta_0\partial_z\ln\theta_0], \tag{11}$$

and the $x$ component of Eq. (10) accordingly becomes

$$\partial_z(\bar{\lambda}^2\partial_z\delta B_x)-\delta B_x\left[k^2\bar{\lambda}^2+1-\lambda^2\frac{k^2}{\omega^2}\partial_z\ln n_0\left(g_e-\gamma g_e-\gamma\frac{\theta_0}{m_e}\partial_z\ln\theta_0\right)\right]=0, \tag{12}$$

where

$$\bar{\lambda}^2=\lambda^2(1-i\nu_e/\omega)=(1-i\nu_e/\omega)c^2/\omega_{pe}^2,$$

and $\delta\vec{B}=(\delta B_x,0,0)$ and $\nabla=(0,ik,\partial_z)$ are chosen. In the absence of collisions and isothermality, Eq. (12) is identical to Eq. (5) of Ref. 1 *except* on the boundary at $z=z_0$, where a sign difference arises in the term proportional to $\partial_z\ln n_0\,\partial_z\ln P_0$. In the presence of an acceleration-induced pressure gradient, $\partial_z\ln n_0\,\partial_z\ln P_0<0$ only on the boundary; otherwise, $\partial_z\ln n_0\,\partial_z\ln P_0>0$ everywhere as in Ref. 1. The necessity for this sign difference is precisely the condition for realization of a Rayleigh-Taylor solution.

Away from the boundary, the terms proportional to $\partial_z\ln n_0$ in Eq. (12) may be dropped provided that $(\lambda/L)^2(\omega_{pe}/\omega)^2(k\lambda_D)^2 << 1$, where $\lambda_D$ is the Debye length. We thus write the solution to Eq. (12) within the slab as

$$\delta B_x^i = C\exp[-\bar{k}(z-z_0)], \tag{13}$$

where $\bar{k}=(k^2+\bar{\lambda}^{-2})^{1/2}$, and outside the slab $(z<z_0)$ as

$$\delta B_x^e = C\exp[k(z-z_0)]. \tag{14}$$

In writing Eqs. (13) and (14), we have assumed continuity of $\delta B_x$ and $\theta_0$ at the boundary $(z=z_0)$ and localization of $\delta B_x$ near the boundary.

Multiplying Eq. (12) by $n_0/\lambda^2$ and integrating across the boundary leads to a boundary condition. By use of Eqs. (13) and (14), the following dispersion relation is found:

$$\omega^2=-kg_e(\gamma-1-\gamma A)k\lambda^2/\bar{k}\bar{\lambda}^2, \tag{15}$$

where $A=-(\partial_z\theta_0|_{z=z_0})/m_e g_e$ is typically small compared to 1. For $\nu_e/|\omega| << 1$, Eq. (15) exhibits a familiar Rayleigh-Taylor behavior:

$$\omega^2=-kg_e\left(\frac{(\gamma-1-\gamma A)}{[1+(k\lambda)^{-2}]^{1/2}}\right)^{1/2}, \tag{16}$$

but with $g_e$ associated strictly with *electron* inertia. This difference alone represents an enhancement over the usual ion-inertial version[8] by a factor $(m_i/m_e)^{1/2} >> 1$.

For $|\omega|/\nu_e << (k\lambda)^2 << 1$, Eq. (15) becomes

$$\omega=-(ikg_e/\nu_e)(\gamma-1-\gamma A), \tag{17}$$

with stability for $\gamma-1-\gamma A>0$. For $(k\lambda)^2 << |\omega|/\nu_e << 1$, we find

$$\omega=[-i(k^4\lambda^2 g_e^2/\nu_e)(\gamma-1-\gamma A)^2]^{1/3}, \tag{18}$$

with growth rate

$$\Gamma=\tfrac{1}{2}[(k^4\lambda^2 g_e^2/\nu_e)(\gamma-1-\gamma A)^2]^{1/3}. \tag{19}$$

In the University of California–Irvine gas-puff $Z$ pinch, for example, $g_e=B_0^2/8\pi n_0 m_e L$ is on the order of $10^{16}$ cm sec$^{-2}$ for $B_0\sim 10$ kG, $L\sim 1$ cm, and $n_0\sim 10^{17}$ cm$^{-3}$ in the initial implosion stage.[9]

Since $\nu_e \sim 10^8\ \mathrm{sec}^{-1}$ at 10 eV, use of Eq. (7) yields a growth time on the order of 1 nsec for $\gamma = \frac{5}{3}$ and $k\lambda \sim 1$.

In an ablatively accelerated laser plasma, $g$ varies as $\bar{u}(r/3L)\partial_t \ln M(t)$, where $\bar{u}$ is an average mass ablation speed, $r$ is a pellet radius, and $M(t)$ is the total mass of the remaining pellet at time $t$. Using $\bar{u} \sim 10^7\ \mathrm{cm\ sec}^{-1}$, $r/3L \sim 10$, and $[\partial_t \ln M(t)]^{-1} \sim 200$ psec yields a value for $g_e = g(m_i/m_e)$ on the order of $10^{21}\ \mathrm{cm\ sec}^{-2}$. Typically, $n_0 \sim 5\times10^{22}\ \mathrm{cm}^{-3}$, $\theta_0 \sim 200$ eV, $\nu_e \sim 10^{15}\ \mathrm{sec}^{-1}$ near the ablation surface, and Eq. (17) yields a growth time $\Gamma^{-1}$ on the order of a few picoseconds for $k\lambda \sim 0.2$ and $\gamma = \frac{5}{3}$.

This growth time is far too short for the stabilizing influence of mass convection through the unstable zone to occur. For the usual ion-inertial Rayleigh-Taylor instability, the associated growth time is on the order of the convection time (50–100 psec) through the unstable region and only a few $e$-folding times are allowed at most.[8]

An alternative mechanism for laser-produced plasma surface acceleration may be related to the rapid expulsion of fast electrons from the critical surface. Modeling this process by an effective scalar potential variation over a scale length $d$ gives an expression for the initial, inward acceleration of ions[10]: $g \simeq \theta_0/m_i d$, where $\theta_0$ is the average energy of the ejected electrons on the order of 10 keV. For $d$ taken as a critical surface scale length on the order of 50 $\mu$m, $g_e$ is found again to be on the order of $10^{21}\ \mathrm{cm\ sec}^{-2}$.

In summary, we have demonstrated the existence of an interchange-type instability associated with electron inertia. Fast-growing magnetic perturbations localized near the plasma surface are the salient features of this instability. The spontaneously generated magnetic fields observed in laser-produced plasmas may be related to this instability.

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