Collisionless Spin Waves in Liquid ^{3}He

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Observations by cw NMR of standing spin-wave resonances in normal liquid ^{3}He are found to agree well with Leggett's theory. In the superfluid A phase the resonances show an alternating pattern of frequency shifts, which disappears in the B phase.

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A long-standing prediction[1] of the Landau Fermi-liquid theory is the existence of spin waves in normal liquid ^{3}He in a magnetic field. The spins interact via the Landau molecular field, so that the spin current precesses about the magnetization. The spin waves should be undamped when the quasiparticle collision time τ_D is longer than the precession period. This type of collective mode was observed for the electrons in nonferromagnetic metals some time ago,[2] but until recently the only evidence for its existence in normal ^{3}He and ^{3}He/^{4}He mixtures was the observation by Corruccini *et al.*[3] of the Leggett-Rice effect.[4,5] This is a variation of the effective spin diffusion coefficient with frequency and spin-tipping angle. Recently, standing spin-wave modes were observed in normal ^{3}He[6] and in degenerate ^{3}He/^{4}He mixtures,[7] but no quantitative explanation of the mode structure was presented.

In this Letter we report a study of spin waves in liquid ^{3}He confined to a rectangular NMR coil. We compare the spectrum of standing spin-wave modes with numerical solutions of the equation of motion for the nuclear magnetization given by Leggett.[5] The theory for the normal liquid is convincingly confirmed. On entering the superfluid phases, the modes are shifted in frequency and damped. The damping and shifts of the modes may provide a useful new way of studying the superfluid.

Leggett[5] has derived the general equations[8] for the evolution of the magnetization $\gamma\hbar\vec{M}(\vec{r},t)$ in a normal Fermi liquid, valid in both the hydrodynamic and collisionless regimes. We consider a cw NMR experiment with a static field $H_0(\vec{r})$ which is everywhere parallel to the z axis, and define the local Larmor frequency $\Omega_0(\vec{r})=\gamma H_{0z}(\vec{r})$. The two components of the magnetization transverse to the static field can be combined into a single complex number, $M^+=M_x+iM_y$. The equation of motion for the transverse magnetization is then

$$\frac{\partial M^+}{\partial t}=-i\Omega_0M^++i\gamma M_zH_1^+ +\frac{D_0(1+i\mu M_z)}{1+\mu^2M^2}\nabla^2M^+.$$

This equation includes the precession of the magnetization about the rf field $\vec{H}_1(\vec{r},t)$, with $H_1^+=H_{1x}+iH_{1y}$. The quantity $D_0=(v_F^2/3)(1+F_0^a)\times\tau_D$ is the diffusion constant for the hydrodynamic regime, and $\mu=-\gamma^2\hbar\lambda\tau_D/\chi(0)$. Here v_F is the Fermi velocity, $\chi(0)$ is the static susceptibility, and $\lambda=(1+F_0^a)^{-1}-(1+F_1^a/3)^{-1}$, where F_0^a and F_1^a are Fermi-liquid parameters. Higher-order parameters have no effect.[5]

In our experiments the fractional variation of $\Omega_0(\vec{r})$ over the sample is small and $|M^+|<<M_z$, and so the equation of motion simplifies to

$$iD\nabla^2M^++\Omega_0M^+=i\frac{\partial M^+}{\partial t}+\gamma M_zH_1^+. \tag{1}$$

This introduces the field-dependent complex diffusion coefficient

$$D=D_0/(1+i\lambda\omega_0\tau_D), \tag{2}$$

where ω_0 is the mean value of $\Omega_0(\vec{r})$. The relaxation time τ_D varies with temperature as T^{-2}, and so D is real at high temperatures, and (1) reduces to the ordinary diffusion equation. For temperatures sufficiently low that $\lambda\omega_0\tau_D>>1$ (the collisionless regime), D is purely imaginary and (1) is a Schrödinger equation for $M^+(\vec{r},t)$ (driven by the transverse field H_1^+), in which the static field $\Omega_0(\vec{r})$ is the potential. The substitutions $M^+(\vec{r},t)=m^+(\vec{r})e^{-i\omega t}$, $H_1^+(\vec{r},t)=h_1^+(\vec{r})e^{-i\omega t}$ lead to the dynamic susceptibility averaged over the sample,

$$\chi(\omega) = \gamma\hbar\left(\int h_1^{+*} m^+ d^3r\right)/\left(\int |h_1^+|^2 d^3r\right)$$
$$= \chi(0)\sum_n c_n\omega_n/(\omega_n - \omega). \quad (3)$$

Here the ω_n are the complex eigenfrequencies of (1) and the c_n are computed from the corresponding eigenfunctions and $h_1^+(\vec{r})$.

The boundary conditions assumed for $m^+(\vec{r})$ have a large effect upon $\chi(\omega)$ in the collisionless regime. We have tried both zero spin current (normal component of $\nabla m^+ = 0$) and zero spin ($m^+ = 0$) at the walls. The latter corresponds to strong relaxation of the transverse magnetization at the boundaries. To compute $\chi(\omega)$ from (3), $h_1^+(\vec{r})$ and $m^+(\vec{r})$ were expanded in a series of functions which solved $\nabla^2 m^+ = \text{const} \times m^+$ and satisfied the boundary conditions.

The experimental apparatus[6,9] consists of an epoxy tower, 6.2 mm in internal diameter, containing the liquid ^{3}He and an NMR coil wound on a quartz former 2.8-mm-long with a 2×2-mm^2 square hole. The coil is perpendicular to the axis of the tower, and to the static magnetic field of a small shielded superconducting solenoid around the tower. The NMR is detected at fixed frequency, while the static field is swept with use of a flux transformer in series with the solenoid. A separate set of windings allows the gradient along the static field to be varied. Above about 5 mK, the NMR line is roughly rectangular as expected for a nearly uniform field gradient. Between 0 and 10 bars, as the temperature is lowered, a series of peaks appears[6] corresponding to standing spin-wave modes. The peaks are sharpest at zero pressure and just above the superfluid transition temperature T_c (about 1 mK at $p = 0$).

Figure 1 shows theoretical and experimental absorption lines for a series of field gradients. To simplify the theory, the sample was assumed to be a rectangular prism $2\times 2\times 6.2$ mm^3, thus ignoring the complicated shape of the vertical channels at the open ends of the coil. Relatively good agreement with experiment is still expected because the main peaks correspond approximately to Airy modes confined between the top and bottom surfaces of the coil former. The frequencies of the main peaks are compared with theory in Fig. 2. For the static field distribution we assumed $\Omega_0 = A + Bz + C(z^2 - x^2/2 - y^2/2)$, where z is the static field direction and x is the coil axis. The parameters A, B, and C for both main and gradient coils were fixed by the high-temperature line shapes, leaving only $\tau_D T^2$ and λ adjustable.

The best fit to the data is obtained with zero–spin-current boundary conditions. This fit, shown in the figures, gives $\tau_D T^2 = (3.0 \pm 0.6) \times 10^{-7}$ sec mK^2 and $\lambda = 2.41 \pm 0.12$, with use of the values[10] $v_F = 5.99\times 10^3$ cm/sec, $F_0^a = -0.70$. When scaled

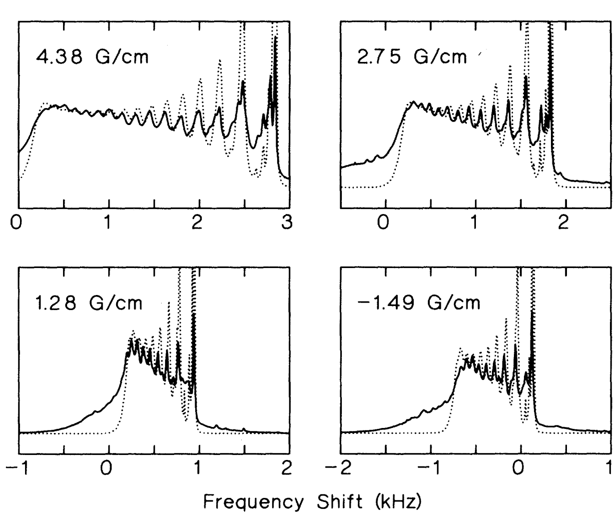

FIG. 1. NMR absorption at 2 MHz in normal liquid ^{3}He at zero pressure and $T/T_c = 1.01$. The vertical scale is different for each field gradient. Solid line is experiment; dotted line is theory. Experimental field shifts have been converted to equivalent frequency shifts with use of the ^{3}He gyromagnetic ratio.

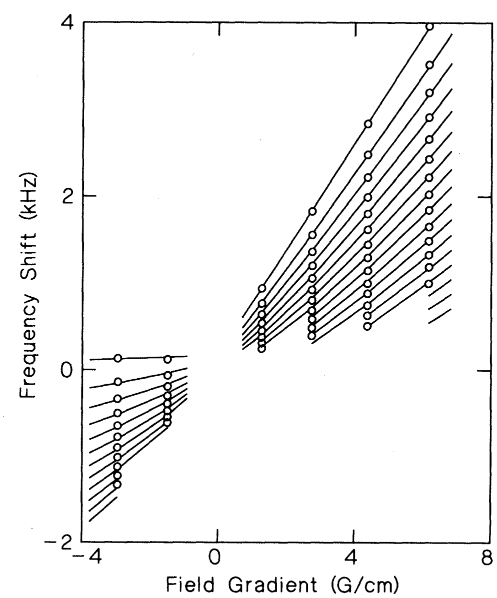

FIG. 2. Frequencies of the main series of spin-wave peaks as a function of field gradient, at a Larmor frequency of 2 MHz. Circles are experimental; lines are calculated.

to the same values for v_F and F_0^a, the results of Corruccini *et al.*[3] from the Leggett-Rice effect are $\tau_D T^2 = (3.26 \pm 0.33) \times 10^{-7}$ sec mK and $\lambda = 2.20 \pm 0.11$. Rough limits for F_1^a may be derived from our results: $-0.1 < F_1^a < 1.0$. A somewhat more negative value for F_1^a is suggested by analysis of finite-temperature contributions to the thermal conductivity[11] and to the specific heat.[10]

Data were also taken at 1 MHz and 4 MHz. Typical 1-MHz data are shown in Fig. 3. The predicted variation of the peak separations is verified, confirming the ω_0 dependence of the complex diffusion coefficient in (2). At 4 MHz, the main spin-wave peaks were partly obscured by the fine structure due to the quadratic part of the static field, but the theory was again confirmed.

Although the agreement between theory and experiment for the normal liquid is excellent, particularly for the frequencies of the spin-wave peaks, a similar experiment failed to show spin waves in the normal liquid. This was in a square NMR coil potted in epoxy over aluminum foil which was later etched away. However, normal-phase spin waves were observed in earlier experiments[6] using a circular-section coil potted in epoxy over a Teflon form. These results suggest that damping may occur as a result of transverse spin relaxation at the boundary, depending on the preparation of the surface.

When the ^{3}He is cooled into the superfluid phases, the spin waves remain but they are shifted in frequency and damped (more so in the A phase), almost disappearing by $T/T_c = 0.7$ (at zero pressure). Figure 4 shows the frequency shifts of the peaks in the superfluid phases, for Larmor frequencies of 2 and 4 MHz. The shifts are continuous through the second-order transition at T_c, but show discontinuities at the first-order A-B transition. In the A phase the "even" peaks, corresponding to the ground state (the highest-frequency peak) and the even-numbered excited states, have much smaller frequency shifts than the "odd" peaks. This may reflect an interaction between the superfluid texture and the spatial structure of the modes. The shift averaged over all peaks is roughly the same as the A-phase longitudinal shift recently measured by Berg, Engel, and Ihas.[12] In ^{3}He-B, all the peaks shift together to higher frequency.

The complex diffusion coefficient D for the superfluid phases has been calculated by Combescot.[13] When it is substituted in (1), the predicted shifts are in fair agreement with the results for the B phase (Fig. 4). The theory also predicts increased damping as the temperature is lowered. Variation

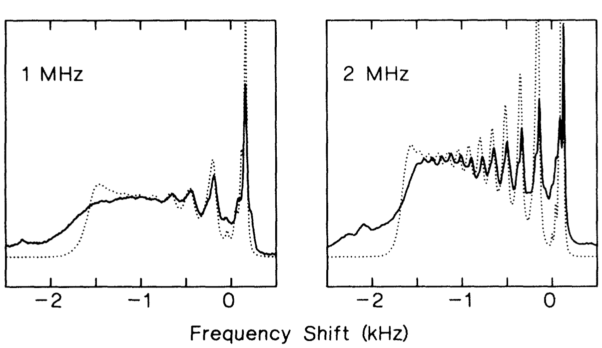

FIG. 3. Experimental (solid) and calculated (dotted) absorption line shapes at Larmor frequencies of 1 and 2 MHz. The field gradient is approximately -3.0 G/cm for both cases.

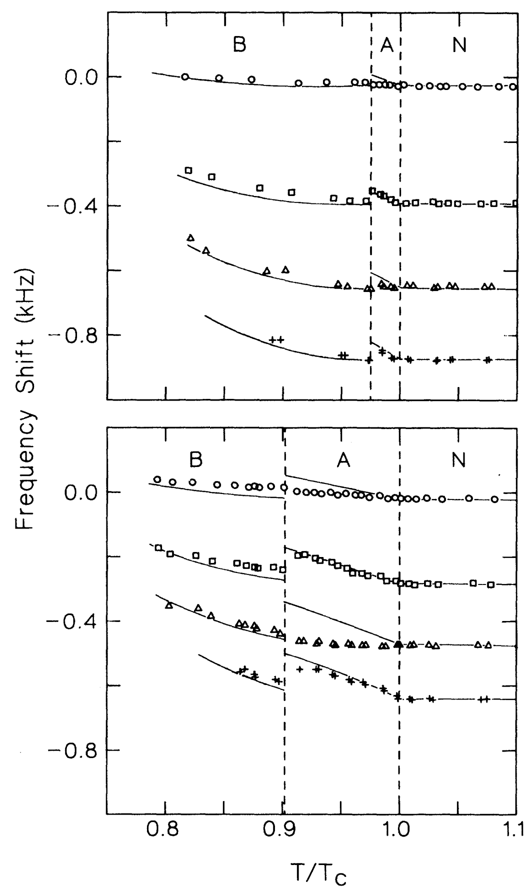

FIG. 4. Frequencies measured from an arbitrary origin of the main spin-wave peaks in the normal liquid (N) and the A and B phases, as a function of reduced temperature. The Larmor frequencies and field gradients are (top) 2 MHz and 4.4 G/cm, and (bottom) 4 MHz and 4.1 G/cm. Circles are the highest-frequency or ground-state peak; squares are the first excited state, etc. The curves are theoretical, but include the experimental longitudinal shift for the A phase (Ref. 12).

of D cannot account for the alternating shifts in the A phase.

When the temperature of our sample is lowered still further ($T/T_c < 0.5$), spin waves reappear[6] as a large number of peaks in the absorption line. In this regime[14] the texture has a large effect on the spin-wave modes, which exist even in zero applied field.

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