Pion Interferometry for Exploding Sources

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A formula for the two-pion correlation function is derived for an arbitrary chaotic source when the emission spectrum from each point in space-time is known. The experimental fact that pions with high momentum in the center-of-mass frame are more correlated than low-momentum pions is explained by a collective expansion of the source. A simple model illustrates how the pion correlations can be used to measure the expansion velocity of a nuclear fireball.

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Pion correlations due to the Hanbury Brown–Twiss effect[1,2] provide a method for measuring the space-time dependence of a pion source, such as a fireball formed in a relativistic nucleus-nucleus[3] or hadron-hadron[4] collision. The correlation function $C(\vec{p},\vec{q})$ is defined as

$$C(\vec{p},\vec{q}) = P(\vec{p},\vec{q})/P(\vec{p})P(\vec{q}), \tag{1}$$

where $P(\vec{p}_1 \ldots \vec{p}_n)$ is the probability of observing pions of momenta $\vec{p}_i$ all in the same event. Classically, $C(\vec{p},\vec{q}) = 1$ if the source is chaotic. For quantum theory $C(\vec{p}=\vec{q}) = 2$ because of the symmetrization of the wave function for identical bosons. The width of the correlation function is inversely related to the source size R. Here we derive a formula for the correlation function which is valid for an arbitrary chaotic source. We then apply it to a simple model exhibiting collective expansion and finite temperature. We show how collective expansion can make the source size appear smaller when one looks at pion pairs with a large total momentum $\vec{K} = \vec{p}+\vec{q}$. See Fig. 1.

A state created by a random pion source is described by[5]

$$|\eta\rangle = \exp\{\int d^4x\, \eta(x)\gamma(t)\psi^\dagger(x)\}|0\rangle = \exp\{\int d^3p\, dt\, \eta(\vec{p},t)\gamma(t)\exp(iE_pt)c^\dagger(\vec{p})\}|0\rangle, \tag{2}$$

where $\psi^\dagger(x)$ is the creation operator in the Heisenberg representation and $\gamma(t)$ is a random phase factor insuring that all pions are uncorrelated:

$$\langle \gamma^*(t)\gamma(t')\rangle = \delta(t-t'). \tag{3}$$

With this normalization of $\gamma(t)$, $|\eta(x)|^2$ is the probability of emitting a pion from the space-time point x, and $|\eta(\vec{p},t)|^2$ is the probability per unit time of emitting a pion of momentum $\vec{p}$ and energy E_p, where $\eta(\vec{p},t)$ is the Fourier transform of $\eta(x)$. These states are eigenstates of the destruction operator in the Schroedinger representation, $c(\vec{p})$, and have the property that removing a pion does not change the state. This means that the number of pions is not conserved. The correlation function is

$$C(\vec{p},\vec{q}) = \frac{\langle\eta|c^\dagger(\vec{p})c^\dagger(\vec{q})c(\vec{q})c(\vec{p})|\eta\rangle}{\langle\eta|c^\dagger(\vec{p})c(\vec{p})|\eta\rangle\langle\eta|c^\dagger(\vec{q})c(\vec{q})|\eta\rangle} = 1 + \frac{|\langle\eta|c^\dagger(\vec{p})c(\vec{q})|\eta\rangle|^2}{\langle\eta|c^\dagger(\vec{p})c(\vec{p})|\eta\rangle\langle\eta|c^\dagger(\vec{q})c(\vec{q})|\eta\rangle}. \tag{4}$$

One can calculate the individual matrix elements by noticing that the states $|\eta\rangle$ are eigenstates of $c(\vec{p})$,

$$c(\vec{p})|\eta\rangle = \int \exp(iE_pt)\eta(\vec{p},t)\gamma(t)d^3p\, dt\, |\eta\rangle. \tag{5}$$

Using Eq. (3), this leads to

$$\langle\eta|c^\dagger(\vec{p})c(\vec{q})|\eta\rangle = \int dt\, \exp[i(E_q-E_p)t]\eta^*(\vec{p},t)\eta(\vec{q},t). \tag{6}$$

In analogy to the Wigner functions we define the function

$$g(x,\vec{p}) = \int d^3x'\, \eta^*(\vec{x}+\tfrac{1}{2}\vec{x}',t)\eta(\vec{x}-\tfrac{1}{2}\vec{x}',t)e^{i\vec{p}\cdot\vec{x}'} = \int d^3p'\, \eta^*(\vec{p}+\tfrac{1}{2}\vec{p}',t)\eta(\vec{p}-\tfrac{1}{2}\vec{p}',t)e^{-i\vec{p}'\cdot\vec{x}}. \tag{7}$$

Inverting the Fourier transform and inserting into Eq. (6) yields

$$\langle\eta|c^\dagger(\vec{p})c(\vec{q})|\eta\rangle = \int d^4x\, \exp[i(p-q)\cdot x]g(x,(p+q)/2). \tag{8}$$

By setting $\vec{p}=\vec{q}$ we see that $g(x,\vec{p})$ can be identified as the probability of emitting a pion of momentum $\vec{p}$ from space-time point x. Inserting Eq. (8) into Eq. (4) gives the correlation function,

$$C(\vec{p},\vec{q})=1+\frac{\int d^4x\, d^4x'\, g(x,\tfrac{1}{2}\vec{K})g(x',\tfrac{1}{2}\vec{K})\exp[ik\cdot(x-x')]}{\int d^4x\, d^4x'\, g(x,\vec{p})g(x',\vec{q})} \tag{9a}$$

$$\simeq 1+\int d^4x\, d^4x'\, g(x,\tfrac{1}{2}\vec{K})g(x',\tfrac{1}{2}\vec{K})\exp[ik\cdot(x-x')]\,|\int d^4x\, g(x,\tfrac{1}{2}\vec{K})|^{-2}, \tag{9b}$$

where $\vec{K}=\vec{p}+\vec{q}$ and $k=(\vec{p}-\vec{q},E_p-E_q)$. The latter expression is true only in the semiclassical limit where the spread in the momentum distribution of g is much larger than R^{-1}. In this limit the width of the correlation function is small and we may set $\vec{p}$ and $\vec{q}$ equal to $\frac{1}{2}\vec{K}$. This is a good approximation in heavy-ion collisions where there is sufficient energy to create enough pions for an interferometric analysis. As the beam energy is increased Eq. (9b) becomes a better and better approximation.

The source size can be determined from the curvature of the correlation function at $k=0$. We define the curvature $I(\vec{K},\hat{k})$ by

$$C(\vec{K},\vec{k})=2-\tfrac{1}{2}I(\vec{K},\hat{k})|\vec{k}|^2. \tag{10}$$

We note that the curvature of an instantaneous Gaussian source, $g(x,\frac{1}{2}\vec{K})=\delta(t)\exp\{-r^2/R^2\}$, is just R^2. Expanding Eq. (9b) about $k=0$ shows that the curvature is a measure of the mean squared distance between pions of the same momentum after emission:

$$I(\vec{K},\hat{k})=(\int d^4x\, d^4x'\, g(x,\tfrac{1}{2}\vec{K})g(x',\tfrac{1}{2}\vec{K})\{\hat{k}\cdot[\vec{x}-\vec{x}'-\vec{v}(\tfrac{1}{2}\vec{K})(t-t')]\}^2)[\int d^4x\, g(x,\tfrac{1}{2}\vec{K})]^{-2}, \tag{11}$$

where $\vec{v}(\frac{1}{2}\vec{K})$ is the velocity of a pion with momentum $\frac{1}{2}\vec{K}$. Different directions of $\hat{k}$ explore different dimensions of the source. Also, for $\vec{k}\cdot\vec{K}=0$, the temporal dependence in g can be neglected.

As an example we evaluate exactly the correlation function using Eq. (9b) for a simple case that includes both collective expansion and thermal excitation. Consider a spherical shell of radius R. Pions are emitted from points on the surface with a Boltzmann distribution that is centered about a radial expansion velocity v.[6] They are emitted with a Gaussian time dependence described by a lifetime τ. The momentum distribution is therefore

$$g(x,\vec{p})=\delta(r-R)\exp(-t^2/\tau^2)\exp[-E'(\vec{p},\hat{r})/T], \tag{12}$$

where $E(\vec{p},\hat{r})=(E_p-v\hat{r}\cdot\vec{p})(1-v^2)^{-1/2}$ is the energy of the pion in the frame moving with velocity $v\hat{r}$ at the point $R\hat{r}$ and T is the temperature. Doing the integration over time and space yields the correlation function

$$C(\vec{K},\vec{k})=1+\frac{1}{2Q^2}\exp[-\tfrac{1}{2}(E_p-E_q)^2\tau^2]\frac{\cosh(2y^2-2k^2R^2+2Q^2)^{1/2}-\cos(2k^2R^2-2y^2+2Q^2)^{1/2}}{(\sinh y/y)^2}, \tag{13}$$

where $Q^2\equiv[(y^2-k^2R^2)^2+4(\vec{y}\cdot\vec{k})^2R^2]^{1/2}$, $\vec{y}=\frac{1}{2}\vec{K}\gamma v/T$, and $\gamma=(1-v^2)^{-1/2}$. The time-dependent exponential factor can be factored out from the experimentally determined correlation function once τ has been estimated:

$$C(\vec{K},\vec{k})=1+\exp[-\tfrac{1}{2}(E_p-E_q)^2\tau^2][C'(\vec{K},\vec{k})-1]. \tag{14}$$

The experimentally determined $C'(\vec{K},\vec{k})$ can then be compared to Eq. (13) with $\tau=0$. Averaging the curvature over different directions of $\hat{k}$ and taking the square root gives an effective source size for each momentum $R(K)$. We obtain $R(K)$ from Eq. (13) where τ has been set equal to zero:

$$R(K)=R[(y\tanh y)^{-1}-\sinh^{-2}y]^{1/2}. \tag{15}$$

This may be compared to experimentally determined source sizes obtained by fitting $C'(\vec{K},\vec{k})$ to Gaussian sources of size $R(K)$. Equation (15) shows that $R(K)$ is a monotonically decreasing function that decreases faster as the ratio of energy in collective expansion to thermal energy is increased. If there is no expansion velocity, $R(K)$ is a constant. The physical explanation for this behavior is that faster pions are more likely to be emitted near the point on the shell expanding with velocity in the direction of $\vec{K}$. In Fig. 2 the effec-

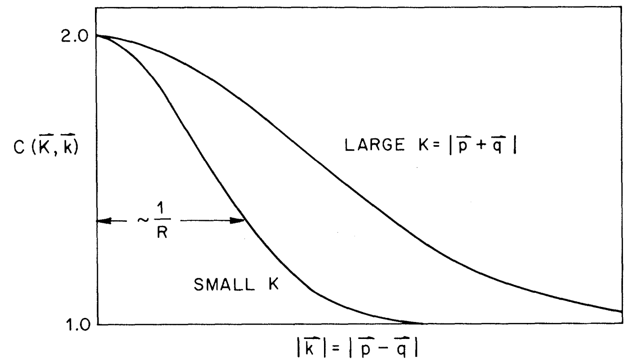

FIG. 1. The width of the correlation function is inversely proportional to the size of the source. Pion pairs with a large total momentum are more correlated, as if they came from a smaller source.

tive source sizes $R(K)$ are shown for a shell-type source of radius 7.75 fm and a temperature-to-expansion-velocity ratio of $T/\gamma v = 100$ MeV. These are compared to experimentally determined values from 1.5-GeV Ar/KCl collisions at the Bevalac.[7] In principle, the shell radius and the temperature-to-expansion-velocity ratio can be chosen to fit the data. For a specific ratio, v and T can be adjusted to best fit the proton and pion momentum distribution. This is a means to measure not only the source size but also the degree to which the source has cooled. We remark that $R(K)$ has also been seen to decrease for larger K in proton-nucleus collisions.[8]

Equation (9b) can be applied to any model of a chaotic source that predicts a momentum distribution for the emission of pions from points in space-time. It could be applied to more realistic three-dimensional models such as a cascade model or a hydrodynamic model. There are other explanations as to why high-momentum pions appear to come from a smaller source. The pion-nucleon cross section falls rapidly above 140 MeV relative energy as a result of the delta resonance. Thus faster pions may have a higher probability for escape during the early stages of a collision when the source is small. A hydrodynamic model's breakup criteria or a cascade model could be modified to take effects such as these into consideration.

We conclude that pion interferometry can be a valuable tool for studying the dynamic expansion of nuclear sources. Data fitting with our simple model can give an estimate of an expansion velocity which can then be compared with dynamical models of hadron or nuclear collisions.

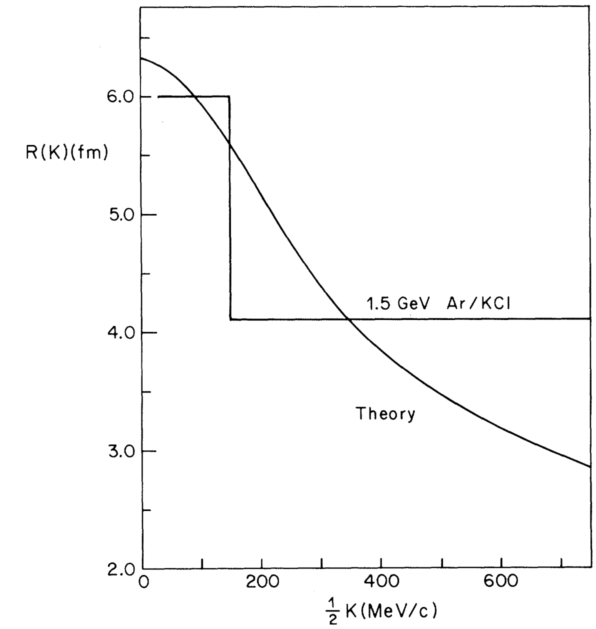

FIG. 2. The effective size of an expanding radial source with $T/\gamma v = 100$ MeV and a radius of 7.75 fm is shown as a function of the total momentum of the two-pion pair. This is compared to experimentally determined values from 1.5-GeV Ar/KCl collisions where the effective source size was estimated for both pions having a momentum less than 150 MeV/c and for both pions having a momentum greater than 150 MeV/c (Ref. 7).

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