Occupation Probabilities of Shell-Model Orbits in the Lead Region

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The occupation probabilities of shell-model orbits in the lead region are estimated by the addition of random-phase-approximation corrections to nuclear-matter results. The occupation probabilities of single-particle states in shells just below and above the Fermi energy are found to be ~0.7 and 0.1, respectively. It is shown that these estimates explain the quenching of the single-particle contribution observed in many elastic and inelastic electron-scattering experiments.

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Substantial empirical evidence indicates that the independent-particle model provides only a partially successful description of some of the simplest nuclear excitations in heavy nuclei. Mean-field theories correctly predict the shape of transition densities but they systematically underestimate the strength of these simple excitations. Various mechanisms have been proposed to explain this quenching of single-particle strength, particularly for the high-spin magnetic transitions in lead. Hamamoto, Lichtenstadt, and Bertsch[1] have studied the quenching due to first-order core polarization, but subsequent studies[2–4] found this effect to be small. While Krewald and Speth[2] could explain the quenching with particle-vibration coupling, self-consistent first- and second-order random-phase-approximation (RPA) calculations[3] could not. The effect of meson exchange currents, left out in these calculations, was estimated by Suzuki and Hyuga[4] and found to be small.

In this Letter we examine the empirical evidence for the quenching of the single-particle strength in both elastic and inelastic scattering measurements. We find that the mechanisms explored so far account only for part of the effect. Reasonable agreement with the data is achieved when the effect of short-range correlations is also included.

In Table I we list transitions to relatively pure shell-model states. The magnitude of the quenching is directly obtained by comparison of the observed and the single-particle form factors for electron scattering. It is central to our argument that in all these cases the observed form factor can be separated, mainly through its momentum dependence, into a quenched single-particle term and a background term:

$$F(q) = QF_{\text{sp}}(q) + F_{\text{bg}}(q). \tag{1}$$

The form factors of the differences in the charge and current densities of neighboring nuclei, measured in the elastic electron-scattering experiments listed in Table II, also have a quenched single-particle part. The quenching Q observed in both elastic and inelastic scattering experiments is ~ 0.6.

We first review the connection between Q, the quenching factor, and the occupation numbers. Consider a transition from a state $|i\rangle$ to the state $|f\rangle$. Let $|i_m\rangle$ and $|f_m\rangle$ denote the independent-particle-model approximations of $|i\rangle$ and $|f\rangle$. The single-particle states occupied in $|i_m\rangle$ are denoted by h_n, $n=1,A$, and the unoccupied states are p_n, $n=1,\infty$. The single-particle part of the transition corresponds to a nucleon being transferred from orbit h to p, so that $|f_m\rangle = p^\dagger h|i_m\rangle$, and the orbitals p and h are given in Table I. The physical states $|i\rangle$ and $|f\rangle$ contain many configurations and they can be represented by

$$|i\rangle = (1+C_i)|i_m\rangle, \tag{2}$$

where C_i is a sum of terms containing products of $p_n^\dagger$ and h_n operators, and

$$|f\rangle = (1+C_f+D_f)|f_m\rangle, \tag{3}$$

where C_f is also a sum of terms containing products of $p_n^\dagger$ and h_n operators, and D_f contains the terms with the operators p and $h^\dagger$. Thus $(1+C_f)|f_m\rangle$ contains those components of $|f\rangle$ in which p is occupied and h is empty, while $D_f|f_m\rangle$ contains those in which p is empty and/or h is occupied. The quenching factor Q is given by

$$Q^2 = \frac{\langle i|h^\dagger p|f\rangle}{\langle f|f\rangle}\frac{\langle f|p^\dagger h|i\rangle}{\langle i|i\rangle}. \tag{4}$$

In a large nucleus we can approximate C_f by C_i. The amplitudes of a-particle, a-hole terms $p_1^\dagger \cdots p_a^\dagger h_1 \cdots h_a$, having $p_{1-a} \neq p$ and $h_{1-a} \neq h$, in C_i and C_f should be identical in the limit $A \to \infty$. The C_i contains terms having operators $p^\dagger$ and/or h. However, C_f can have these with any amplitude, since $p^\dagger|f_m\rangle$ and $h|f_m\rangle$ are zero. Secondly, the operator $D_f^\dagger p^\dagger h = 0$, since $D_f^\dagger$ must contain either $p^\dagger$ or h. Finally the operator $p^\dagger h$

TABLE I. The energy, E, and spin and parity, J^π, of the final state are given in the first two columns. The next three columns give the shell-model description of the transition. The last two columns give the experimental value of Q, the quenching factor, and the reference.

E (MeV)	J^π	h	p	t	Q	Ref.
			^{208}Pb(e,e')			
4.04	7^-	$2f_{5/2}$	$2g_{9/2}$	N	0.51 ± 0.05	5
6.10	12^+	$1i_{13/2}$	$1i_{11/2}$	N	0.65 ± 0.04	6
6.43	12^-	$1i_{13/2}$	$1j_{15/2}$	N	0.71 ± 0.05	7
6.74	14^-	$1i_{13/2}$	$1j_{15/2}$	N	0.71 ± 0.05	7
7.06	12^-	$1h_{11/2}$	$1i_{13/2}$	P	0.71 ± 0.05	7
			^{207}Pb(e,e')			
0.57	$\frac{5}{2}^-$	$2f_{5/2}$	$3p_{1/2}$	N	0.65 ± 0.05	8
0.90	$\frac{3}{2}^-$	$3p_{3/2}$	$3p_{1/2}$	N	0.65 ± 0.05	8
1.63	$\frac{13}{2}^+$	$1i_{13/2}$	$3p_{1/2}$	N	0.47 ± 0.05	8
2.34	$\frac{7}{2}^-$	$2f_{7/2}$	$3p_{1/2}$	N	0.55 ± 0.05	8
2.73	$\frac{9}{2}^+$	$3p_{1/2}$	$2g_{9/2}$	N	0.50 ± 0.05	8
3.51	$\frac{11}{2}^+$	$3p_{1/2}$	$1i_{11/2}$	N	0.65 ± 0.05	8

commutes with C_i because C_i does not contain either p or $h^\dagger$. Using these relations we find

$$\langle f|p^\dagger h|i\rangle = \langle i_m|h^\dagger p(1+C_f^\dagger)p^\dagger h(1+C_i)|i_m\rangle$$
$$\simeq \langle i|pp^\dagger h^\dagger h|i\rangle, \qquad (5)$$

and hence

$$\frac{\langle f|p^\dagger h|i\rangle}{\langle i|i\rangle} \simeq n_i(h) - n_i(p) + \frac{\langle i|p^\dagger phh^\dagger|i\rangle}{\langle i|i\rangle}, \qquad (6)$$

where $n_i(q)$ is the occupation number of the single-particle state q (q can be either p or h) in the state $|i\rangle$:

$$n_i(q) = \langle i|q^\dagger q|i\rangle / \langle i|i\rangle. \qquad (7)$$

Equation (6) is exact in the limit $A\to\infty$. The expectation value $\langle i|p^\dagger phh^\dagger|i\rangle/\langle i|i\rangle$ is expected to be small in nuclei, and useful relations are obtained by approximating it with $n_i(p)[1-n_i(h)]$. We thus obtain

$$Q^2 \simeq n_i(h)[1-n_i(p)]n_f(p)[1-n_f(h)]. \qquad (8)$$

The three elastic scattering experiments considered in Table II measure the difference in the charge or current density distributions in nuclei with A and $A-1$ nucleons. If the shell-model approximations for the ground states of nuclei A and $A-1$ are $|A_m\rangle$ and $|(A-1)_m\rangle$, and $|(A-1)_m\rangle = h|A_m\rangle$, then we have

$$Q' = n_A(h) - n_{A-1}(h). \qquad (9)$$

The results of calculations of occupation numbers in nuclear matter[12] are given in Fig. 1. The short-

TABLE II. The Pb-Tl experiment measures the difference in charge distributions, while the elastic magnetic scattering from ^{209}Bi and ^{207}Pb measures the magnetization current distribution generated by the unpaired surface nucleon.

A	$A-1$	h	Q'	Ref.
^{206}Pb	^{205}Tl	$3s_{1/2}$	0.6 ± 0.1	9
^{209}Bi	^{208}Pb	$1h_{9/2}$	0.7 ± 0.1	10
^{208}Pb	^{207}Pb	$3p_{1/2}$	0.7 ± 0.1	11

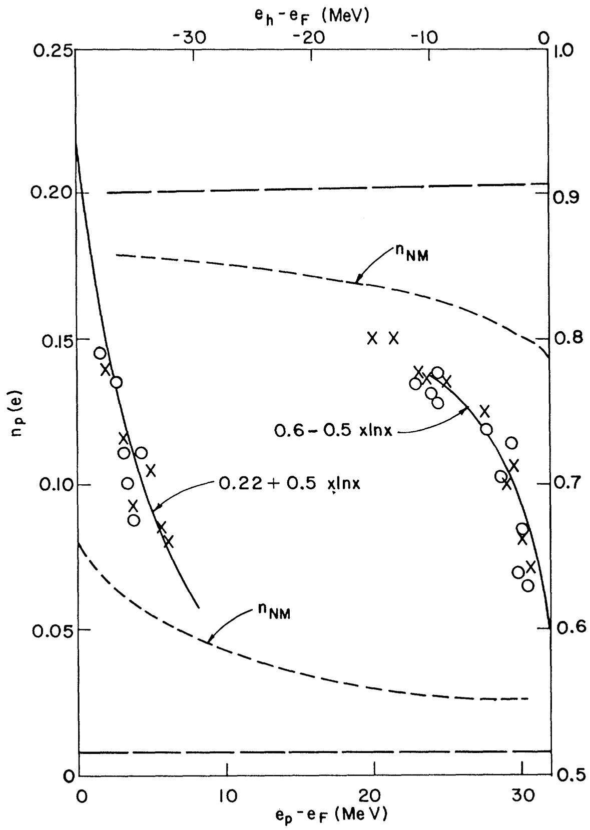

FIG. 1. The broken and dashed curves, respectively, give the $n_{\mathrm{NM}}(e)$ obtained on including short-range and short- plus long-range correlations. The occupation numbers shown by crosses (proton states) and circles (neutron states) are obtained by adding Gogny's δn_{RPA} to the nuclear-matter results. The full curves show the $n(e)$ obtained from Eqs. (10) and (11).

range correlations cause a rather uniform depletion of the occupation of the hole states and a very small occupation of the particle states, as illustrated by the broken line in Fig. 1. This effect of short-range correlations is calculated with the variational method. The effect of long-range correlations is pronounced at the Fermi energy, and it is calculated with correlated-basis perturbation theory.[12] It is mostly due to the tensor forces, and is very energy dependent, so that it is more appropriate to consider $n_{\mathrm{NM}}(k)$ as a function of the single-particle energy[13] $e(k)-e(k_{\mathrm{F}})$. The $n_{\mathrm{NM}}(k/k_{\mathrm{F}})$ has little density dependence,[12] and so we can expect the nuclear matter $n_{\mathrm{NM}}(e-e_{\mathrm{F}})$ to be a good starting approximation for the $n(e-e_{\mathrm{F}})$ in heavy nuclei.

The effect of the nuclear surface is not accounted for in the nuclear-matter calculations. The surface oscillations couple to the single-particle states, deplete the hole states, and populate the particle states near e_{F}. Their effect has been estimated by Gogny[14] using density-dependent effective interactions and the random-phase approximation. The δn_{RPA} also depends mostly on $e-e_{\mathrm{F}}$ as can be seen from Fig. 1. In this work we approximate the occupation probabilities in the lead region with the $n_{\mathrm{NM}}+\delta n_{\mathrm{RPA}}$ shown in Fig. 1.

There is the possibility of double counting in adding the second-order correlated-basis perturbation theory corrections in nuclear matter and the RPA corrections in nuclei to obtain the total effect of the long-range corrections. A more correct treatment would presumably calculate both of these corrections simultaneously using RPA in correlated basis.[15] It is, however, unlikely that there is too much double counting in the present work. Gogny's effective interaction does not have a tensor force, and $\frac{3}{4}$ of the second-order correction in nuclear matter is from the tensor forces. The Hamiltonian used in the nuclear-matter calculations[12] retains only the nucleon degrees of freedom. Calculations[16] using Hamiltonians keeping both the nucleon and the isobar degrees of freedom suggest that in the lead region there is a $\sim 5\%$ probability for the nucleons to be in the isobar state.

We note that the total $n(e\sim e_{\mathrm{F}})$ (Fig. 1) can be well approximated by

$$n(e<e_{\mathrm{F}})=0.6-0.5x\ln x, \tag{10}$$

$$n(e>e_{\mathrm{F}})=0.22+0.5x\ln x, \tag{11}$$

$$x=|e-e_{\mathrm{F}}|/E_0. \tag{12}$$

We use $E_0=40$ MeV as the scale of energy in nuclei. The coefficient of the $x\ln x$ term is given by $2W_0E_0/\pi$, where W_0 is the coefficient in the expansion of the imaginary part of the energy-dependent optical potential[12]:

$$W(e)=W_0(e-e_{\mathrm{F}})^2, \quad e\sim e_{\mathrm{F}}. \tag{13}$$

The above fit to $n(e\sim e_{\mathrm{F}})$ gives $W_0=0.02$ MeV^{-1}, in reasonable agreement with the empirical values $W_0=0.025$ to 0.033 MeV^{-1} obtained from the widths of single-particle states.[17]

The energy dependence of $n(q)$ essentially comes from the two-particle, two-hole components $\alpha(p_1p_2h_1h_2)p_1^\dagger p_2^\dagger h_1h_2$ in the C_i;

$$\alpha(p_1p_2h_1h_2)=-\langle p_1p_2|G|h_1h_2\rangle/[e(p_1)+e(p_2)-e(h_1)-e(h_2)]. \tag{14}$$

In Fermi liquids,[18,12] the $\alpha(p_1,p_2,h_1,h)$ or $\alpha(p,p_1,h_1,h_2)$ for a given h or p have energy denominators starting from $e_F - e(h)$ or $e(p) - e_F$. The vanishing of these denominators as $e(q) \rightarrow e_F$ causes the $x \ln x$ singularity in $n(e)$. In finite nuclei these denominators cannot vanish, and it is necessary to use a lower cutoff for the $|e(q) - e_F|$. The average value of the $n(h)$ for states in the shell just below the Fermi energy is $\sim 0.7 \pm 0.1$, and we denote it by $n(-)$; and $n(+) \sim 0.1 \pm 0.05$ is the average of $n(p)$ in the shell just above. The renormalization constant[19] $Z = n(e_F - \epsilon) - n(e_F + \epsilon)$, $\lim \epsilon \rightarrow 0$, should be taken as $n(-) - n(+)$ in nuclei, and the present calculation gives it as 0.6 ± 0.1. This value of Z in nuclei is also suggested by the empirical value of 1.5 for the e mass $(=1/Z)$ of nucleons in nuclei.[20] Our value of Z is somewhat smaller than that of Li and Klemt[21] who incorporate only RPA and tensor correlations in their calculation.

The occupation numbers given in Fig. 1 are not sufficient to calculate the Q and Q'. For example, we need to know the $n_f(q)$ in the final (excited) states to determine the Q. However, the states considered in Tables I and II are supposedly "simple shell-model" states that do not have significant mixing of nearby configurations. For such states it is reasonable to use the approximations

$$n_i(h) \sim n_f(p) \sim n_A(h) \sim n(-) \qquad (15)$$

for the "occupied orbits," and

$$n_i(p) \sim n_f(h) \sim n_{A-1}(h) \sim n(+) \qquad (16)$$

for the "unoccupied orbits" in shell-model terminology. We then obtain $Q \sim 0.63$, and $Q' \sim 0.6$ with a possible error of ~ 0.1. This result indicates that much of the quenching of the single-particle contribution observed in electron-scattering experiments is due to partial occupations of shell-model orbits.

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