One-Dimensional Electron Localization and Superconducting Fluctuations in Narrow Aluminum Wires

P. Santhanam, S. Wind, and D. E. Prober
Section of Applied Physics, Yale University, New Haven, Connecticut 06520
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We report magnetoresistance studies on narrow aluminum wires of width 0.2 μm < W < 0.6 μm which demonstrate clearly for the first time the one-dimensional localization effects predicted by Thouless, where the theory is generalized to include spin-orbit scattering. The electron inelastic-scattering rate is the same as in two-dimensional films, determined by known inelastic mechanisms. The dependence of resistance on temperature for 2 K < T < 20 K is also well explained by the theory.

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Many experiments in recent years have attempted to verify the predictions by Thouless[1] of localization effects in narrow metallic wires. These and other predictions of scaling theory have been justified by microscopic calculations, and some of the predictions for two-dimensional (2D) systems have been verified by experiments on metal films or MOSFETs.[2] However, despite significant experimental effort, there has not been a clear verification of the predictions for one-dimensional metallic wires. In the earliest experiments,[3,4] it appeared that the results could be interpreted in terms of localization theory. However, very large inelastic-scattering rates, much larger than usual electron-phonon rates, were required. These large inelastic-scattering rates remain unexplained.[5] Furthermore, in these experiments very small magnetoresistance was observed; subsequent theory[6] predicted much larger magnetoresistance. In later experiments on narrow metallic wires localization effects either were absent[7] or were characteristic of a *three*-dimensional system.[8] Thus, significant issues on electron transport in *1D* systems remain open—as to whether real one-dimensional systems differ subtly (but critically) from the model understood by theory, and whether new pathways for electron energy loss (inelastic scattering) occur in such lower-dimensional metals.[5] In two-dimensional metal films, in contrast, the predictions of localization theory are well confirmed; but inelastic-scattering rates in many cases are large and to date unexplained.[2a]

In this Letter we report new experiments which confirm unambiguously, and for the first time, the localization predictions of Thouless for metallic wires, using magnetoresistance measurements. We have extended the 1D theory to include spin-orbit scattering. The inelastic-scattering rates inferred are in excellent *quantitative* agreement with rates determined independently for 2D metal films with the same film properties. The inelastic mechanisms are well understood from previous 2D studies as being due to electron-phonon and electron-electron scattering.[9,10] Thus we find that there are no new inelastic mechanisms, at least for the wires studied. In addition, the theoretical understanding of such 1D systems does, in fact, appear to be in good order.

We also find that most previous experiments were not in the fully 1D regime discussed by Thouless; rather, they were in a regime of mixed dimensionality. We discuss this further at the end of this paper.

The aluminum wires studied here had widths 0.2 $\mu\mathrm{m} \leq W \leq 0.6$ μm, lengths ~200 μm, and thickness 150–250 Å. Sheet resistances were $R_\square \sim 1\ \Omega$, comparable to the 2D films studied previously.[9] The wires were fabricated on silicon wafers by x-ray lithography followed by liftoff processing. Films were deposited by thermal evaporation. Wide (2D) films were also deposited, at the same time, to allow comparison of their properties. Resistance changes were measured in a magnetic field normal to the film with an ac bridge.[9] Properties of the wires are given in Table I.

The experiments measure the resistance change as a function of temperature or magnetic field. The total resistance at a *fixed* temperature and field is

$$R(T,H) = R_0 + \Delta R^{\mathrm{ph}}(T) + \Delta R^{\mathrm{Loc}}(T,H) + \Delta R^{\mathrm{MT}}(T,H).$$

R_0 is the classical (Drude) contribution due to temperature-independent elastic scattering. The classical electron-phonon contribution ΔR^{ph} causes R to increase with T; it is independent of H. Quantum contribu-

TABLE I. Sample parameters. $R_\square$ at 4.5 K.

Sample	$R_\square$ (Ω)	d (Å)	W (μm)	l_{so} (μm)	l_i(4.5 K) (μm)
A	0.9	250	0.20	0.48	1.47
B	1.2	240	0.24	0.57	1.28
C	2.8	150	0.60	0.32	0.98
D[a]	4.5	150	0.40	0.30	0.78

[a]This is sample S562 of Ref. 11.

tions are from localization and from Maki-Thompson superconducting fluctuations. ($T_c \sim 1.4$ K for our Al wires.) Other contributions are negligible, as in the 2D case.[9] For a 1D system with no spin-orbit scattering, $\Delta R^{\rm Loc}$ is[6]

$$\frac{\Delta R^{\rm Loc}(T,H)}{R}=\frac{R_\square e^2 l_i}{\pi\hbar\, W}\left[1+\frac{H^2}{48H_iH_W}\right]^{-1/2}=\frac{R_\square e^2}{\pi\hbar}\left[\frac{H_W}{H_i}\right]^{1/2}\left[1+\frac{H^2}{48H_iH_W}\right]^{-1/2}\equiv f_1(H,H_i), \tag{1}$$

where $l_i=(D\tau_i)^{1/2}$ is the inelastic diffusion length, D the diffusion constant, and $\tau_i(T)$ the inelastic time. $H_i=\hbar c/4eD\tau_i$ is the inelastic "scaling" field and $H_W\equiv\hbar_c/4eW^2$. The criterion for the system to be 1D is that $W<l_i$. The zero-field value of Eq. (1) is just Thouless's original result, to which most previous experimental data on $R(T)$ were compared.

To include the effects of spin-orbit scattering, we have calculated[11] that, similar to the 2D case,[12] the full result for a 1D system consists of two terms.

$$\frac{\Delta R^{\rm Loc}(T,H)}{R}=\frac{3}{2}f_1(H,H_2)-\frac{1}{2}f_1(H,H_i), \tag{2}$$

where a new scaling is introduced: $H_2=H_i+\frac{4}{3}H_{so}$ with $H_{so}=\hbar c/4eD\tau_{so}$, the spin-orbit field. We thus find a new rate, $\tau_2^{-1}\equiv\tau_i^{-1}+\frac{4}{3}\tau_{so}^{-1}$, and a new scaling length, $l_2=(D\tau_2)^{1/2}$. In Eq. (2), the term multiplied by $\frac{3}{2}$ is the "triplet" term; this term is sensitive to spin-orbit scattering, and the length scale for determining sample dimensionality is l_2. The second ("singlet") term is unaffected by spin-orbit scattering and has a length scale of l_i. Thus, for the sample to be fully one-dimensional [and for Eq. (2) to be applicable], we require that $W<l_i$ *and* l_2. Equation (1) is applicable only when $H_{so} \ll H_i$ ($l_{so} \gg l_i > W$). Experimentally, this is much more difficult to achieve. In interpreting previous experiments, it was assumed that l_i was the only relevant length scale for determing localization dimensionality. It was not known that the length l_2 also determines sample dimensionality.

The Maki-Thompson contribution[13] is, for 1D samples,[11]

$$\Delta R^{\rm MT}(T,H)/R=-\beta(T/T_c)f_1(H,H_i). \tag{3}$$

β is the electron interaction parameter introduced by Larkin[13]; it is independent of sample dimensionality.[12] β increases as $T\rightarrow T_c$. The length scale determining sample dimensionality for the MT term is l_i.

The magnetoresistance (MR) is defined as $\delta R = R(T,H)-R(T,H=0)$. Only the localization and MT terms depend on field (for low fields), so that δR is given as

$$\delta R(T,H)=\delta R^{\rm Loc}(T,H)+\delta R^{\rm MT}(T,H) \tag{4}$$

with

$$\delta R^{\rm Loc}(T,H)=[\Delta R^{\rm Loc}(T,H)-\Delta R^{\rm Loc}(T,H=0)];$$

$\delta R^{\rm MT}(T,H)$ is defined similarly.

To test the fully 1D theoretical prediction we show in Fig. 1 the normalized magnetoresistance for sample A. The theoretical expression, Eq. (4), is shown by solid lines. Fitting was done by choosing H_i and H_{so} and using the results of Larkin for $\beta(T/T_c)$. We show MR data only up to 300 G, as $\beta(T/T_c)$ is depressed in large fields, and also Eq. (2) is valid only for $H<12H_W$.[11] We find that the MR obeys the *one-dimensional* form over the entire temperature range 1.8 to 15 K. Fits to the 2D or mixed-dimensional form (see below) were not satisfactory. H_{so} is found to be independent of temperature, as for 2D films.

The inelastic-scattering length $l_i(T)$ is also determined from experiment, and is plotted in Fig. 2 for wire A. For comparison we show $l_i(T)$ for the codeposited wide (2D) film. The agreement is excellent, further confirming our use of the fully 1D analysis. Similar agreement is seen for the other

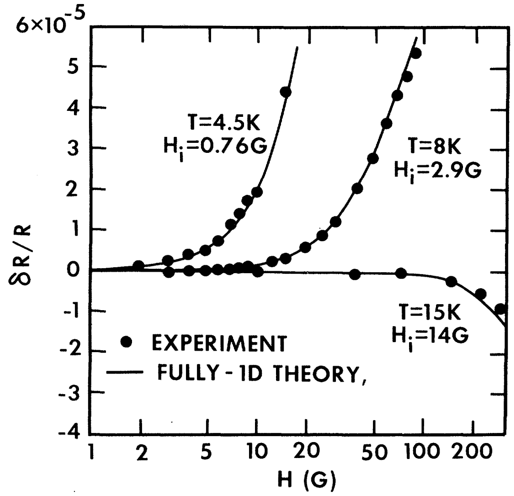

FIG. 1. Normalized magnetoresistance, sample A. $H_{so} = 7$ G for theory curves.

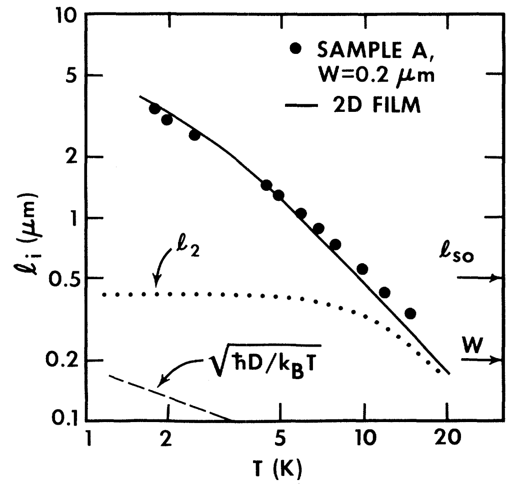

FIG. 2. Inelastic diffusion length vs temperature. The solid line is the experimentally determined l_i for the codeposited wide film. Up to 15 K, $W < l_2$ and l_i as required for fully one-dimensional behavior. l_{so} is the spin-orbit scattering length, $(D\tau_{so})^{1/2}$.

fully 1D wire, sample B.

The agreement of inelastic scattering rates in Fig. 2 is the first reported agreement of inelastic rates for samples of different localization dimensionality. This result is in fact sensible. The dimensional size scales for electron-phonon inelastic scattering and for dirty-limit electron-electron scattering are, respectively, the phonon wavelength and the quantum diffusion length $(\hbar D/k_B T)^{1/2}$. These are both less than the wire width, so that inelastic mechanisms for our wires and for 2D films should be the same. As seen in Fig. 2, wires of width ≤ 0.1 μm would be required to study the one-dimensional electron-electron inelastic mechanism.

Previous experiments measured primarily the resistance change with temperature. For our samples, MR data and resulting τ_i values provide a more direct test of localization theory. Still, to verify the $R(T)$ prediction, in Fig. 3 we plot $R(T)$ for sample A. We expect that the localization and MT contributions to $R(T)$ will be 1D up to 15 K. The electron-phonon contribution is well fitted by $\Delta R^{ph}/R = C_{ph} T^3$ for both the wires and our wide 2D samples, independent of H. Reference 7 also reports a similar term.[14]

Theoretical plots of $R(T)$ are also given in Fig. 3. The parameters of the theoretical curves, H_i and H_{so}, are taken from the 1D magnetoresistance studies. The values are *not* adjustable here. C_{ph} was determined from the 2D film. In Fig. 3 there is very good *quantitative* agreement over the full temperature range, further confirming the 1D theoretical prediction. At low temperatures the MT fluctuation term is dominant. The agreement of theory and experiment at low temperatures confirms that l_i is the relevant length scale for the MT term.

In addition to the two fully 1D wires we have studied two wider wires, C and D. For these samples $l_2 < W < l_i$ at low temperatures. The localization magnetoresistance is thus of mixed dimensionality: The triplet contribution to the MR is 2D, since $l_2 < W$.[11] We find that the behavior of samples C and D is well described only by the MR formula with this *two-dimensional* triplet term, the mixed-dimensionality result. Here again, the

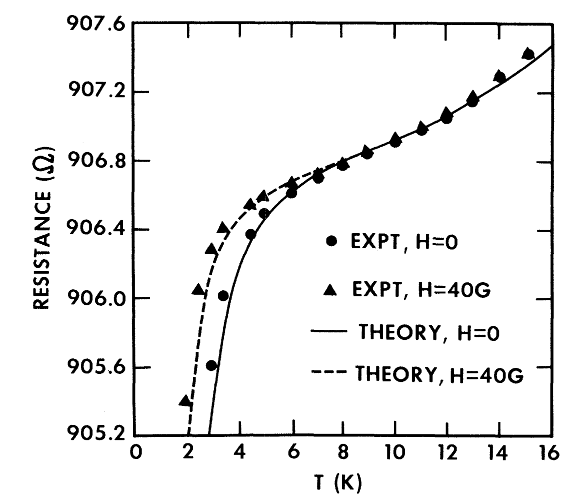

FIG. 3. Resistance as a function of temperature, sample A. Theory curves were matched to the data at 6 K.

agree with those of 2D films.

We now return to the questions raised by earlier 1D studies.[3,4] The original theoretical analyses were done with Eq. (1), which is for samples in the fully 1D regime with negligible spin-orbit scattering. However, we estimate that the spin-orbit scattering rates were very large[15–17] for the alloy wires, so that all those wires were actually in the regime of *mixed* dimensionality for localization. Thus, the *conclusions of the early studies regarding large inelastic rates must be revised.* More extensive MR data will, however, be required for unambiguous extraction of τ_i values. The smallness of the observed resistance rise with decreasing T, or the apparent absence of localization effects,[7] may have been due to magnetic scattering, which is not significant for our Al films.[9]

Recently, narrow silicon inversion-layer wires have also been studied.[18] A *one-dimensional* electron-electron inelastic mechanism was suggested by the data. The observed temperature dependence, $\tau_i^{-1} \propto T^{1/2}$, may be expected theoretically. However, the theory is not sufficiently developed to confirm the experimental magnitude of τ_i. Also, no independent experimental confirmation of this 1D rate is possible from 2D samples. For the present work, we find that the *quantitative* agreement on rates in Fig. 2 is a critical element for rigorous confirmation of the 1D localization theory. We conclude that the localization predictions are now fully confirmed for 1D systems.

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