Scalar Boson Masses in Dynamically Broken Gauge Theories

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Assuming that nonperturbative QCD generates the chiral-limiting quark mass and the spinless π and σ mesons, we obtain a gap equation which can be solved for the scalar mass $m_\sigma \approx 2m_{qk}$ in a manner consistent with the original Nambu–Jona-Lasinio approach. A similar relation is obtained within a hypercolor scenario between the dynamically generated component of the "hyperquark" mass and the mass of the Higgs analog.

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The 0^+ sigma mass in the strong-interaction, spontaneously broken σ model[1] and the analog Higgs mass in the spontaneously broken electroweak Glashow-Salam-Weinberg[2] (GSW) model are undetermined mass scales. However, in the context of quantum chromodynamics (QCD) but in the Nambu–Jona-Lasinio[3] spirit of dynamical breakdown of chiral symmetry, it has been suggested[4] that the scalar $\bar{q}q$ binding equations are the same as the pseudoscalar $\bar{q}q$ binding equations for the massless pion (which in turn are identical[5] to the dressing equations giving the quark a nonperturbative dynamically generated mass $m_{\rm dyn}$) provided

$$m_\sigma = 2m_{\rm dyn}, \tag{1}$$

where m_σ is the pure nonstrange sigma mass in the chiral limit.

In this paper we attempt to reinforce the result (1) by returning to the renormalized σ model and converting the corresponding scalar quark-loop tadpole graph to a gap equation which has the self-consistent solution

$$m_\sigma/m_{\rm dyn} \approx [\pi/\alpha_s(m_\sigma^2)]^{1/2} \tag{2a}$$

$$\approx 2.05, \tag{2b}$$

for $\alpha_s(1\ \text{GeV}) \approx 0.50$, $\alpha_s(0.63\ \text{GeV}) \approx 0.75$, and $m_\sigma \approx 630$ MeV. A similar argument is presented within the framework of hypercolor,[6] suggesting a relation analogous to (2) between the dynamically generated component of the hyperquark mass (m_F) and the mass of the GSW Higgs analog (m_H):

$$m_H \approx 2m_F, \tag{3}$$

as suggested by Scadron.[7]

If we concentrate first on the strong-interaction problem, chiral symmetry is broken when the quark acquires a nonperturbative, chiral-limiting dynamical mass, $m_{\rm dyn}$. A reasonable model is the renormalization-group improved version of the self-energy graph of Fig. 1 for multigluon exchanges,[8] leading to $\delta m_{\rm dyn} = m_{\rm dyn} = \Lambda e^{1/6} \sim 300$ MeV. This mass runs with momentum p in the deep-Euclidean region according to[9,10]

$$m_{\rm dyn}(p^2) = \frac{M^2}{p^2} m_{\rm dyn}(M^2) \left[\frac{\ln M^2/\Lambda^2}{\ln p^2/\Lambda^2}\right]^{1-d}, \tag{4}$$

for $d = 12(33-2n_f)^{-1}$, but is anchored to the pole position $m_{\rm dyn} = m_{\rm dyn}(p^2 = m_{\rm dyn}^2)$ in a gauge-parameter independent manner.[11]

Although it is not possible to compute the nonperturbative mass $m_{\rm dyn}$, one can use perturbative techniques to relate $m_{\rm dyn}$ to other nonperturbative quantities such as the quark condensate $\langle(\bar{q}q)_M\rangle_0$ at some renormalization mass cutoff M. In particular, the closed quark loop in Fig. 2 corresponds to the large-M Feynman integral for N_c colors[7,10,11]:

$$\langle(\bar{q}q)_M\rangle_0 = -\frac{iN_c \times 4}{(2\pi)^4}\int^M d^4p\, \frac{m_{\rm dyn}(p^2)}{p^2 - m_{\rm dyn}^2(p^2)}, \tag{5a}$$

$$\approx -\frac{N_c}{4\pi^2 d} m_{\rm dyn}(M^2) M^2 \ln(M^2/\Lambda^2). \tag{5b}$$

Combining (5b) with the asymptotic freedom relation[12] $\alpha_s(p^2) = \pi d/\ln(p^2/\Lambda^2)$ and (4) down to the freezeout region[13] of $M \sim 1$ GeV, below which (4) becomes $m_{\rm dyn}(M^2) = m_{\rm dyn}^3/M^2$, we obtain

$$-\langle(\bar{q}q)_M\rangle_0 = N_c m_{\rm dyn}^3/4\pi\alpha_s(M^2). \tag{6}$$

Equations (5b) and (6) are manifestly compatible with the operator product determination[9] of $m_{\rm dyn}$. Moreover, Eq. (6) implies that $m_{\rm dyn} \simeq 319$ GeV for empirical values of $\langle(\bar{q}q)_{1\,\text{GeV}}\rangle_0$ [$\simeq(-249$

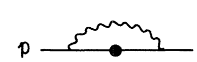

FIG. 1. Quark self-energy through nonperturbative gluon exchanges.

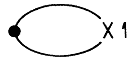

FIG. 2. Quark condensate regularized by a nonperturbative quark propagator at cutoff mass scale M.

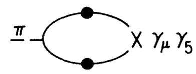

FIG. 3. Pion decay constant in $\langle 0|A_\mu|\pi\rangle = if_\pi q_\mu$ obtained using nonperturbative quark propagators.

MeV)^3],[14] $\alpha_s(1\text{ GeV})$ [$\simeq 0.50$], and N_c [$=3$]. This estimate of m_{dyn} is close to the weak-binding limit $m_N/3 \approx 313$ MeV and to the 310-MeV estimate obtained from the δ–nonstrange-η mass difference.[4,7] Thus we shall assume $m_{\text{dyn}} = 315 \pm 5$ MeV in what follows.

Another parameter needed in this analysis is the coupling of the nonstrange σ meson to the quarks, which in the sigma model of chirally invariant σ and π coupling to quarks satisfies the Goldberger-Treiman relation at the quark level,[5,10,15]

$$g_{\sigma qq} = g_{\pi qq} = m_{\text{dyn}}/f_\pi \approx 3.5, \tag{7}$$

for the chiral-limiting values of $m_{\text{dyn}} \approx 315$ MeV and $f_\pi \approx 90$ MeV. This dimensionless scale can be reinforced by the quark loop for the pion decay constant depicted in Fig. 3, which leads to the chiral-limiting value

$$if_\pi = g_{\pi qq} \frac{N_c \times 4}{(2\pi)^4} \int d^4p \, \frac{m_{\text{dyn}}(p^2)}{[p^2 - m_{\text{dyn}}^2(p^2)]^2}. \tag{8}$$

When we apply (3) for large $|p^2|$ down to 1 GeV^2 and $m_{\text{dyn}}(p^2) = m_{\text{dyn}}^3/p^2$ from 1 GeV^2 to $p^2 = m_{\text{dyn}}^2$, while holding m_{dyn} constant[16] for $p^2 = m_{\text{dyn}}^2$ to $p^2 = 0$, the mass scale m_{dyn} factors out of the integral in (8). The latter is then approximately the Euclidean value $i\pi^2 m_{\text{dyn}}$. Replacing f_π by $m_{\text{dyn}}/g_\pi qq$ means that $m_{\text{dyn}} \neq 0$ therefore cancels out of (8), leading to the "gap equation" solution[7,10,11]

$$g_{\sigma qq} = g_{\pi qq} \approx 2\pi/\sqrt{N_c} \approx 3.6 \tag{9}$$

for $N_c = 3$. Such an approach using Fig. 3 to calculate the nonperturbative pion decay constant was first suggested by Pagels and Stokar[17] and the result (9) was first obtained by Cornwall[18] with a somewhat different prescription for the p^2 dependence of $m_{\text{dyn}}(p^2)$ than (4). But, in any case, we believe the significant point is that (9) is in close agreement with (7). An alternative scheme of light-plane wave functions[19] can replace the Goldberger-Treiman couplings, but the end result remains essentially unchanged.

To proceed to the estimate of the σ mass in this sigma model (at the quark level), we consider the scalar-meson quark-tadpole graph equation of Fig. 4, $\delta m_{\text{dyn}} = m_{\text{dyn}}$, as representing the generation of m_{dyn}. This yields $m_{\text{dyn}} = g_{\sigma qq}^2(-m_\sigma^{-2})\langle(\bar{q}q)_{m_\sigma}\rangle_0$, but since $\langle\bar{q}q\rangle_0 \propto m_{\text{dyn}}^3$ from (6), $m_{\text{dyn}} \neq 0$ again cancels out,[20] leading to a new gap equation for the σ mass:

$$m_\sigma = \frac{m_{\text{dyn}}^2}{f_\pi}\left(\frac{N_c}{4\pi\alpha_s(m_\sigma^2)}\right)^{1/2}. \tag{10}$$

It is important to stress that the divergent pion or sigma-meson tadpole loop contributions to the fermion mass m_{dyn} can be neglected in Fig. 4 because when summed to all orders, such perturbative tadpole graphs vanish when the scalar field is redefined consistent with spontaneous symmetry breakdown.[20-22] On the other hand, the nonperturbative quark condensate in the tadpole graph of Fig. 4 occurs in the renormalized effective-field theory and must be self-consistently accounted for, as in (10). On further employment of $m_{\text{dyn}}/f_\pi \approx 2\pi/\sqrt{N_c}$ from (7) and (9), the color factor N_c is eliminated from (10), giving our primary result (2). If we then freeze out $\alpha_s \approx 0.50$ at $M \sim 1$ GeV, (2) or (10) requires $m_\sigma \approx 760$–790 MeV. But if we allow α_s to continue to rise as $\pi d/\ln(p^2/\Lambda^2)$ and freeze out at the lower value of m_σ, then the self-consistent solution of (2) is $m_\sigma \approx 630$ MeV for $N_c = n_f = 3$ which is essentially identical with the Bethe-Salpeter binding-equation result (1) for $m_{\text{dyn}} \approx 315$ MeV. Such a nonstrange σ mass scale is not only compatible with the findings of nuclear physicists for the $^3S_1$ NN potential,[23] but also with recent data analyses of $\pi\pi \to \pi\pi, K\bar{K}$ scattering,[24] $\pi^0\pi^0$ scattering,[25] and the $\gamma\gamma \to \pi^+\pi^-$ transition.[26]

The mass-gap equation $\delta m_{\text{dyn}} = m_{\text{dyn}}$ for m_{dyn} in Fig. 1 and for m_σ in Fig. 4 as given by (2) or (10) are both in the spirit of the four-fermion gap equation of Nambu and Jona-Lasinio,[3] except that neither m_{dyn} nor m_σ could be computed in the latter model. The fact that the *color number* N_c cancels

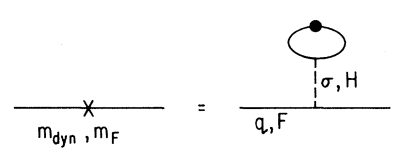

FIG. 4. Scalar boson nonperturbative tadpole graphs used to generate the nonperturbative fermion mass gap.

out in the strong interaction ratio $m_\sigma/m_{\rm dyn}$ in both (1) and (2), even though $N_c=3$ was so crucial in the observed scale of f_π or $g_{\pi qq}$ in (9) and $\langle \bar{q}q\rangle_0$ in (6), leads us to believe that a similar analysis might also apply to the scalar Higgs to fermion mass ratio m_H/m_F in electroweak interactions.

In the absence of fundamental scalar fields or any other weak symmetry-breaking mechanism, the Goldstone pions accompanying σ would be incorporated into the longitudinal degrees of freedom of the $W^\pm$ and Z, thereby enabling us to relate $m_{\rm dyn}$, the fundamental QCD mass scale associated with the breakdown of global chiral symmetry, with mass scales associated with the breakdown of local $SU(2)\times U(1)$ gauge symmetry. Moreover, the σ meson in this picture would be identified with the scalar Higgs particle of the GSW model. Of course, the W and Z masses obtained in this symmetry-breaking scenario are unphysically small, thereby motivating the hypothesis of hypercolor,[6] an additional set of strong interactions providing linkage between mass scales associated with a non-QCD global chiral-symmetry breakdown and local weak-interaction symmetry breakdown. Thus, nonperturbative masses are scaled up from QCD by a factor $F_W/f_\pi \sim 2500$, where $f_W=2m_W/g_W=\sqrt{2}\langle\phi_H\rangle_0$ is the vacuum expectation value order parameter for $SU(2)\times U(1)$ breakdown.

We note here that GSW weak forces themselves can lead to weak-symmetry breakdown in a theory containing massless fundamental scalars; such symmetry breakdown is perturbative, spontaneous, and characterized by a Higgs mass of 10 GeV.[22]

We further note that not *all* GSW weak forces are necessarily perturbative at near teraelectronvolt energies. The scalar-field self-interaction of spontaneously broken $SU(2)\times U(1)$ is characterized by a coupling constant λ (proportional to m_H^2/m_W^2) that *cannot* be asymptotically free.[27] If $m_H \lesssim m_W$, then λ can be shown to remain perturbatively small up to mass scales comparable to that of grand unification. However, if m_H is several times larger than m_W, then $\lambda(f_W)$ is no longer small; for $m_H \sim 7m_W$ the one-loop running value of λ diverges at 1 TeV. These results suggest that a perturbative Higgs mass of order f_W is indicative of nonperturbative physics in the weak interaction sector.

The dynamical fermion mass m_F manifesting chiral symmetry breakdown of hypercolor (HC) is therefore obtained from Fig. 4 as follows: $m_F=g_{HFF}^2(-m_H^{-2})\langle(\bar{F}F)_M\rangle_0$. If the HC group is $SU(N)_{HC}$ with n_{HC} "hyperflavors," then (4) again holds with

$$d=9(N^2-1)[2N(11N-2n_{HC})]^{-1}$$

in which case (6) is replaced by the fermion condensate:

$$-\langle(\bar{F}F)_M\rangle_0=\frac{2N^2m_F^3}{3(N^2-1)\pi\alpha_{HC}(M^2)}, \qquad (11a)$$

$$\alpha_{HC}(M)=12\pi/[(11N-2n_{HC})\times\ln(M^2/\Lambda_{HC}^2)], \qquad (11b)$$

the latter the direct generalization of $\alpha_s(M)$ in QCD. Using (11a) and the weak Goldberger-Treiman relation $g_{HFF}=\sqrt{2}m_F/f_W$, we obtain the GSW weak analog of (10):

$$m_H=\frac{2Nm_F^2}{f_W[3\pi(N^2-1)\alpha_{HC}(m_H^2)]^{1/2}}. \qquad (12)$$

To convert (10) to (2) it was necessary to use $g_{\sigma qq}\approx 2\pi/\sqrt{3}$, a value reinforced by (7). In the absence of detailed phenomenological information about hypercolor necessary for an analog calculation to (8), we shall assume that $m_H=2m_F$ and (eventually) establish that $g_{HFF}\approx g_{\sigma qq}$ as a self-consistency check. We further utilize the dynamical symmetry breaking relation[8] $\Lambda_{HC}=m_Fe^{-16}\approx 0.42m_H$. To obtain a scale for α_{HC}, we assume that $SU(N)_{HC}$, SU(3), and $SU(2)\times U(1)$ are subgroups of a grand unified (GU) theory subject to the one-loop relations[28]

$$\sin^2\theta_W=\tfrac{1}{6}+5\alpha(m_W^2)/9\alpha_s(m_W^2), \qquad (13a)$$

$$\frac{\alpha_s(m_W^2)}{\alpha(m_W^2)}=\frac{8}{3}\left(1-\frac{11\alpha(m_W^2)}{\pi}\ln\frac{M_{GU}}{m_W}\right)^{-1}, \qquad (13b)$$

appropriate for the symmetric embedding of all $SU(N)$ groups within a simple grand unification group. If $\sin^2\theta_W\approx 0.22$, then (13) leads to $M_{GU}\approx 10^{14}$ GeV, $\alpha_{HC}(M_{GU})\approx 0.023$. These relations can be combined with (11b) and (12) to obtain a single equation for m_H by eliminating factors of $11N-2n_{HC}$. We then find for the unification-motivated choices $N=3$ or 4 that $m_H\approx 1.2$ TeV $(=2m_F)$, $\Lambda_{HC}\approx 520$ GeV, and consequently that $g_{HFF}\approx 3.5\approx g_{\sigma qq}$.[29]

In conclusion, we suggest that the chiral-limiting relation for the nonstrange σ mass, $m_\sigma=2m_{\rm dyn}\approx[\pi/\alpha_s(m_\sigma^2)]^{1/2}m_{\rm dyn}$, is of fundamental significance in QCD and is the direct parallel of the superconductivity result[30] $m_{C^+}=2\Delta$, where Δ is the gap energy and m_{C^+} is the charge-conjugation–even electron-hole vibrational amplitude mode. There may even be an analog state for the valence nuclear pairing problem.[31] Thus it would not be surprising that $m_H=2m_F$ also relates masses of the electroweak scalar Higgs to the dynamicaly generated component of hyperquark masses.[7]

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Note added.—Elsewhere[32] it is shown that the required link between the nonperturbative mass in QCD, $m_{\text{dyn}}(p^2)$, and the Nambu-Goldstone (pion) wave function is severed when the Bethe-Salpeter equation for the latter breaks down. This occurs when spontaneous symmetry breaking becomes impossible[32] for $\alpha_{\text{eff}} = C_2(R)\alpha_s > \pi/3 \cong 1$. Such a "supercritical" coupling can be qualitatively understood[33] on the basis of the uncertainty principle; $p \sim 1/r$ when forming the $q\bar{q}$-pion energy, and $E_\pi = p - \alpha_{\text{eff}}/r \sim p(1-\alpha_{\text{eff}})$. Then $E_\pi > 0$ is physical for $\alpha_{\text{eff}} < 1$ corresponding to having $\alpha_s < \pi/4$ [$C_2(R) = \frac{4}{3}$ for $N_c = 3$]. Therefore, if we maximize α_s at this supercritical value of $\pi/4$, then our fundamental equation (2) requires m_σ to be *exactly* $2m_{\text{dyn}}$ [Eq. (1)].

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